Higgsing the stringy higher spin symmetry

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Stringy symmetries at tensionless point

In the context of the AdS_3/CFT_2 correspondence, the symmetric product orbifold CFT of the D1-D5 system is dual to string theory on $AdS_3 \times S^3 \times \mathbb{T}^4$ at the tensionless point.

[Gaberdiel & Gopakumar, '14]

The symmetric orbifold CFT has an infinite tower of massless conserved higher spin (HS) currents, a closed subsector of which are dual to the HS fields of the Vasiliev theory.

This work: we consider deformation of the symmetric orbifold CFT which corresponds to switching on the string tension and study the behaviour of symmetry generators of the theory.

Outline

- Symmetric orbifold CFT and the stringy symmetries
- Higgsing stringy symmetries
- Results
- Summary

D1-D5 system

	0	1	2	3	4	5	6	7	8	9
N_1 D1	–	–	·	·	·	·	·	·	·	·
N_5 D5	–	–	–	–	–	–	·	·	·	·
		$\downarrow$ S^1	$\underbrace{\qquad}_{\mathcal{M}}$							

where $\mathcal{M}$ is $\mathbb{T}^4$ or $K3$.

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N_5 D5	–	–	–	–	–	–	·	·	·	·
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where $\mathcal{M}$ is $\mathbb{T}^4$ or $K3$.

In the limit where size of $\mathbb{T}^4 \ll$ size of S^1, worldvolume gauge theory of D branes is a 2d field theory that lives on S^1.

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N_1 D1	–	–	·	·	·	·	·	·	·	·
N_5 D5	–	–	–	–	–	–	·	·	·	·
		↓ S^1	$\mathcal{M}$ (2–5)							

where $\mathcal{M}$ is $\mathbb{T}^4$ or $K3$.

In the limit where size of $\mathbb{T}^4 \ll$ size of S^1, worldvolume gauge theory of D branes is a 2d field theory that lives on S^1.

It flows in IR to a CFT described by a sigma model whose target space is a resolution of symmetric product orbifold

[Vafa, '95]

$$Sym_{N+1}(\mathbb{T}^4) = (\mathbb{T}^4)^{N+1}/S_{N+1}, \qquad (N+1 = N_1 N_5).$$

$\mathrm{AdS}_3/\mathrm{CFT}_2$

String theory on $AdS_3 \times S^3 \times \mathbb{T}^4$ is dual to symmetric product orbifold CFT. [Maldacena, '97]

Free orbifold point is the analogue of free Yang Mills theory for the case of D3 branes.

AdS_3/CFT_2

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Symmetric product orbifold CFT

- Generators of left-moving superconformal algebra: L_n, G_r^α, and J_n^I (similar for right-moving generators).
- At the orbifold point, we have a free CFT of $2(N+1)$ complex bosons and $2(N+1)$ complex fermions and their conjugates:

$$\partial\phi_a^i, \;\; \partial\bar{\phi}_a^i, \;\; \psi_a^i, \;\; \bar{\psi}_a^i, \qquad i \in \{1,2\}, \;\; a \in \{1, \cdots, N+1\},$$

plus right-moving counterparts. S_{N+1} acts by permuting $N+1$ copies of $\mathbb{T}^4$

Higher spin embedding

The perturbative part of the HS dual coset CFT forms a closed subsector of the symmetric orbifold CFT.

[Gaberdiel & Gopakumar, '14]

All states of the symmetric orbifold CFT are organised in terms of representations of the HS $\mathcal{W}_\infty^{(\mathcal{N}=4)}[0]$ algebra.

The chiral algebra of symmetric orbifold CFT is written as

$$Z_{vac,stringy}(q,y) = \sum_{\Lambda} n(\Lambda)\ \chi_{(0;\Lambda)}(q,y).$$

Original $\mathcal{W}_\infty^{\mathcal{N}=4}$ algebra

$$\big(\mathcal{N}=4\big) \;\oplus\; \bigoplus_{s=1}^{\infty} R^{(s)}, \qquad R^{(s)}: \quad \begin{array}{rc} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{3}). \\ s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\ s+2: & (\mathbf{1},\mathbf{1}) \end{array}$$

Free field realisation of HS fields dual to Vasiliev theory is in terms of neutral bilinears:

$$\sum_{a=1}^{N+1} P_a^1 P_a^2, \qquad P_a^1 \in \{\partial^{\#}\phi^i, \partial^{\#}\psi^i\}, \quad P_a^2 \in \{\partial^{\#}\bar{\phi}^i, \partial^{\#}\bar{\psi}^i\}.$$

Stringy HS fields

HS fields of symmetric orbifold theory come from the untwisted sector of orbifold. Their single particle symmetry generators are:

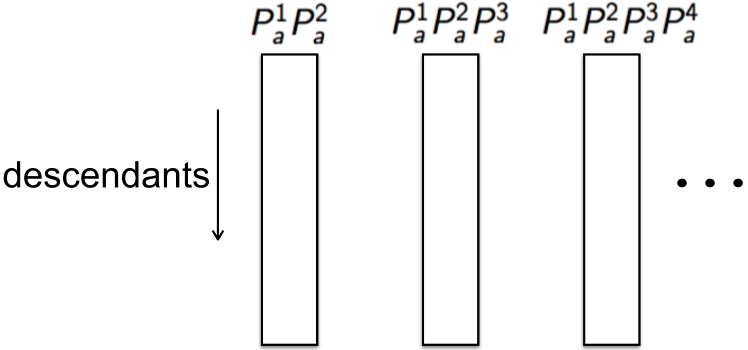
$$\sum_{a=1}^{N+1} P_a^1 \cdots P_a^m,$$

where P_a^j is one of the 4 bosons/fermions or their derivatives in the a^{th} copy.

They fall into additional $\mathcal{W}_\infty^{\mathcal{N}=4}$ representations: hugely extend coset $\mathcal{W}$ algebra

$$\mathcal{W}_\infty^{\mathcal{N}=4} \oplus \bigoplus_{n,\bar{n}} (0; [n, 0, \cdots, 0, \bar{n}]), \qquad m = n + \bar{n}.$$

Stringy HS fields



[Gaberdiel & Gopakumar, '15]

Example: cubic generators ($m = 3$)

$$P^1_a, P^2_a, P^3_a \in \{\partial^\# \phi^i, \partial^\# \psi^i\} \quad \text{or} \quad P^1_a, P^2_a, P^3_a \in \{\partial^\# \bar{\phi}^i, \partial^\# \bar{\psi}^i\},$$

lie in the multiplets

$$(0;[3,0,\cdots,0,0]), (0;[0,0,\cdots,0,3]) : \bigoplus_{s=2}^{\infty} n(s) \left[R^{(s)}(\mathbf{2},\mathbf{1}) \ \oplus \ R^{(s+3/2)}(\mathbf{1},\mathbf{2}) \right] ,$$

where $\dfrac{q^2}{(1-q^2)(1-q^3)} = \sum_{s=2}^{\infty} n(s) q^s$, and

$$R^{(s)}(\mathbf{2},\mathbf{1}) : \begin{array}{cc} s : & (\mathbf{2},\mathbf{1}) \\ s+\frac{1}{2} : & (\mathbf{3},\mathbf{2}) \oplus (\mathbf{1},\mathbf{2}) \\ s+1 : & (\mathbf{4},\mathbf{1}) \oplus (\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{3}), \\ s+\frac{3}{2} : & (\mathbf{3},\mathbf{2}) \oplus (\mathbf{1},\mathbf{2}) \\ s+2 : & (\mathbf{2},\mathbf{1}) \end{array} \quad R^{(s)}(\mathbf{1},\mathbf{2}) : \begin{array}{cc} s : & (\mathbf{1},\mathbf{2}) \\ s+\frac{1}{2} : & (\mathbf{2},\mathbf{3}) \oplus (\mathbf{2},\mathbf{1}) \\ s+1 : & (\mathbf{1},\mathbf{4}) \oplus (\mathbf{1},\mathbf{2}) \oplus (\mathbf{3},\mathbf{2}). \\ s+\frac{3}{2} : & (\mathbf{2},\mathbf{3}) \oplus (\mathbf{2},\mathbf{1}) \\ s+2 : & (\mathbf{1},\mathbf{2}) \end{array}$$

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Higgsing of stringy symmetries

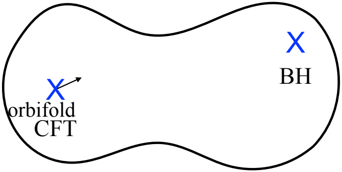
- At the tensionless point, the symmetry algebra is *much* bigger than $\mathcal{N} = 4$ superconformal algebra + algebra of Vasiliev HS theory.

- As string tension is switched on, HS symmetries are broken. Expect that Regge trajectories emerge: Vasiliev fields fall into the leading trajectory. Higher trajectories correspond to additional HS fields — which become massless at tensionless point.

- We examine this picture by switching on string tension and studying behaviour of symmetry generators of symmetric orbifold CFT.

Higgsing of stringy symmetries

- Switching on tension corresponds to deforming CFT away from orbifold point by an exactly marginal operator Φ, which belongs to twist-2 sector.



- Φ is the super-descendant of BPS ground state: $\propto G_{-1/2}\tilde{G}_{-1/2}|\Psi_2\rangle$, and preserves the two $SO(4)$ symmetries.

Symmetries broken?

First order deformation analysis: criterion for spin s field $W^{(s)}$ of the chiral algebra to remain chiral under deformation by Φ

[Cardy, '90; Fredenhagen, Gaberdiel, Keller, '07;

Gaberdiel, Jin, Li, '13]

$$\mathcal{N}(W^{(s)}) \equiv \sum_{l=0}^{\lfloor s+h_\Phi \rfloor -1} \frac{(-1)^l}{l!} (L_{-1})^l \, W^{(s)}_{-s+1+l} \, \Phi = 0,$$

where

$$\partial_{\bar{z}} W^{(s)}(z,\bar{z}) = g\,\pi\,\mathcal{N}(W^{(s)}).$$

$\mathcal{N} = 4$ superconformal algebra is preserved, while HS currents are not conserved: gigantic symmetry algebra is broken down to the $\mathcal{N} = 4$ SCA.

Conformal perturbation theory

Compute relevant anomalous dimensions and determine masses of the corresponding fields.

Consider adding a small perturbation to the action of free CFT.
The normalised perturbed 2pf is:

$$\left\langle W^{(s)i}(z_1) W^{(s)j}(z_2) \right\rangle_\Phi = \frac{\left\langle W^{(s)i}(z_1) W^{(s)j}(z_2) e^{\delta S} \right\rangle}{\left\langle e^{\delta S} \right\rangle}, \quad \delta S = g \int d^2 w \, \Phi(w, \bar{w}) \, .$$

Upon expanding in powers of g, we have

$$\left\langle W^{(s)i}(z_1) W^{(s)j}(z_2) \right\rangle_\Phi - \left\langle W^{(s)i}(z_1) W^{(s)j}(z_2) \right\rangle =$$

$$\frac{g^2}{2} \left(\int d^2 w_1 \, d^2 w_2 \left\langle W^{(s)i}(z_1) \, W^{(s)j}(z_2) \, \Phi(w_1, \bar{w}_1) \, \Phi(w_2, \bar{w}_2) \right\rangle \right.$$

$$\left. - \int d^2 w_1 \, d^2 w_2 \left\langle W^{(s)i}(z_1) \, W^{(s)j}(z_2) \right\rangle \left\langle \Phi(w_1, \bar{w}_1) \, \Phi(w_2, \bar{w}_2) \right\rangle \right) + \mathcal{O}(g^3) \, .$$

Anomalous dimensions

2pf of quasiprimary operators is of the form

$$\left\langle W^{(s)i}(z_1) W^{(s)j}(z_1) \right\rangle_\Phi \sim \frac{c^{ij}}{(z_1 - z_2)^{2(s+\gamma^{ij})} \, (\bar{z}_1 - \bar{z}_2)^{2\bar{\gamma}^{ij}}} \,,$$

where for small γ^{ij} reads

$$\approx \frac{c^{ij}}{(z_1 - z_2)^{2s}} \left(1 - 2\gamma^{ij} \ln(z_1 - z_2) - 2\bar{\gamma}^{ij} \ln(\bar{z}_1 - \bar{z}_2) + \cdots \right) .$$

Read coefficient of the log term in perturbed 2pf.

Anomalous dimensions

To first order, γ_{ij} is given by 3 point function

$$\left\langle W^{(s)i}(z_1)\, \Phi(w_1, \bar{w}_1)\, W^{(s)j}(z_2) \right\rangle$$

which vanishes: Φ has $h_\Phi = \bar{h}_\Phi = 1$ while W's have $\bar{h}_W = 0$.

Leading correction to the 2pf appears at second order:

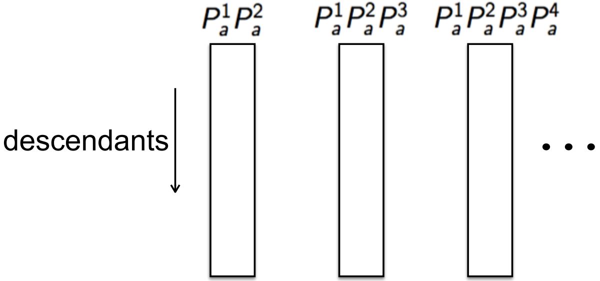
$$\gamma^{ij} = g^2 \pi^2 \left\langle \mathcal{N}(W^{(s)i})\ \mathcal{N}(W^{(s)j}) \right\rangle,$$

$$\mathcal{N}(W^{(s)}) \equiv \sum_{l=0}^{\lfloor s+h_\Phi \rfloor -1} \frac{(-1)^l}{l!} (L_{-1})^l\, W^{(s)}_{-s+1+l}\, \Phi = 0.$$

Operator mixing

In general, matrix γ_{ij} is not diagonal: need to diagonalise it to extract anomalous dimensions.

- In general, fields within each family, $m = 2, 3, \cdots$, mix (multiplicities $n(s) > 1$).
- There is also mixing present between fields from different families.



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Vasiliev HS fields:

$$W^{(s)} = \sum_{q=0}^{s-2} (-1)^q \binom{s-1}{q} \binom{s-1}{q+1} \partial^{s-1-q} \bar{\phi}^1 \partial^{q+1} \phi^2,$$

$$\gamma^{ij} = g^2 \pi^2 \left\langle \mathcal{N}(W^{(s)i}) \ \mathcal{N}(W^{(s)j}) \right\rangle.$$

Vasiliev HS fields:

$$W^{(s)} = \sum_{q=0}^{s-2} (-1)^q \binom{s-1}{q} \binom{s-1}{q+1} \partial^{s-1-q} \bar{\phi}^1 \partial^{q+1} \phi^2.$$

The diagonal elements γ^{ii} can be computed analytically and in closed form:

$$\gamma^{(s)} = \frac{g^2 \pi^2 \sum_{p=0}^{s} (-1)^{s-p} \binom{2s}{s-p} P_2(s,p)}{(N+1)\, E_2(s)},$$

where

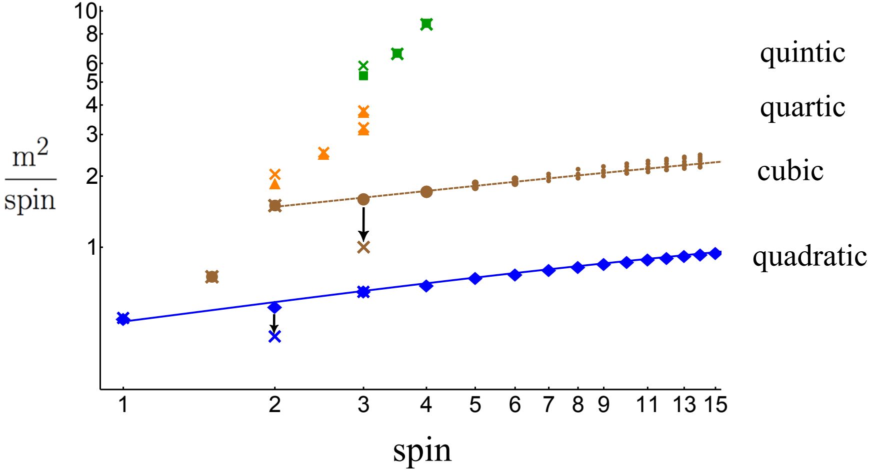
$$E_2(s) = \sum_{q=0}^{s-1} \sum_{p=0}^{s-1} (-1)^{s+1+p+q} \binom{s}{q} \binom{s}{q+1} \binom{s}{p} \binom{s}{p+1}$$
$$\times \left((-2)_{(q)} (-2-q)_{(s-p-1)} (-2)_{(s-q-1)} (q-s-1)_{(p)} \right) ,$$

$$P_2(s,p) = \sum_{n=3/2}^{p-3/2} n(p-n) f(s,p,n) f(s,-p,n-p)$$
$$+ \tfrac{3}{2} \, (-1)^{s+1} \, \Theta(p-2) f(s,p,1/2) f(s,-p,-1/2) \, (p-1/2)$$
$$+ \tfrac{1}{2} \, \delta_{p,1} \, f(s,1,1/2) f(s,-1,-1/2) \, ,$$

$$f(s,p,n) = \sum_{q=0}^{s-1} (-1)^q \binom{s}{q} \binom{s}{q+1} (-1-p+n)_{(s-q-1)} \, (-1-n)_{(q)}.$$

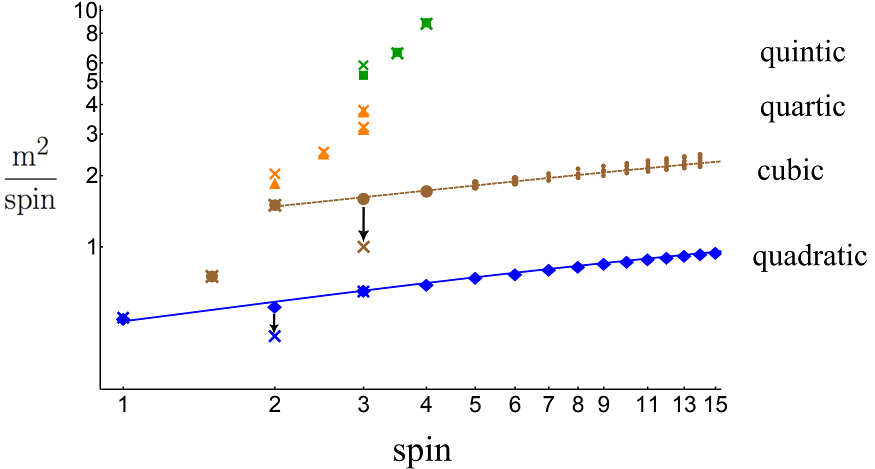
Regge trajectories

- Vasiliev HS generators correspond to the leading Regge trajectory (blue diamonds); have lowest masses for a given spin.
- Cubic generators describe the first sub-leading Regge trajectory (brown circles).



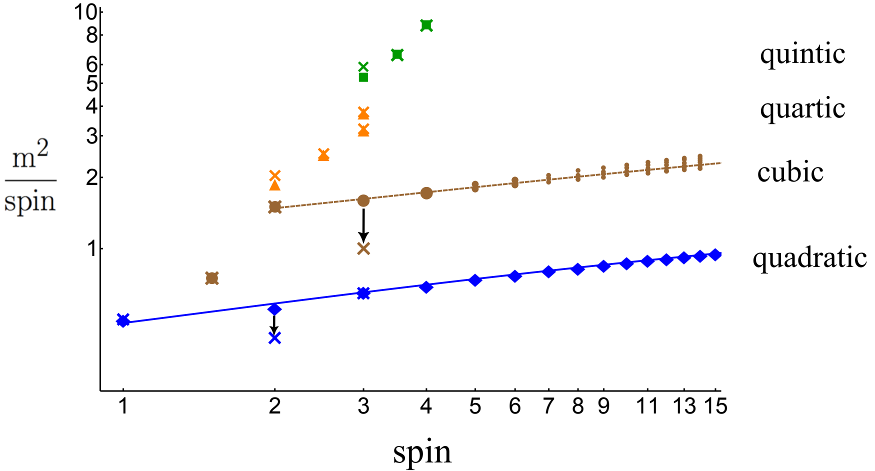
Regge trajectories

- Diagonalisation of complete mixing matrix becomes complicated as spin increases: we have solved it completely for low-lying fields (X's).
- For cubic generators, we perform partial diagonalisation at larger spin where we only diagonalise γ_{ij} among the fields of $m = 3$.



Regge trajectories

- Diagonal entries of Regge trajectories behave as $\gamma^{(s)} \cong a \log s$ at large spin, with dispersion relation $E(s) \cong s + a \log s$. This suggests that symmetric orbifold CFT is dual to an AdS_3 background with pure RR flux. [Loewy, Oz, '03; David, Sadhukhan, '14]



Summary:

- Computed anomalous dimensions of the HS generators of symmetric orbifold CFT as the string tension is switched on.
- HS fields of original $\mathcal{W}_\infty^{(\mathcal{N}=4)}$ algebra form a decoupled subsector at tensionless point. As tension is switched on, they couple with stringy symmetry generators.

Future directions:

- Solve for exact anomalous dimensions for higher spins and determine shape of dispersion relations.
- Derive anomalous diemensions for symmetric product orbifold of K3.
  [Baggio, Gaberdiel, and Peng, '15]
- Compute the anomalous dimensions from the dual AdS viewpoint.