On a Four Dimensional Formulation of Dimensionally Regulated Amplitudes

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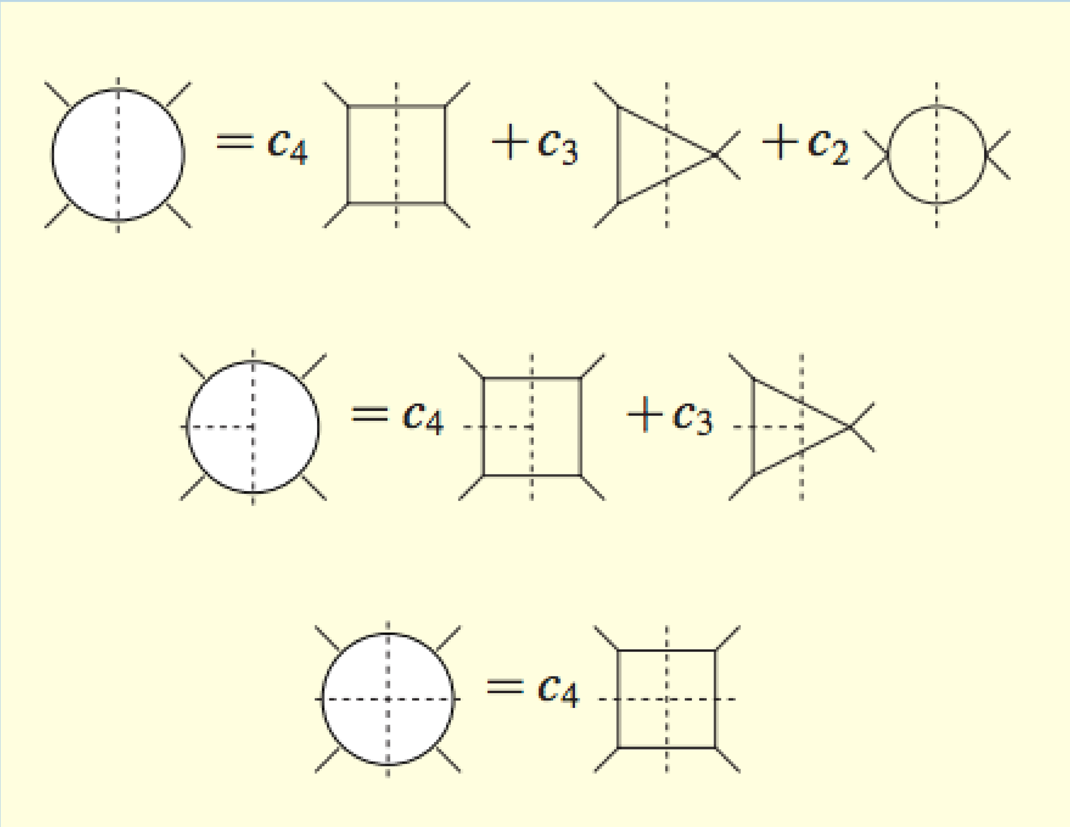
Abstract

We introduce [1] a pure four-dimensional formulation (FDF) of the d-dimensional regularization of one-loop scattering amplitudes where we present an explicit representation of the polarization and helicity states of the four-dimensional particles propagating in the loop. FDF is an operational realization of the Four Dimensional Helicity scheme (FDH).The constructed internal states with easily implemented rules allow for a four-dimensional, unitarity-based construction of d-dimensional amplitudes. Generalized unitarity within the FDF does not require any higher-dimensional extension of the Clifford and the spinor algebra.

Generalized Unitarity

From the reduction theorem any one-loop dimensionally regularized scattering amplitude is decomposed in a cut-constructible part and in a rational part expressed in terms of scalar integrals in $d = 4 - 2\epsilon$ dimensions. The coefficients c_i are rational functions of the external momenta and polarizations.



The regularized scalar master integrals are perfomed by using polar coordinates in the -2ϵ dimensional subspace:

$$\int \frac{d^{-\epsilon}(\mu^2)}{(2\pi)^{-2\epsilon}} \int \frac{d^4\ell}{(2\pi)^{4-2r}} (\pi\mu^2)^r \frac{f(\ell^\alpha, \mu^2)}{\epsilon(\epsilon-1)} =$$

$$(2-\epsilon)\cdots(r-1-\epsilon) \int \frac{d^4\ell}{(2\pi)^4} \int \frac{d^{2r-2\epsilon}\mu}{(2\pi)^{2r-2\epsilon}} f(\ell^\alpha, \mu^2)$$

The FDH scheme

The FDH scheme defines a d-dimensional vector space embedded in a larger d_s dimensional space, $d_s \equiv (4-2\epsilon) > d > 4$. The loop momenta are considered to be d dimensional. All observed external states are four-dimensional. All unobserved internal states are d_s dimensional.The Lorentz and the Clifford algebra are performed in d_s dimensions, then the limit $d_s \to 4$ is made.

The -2ϵ selection rules

The -2ϵ dimensional quantities are substituted

$$\tilde{g}^{\alpha\beta} \to G^{AB}, \quad \ell^\alpha \to \imath\mu Q^A, \quad \bar{\gamma}^\alpha \to \gamma^5 \Gamma^A$$

the FDF formulation is defined by the rules

$$G^{AB}G^{BC} = G^{AC}, \; G^{AA} = 0, \; G^{AB} = G^{BA},$$

$$\Gamma^A G^{AB} = \Gamma^B, \; \Gamma^A\Gamma^A = 0, \; Q^A\Gamma^A = 1,$$

$$Q^A G^{AB} = Q^B, \; Q^A Q^A = 1$$

Generalized internal legs

Generalized four dimensional subluminal Dirac equations

$$\left(\not{\ell} + i\mu\gamma^5 + m\right) u_\lambda(\ell) = 0, \qquad \left(\not{\ell} + i\mu\gamma^5 - m\right) v_\lambda(\ell) = 0.$$

In the four dimensional helicity formalism with the usual light-cone decomposition

$$u_+(\ell) = \left|\ell^\flat\right\rangle + \frac{(m - i\mu)}{[\ell^\flat q_\ell]} |q_\ell], \quad u_-(\ell) = \left|\ell^\flat\right] + \frac{(m + i\mu)}{\langle \ell^\flat q_\ell\rangle} |q_\ell\rangle,$$

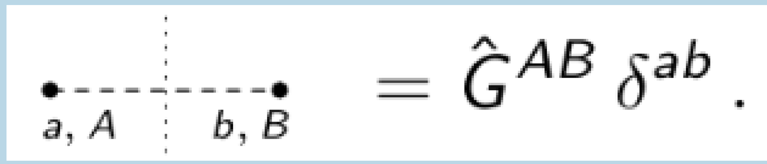
$$v_-(\ell) = \left|\ell^\flat\right\rangle - \frac{(m - i\mu)}{[\ell^\flat q_\ell]} |q_\ell], \quad v_+(\ell) = \left|\ell^\flat\right] - \frac{(m + i\mu)}{\langle \ell^\flat q_\ell\rangle} |q_\ell\rangle.$$

By the same light cone projection the polarization vectors for a μ-massive vector particle are

$$\varepsilon_+^\alpha(\ell) = -\frac{[\ell^\flat|\gamma^\alpha|\hat{q}_\ell\rangle}{\sqrt{2}\mu}, \qquad \varepsilon_-^\alpha(\ell) = -\frac{\langle\ell^\flat|\gamma^\alpha|\hat{q}_\ell]}{\sqrt{2}\mu}, \qquad \varepsilon_0^\alpha(\ell) = \frac{\ell^{\flat\alpha} - \hat{q}_\ell^\alpha}{\mu},$$

$$\sum_{\lambda=\pm,0} \varepsilon_\lambda^\alpha(\ell)\, \varepsilon_\lambda^{*\beta}(\ell) = -g^{\alpha\beta} + \frac{\ell^\alpha\ell^\beta}{\mu^2}.$$

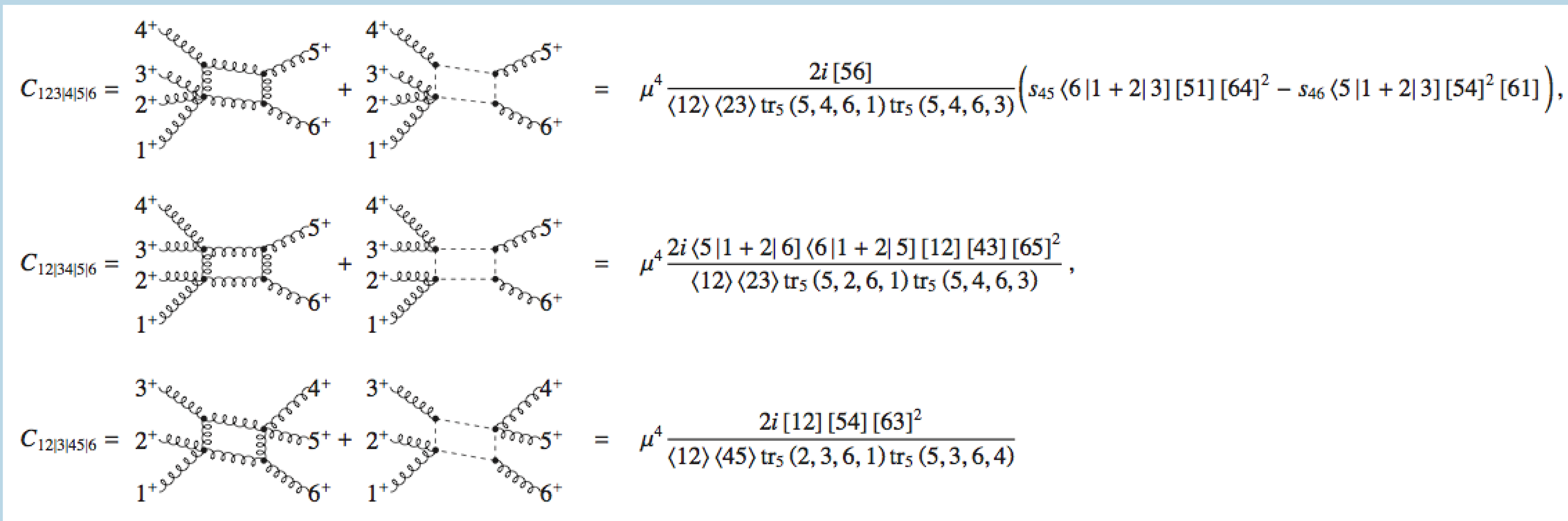
The cut of the scalar propagator is done in terms of the metrics of the -2ϵ dimensional space.



the a and b indices refer to the color group and $\hat{G}^{AB} = G^{AB} - Q^A Q^B$, such a colored scalars represent the -2ϵ components of the d_s dimensional gluon.

Six-gluon amplitudes

The FDF @ work with Ninja [2] is seen in the analytic expression of the six- point all plus amplitude, characterized by the absence of triangles and bubble contributions.

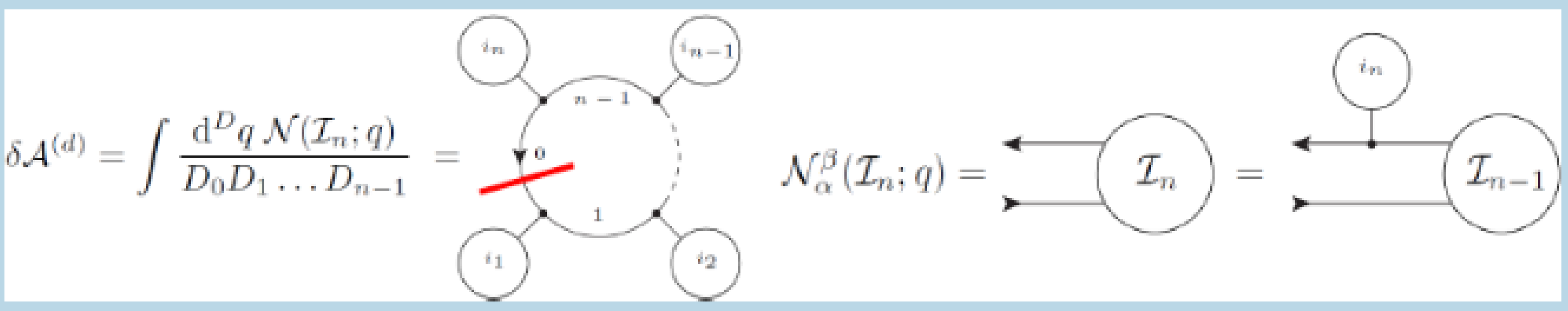


The finite color-ordered amplitude takes the form

$$A_6^{\text{1-loop}}(1^+,2^+,3^+,4^+,5^+,6^+) = c_{123|4|5|6;4}\, I_{123|4|5|6}\left[\mu^4\right] + c_{12|34|5|6;4}\, I_{12|34|5|6}\left[\mu^4\right] + \frac{1}{2} c_{12|3|45|6;4}\, I_{12|3|45|6}\left[\mu^4\right] + \text{cyclic perms.}$$

Generalized Open Loop

The FDF formulation can be generalized to the Open Loop recursive construction [3]



$$\mathcal{N}^\beta{}_\alpha(\mathcal{I}_n, \ell, \mu) = X^\beta_{\gamma\delta}(\mathcal{I}_n, i_n, \mathcal{I}_{n-1})\, \mathcal{N}^\gamma{}_\alpha(\mathcal{I}_{n-1}, \ell, \mu)\, w^\delta(i_n)$$

$$X^\beta_{\gamma\delta} = Y^\beta_{\gamma\delta} + \ell^\nu Z^\beta_{\nu;\gamma\delta} + \mu\, W^\beta_{\gamma\delta}.$$

The FDF can improve the generation of the d-dimensional integrands performed by the packages GoSam and FormCalc. The FDF analytically and numerically beyond one loop is under study.

References

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[3] F. Cascioli, P. Maierhofer, S. Pozzorini, Scattering amplitudes with open loops, Phys.Rev.Lett. 108 (2012) 111601.