mplitude

Boundary Contribution	=	Boundary Operator + External On-shell States	=	Form Factor

What is boundary operator? Let us consider an OPE

$$\mathcal{O}_I(x)\mathcal{O}_J(y)=\sum_K C_{IJ}{}^K(x-y)\mathcal{O}_K(y)$$

x and y are nearby in coordinate space is equivalent to the condition that k_1 and k_2 are far away in momentum space. If we set $k_1=k_L+zq\,, k_2=k_R-zq$, then when z is large

$$\mathcal{O}_I(k_L+zq)\mathcal{O}_J(k_R-zq)=\sum_K C_{IJ}{}^K(k_L+zq)\mathcal{O}_K(k_L+k_R)$$

Boundary operator is defined as the $O(z^0)$ order of the OPE. In the language of correlation function, the OPE is

$$\begin{aligned}&\langle\mathcal{O}_I(k_L+zq)\mathcal{O}_J(k_R-zq)\Phi(k_2)\cdots\Phi(k_{n-1})\rangle\\ =&\sum_K C_{IJ}^K(k_L+zq)\langle\mathcal{O}_K(k_L+k_R)\Phi(k_2)\cdots\Phi(k_{n-1})\rangle\end{aligned}$$

If $\mathcal{O}_I$ and $\mathcal{O}_J$ are two fundamental fields $\Phi(k_n)$ and $\Phi(k_1)$, after LSZ reduction, we get n-point amplitude under $\langle 1|n]$ shift. The boundary contribution under $\langle 1|n]$ shift is $B^{\langle 1|n]}=\langle\Phi(k_2)\cdots\Phi(k_{n-1})|\mathcal{O}^{\langle 1|n]}(k_1+k_n)|0\rangle$

It is a form factor with boundary operator and undeformed particles.

How to obtain the boundary operator? Split the field into hard part Φ^Λ and soft part Φ, under $\langle 1|n]$ shift, the z dependence only enters through

$$\mathcal{Z}(z)=-i\int D\Phi^\Lambda\exp\left(iS_2^\Lambda[\Phi^\Lambda,\Phi]\right)\Phi^\Lambda(k_1+zq)\Phi^\Lambda(k_n-zq)$$

where S_2^Λ contains terms quadratic in Φ^Λ, and the boundary operator is the $O(z^0)$ order of $\mathcal{Z}(z)$ around $z=\infty$. S_2^Λ can be written as

$$S_2^\Lambda=\frac{1}{2}\Phi^\Lambda D(\Phi)\Phi^\Lambda\,,\quad D(\Phi)=D_0+V(\Phi)$$

Finally we can get

$$\mathcal{Z}(z)=\left(1+V(\Phi)D_0^{-1}\right)^{-1}V(\Phi)$$

So once $D(\Phi)$ is known, $\mathcal{Z}(z)$ is known.

How to compute form factor from the boundary contribution of corresponding amplitude?

Example 1

$$L=L_{\rm SYM}-\frac{\kappa}{4N}\,{\rm Tr}(\phi^I\phi^J)\,{\rm Tr}(\phi^K\phi^L)$$

The boundary operator under $\langle\phi_{n+1}|\phi_{n+2}]$ shift is

$$\mathcal{O}^{\langle\phi_{n+1}|\phi_{n+2}]}=\frac{N}{2}\kappa\,{\rm Tr}(\phi_K\phi_L)+2g^2N\delta^{IJ}\,{\rm Tr}(A\cdot A+\phi_K\phi_L)$$

The traceless part is proportional to the bilinear half-BPS scalar operator.

$$A_{n;2}(\{g^+\},\phi_i,\phi_j;\phi_{n+1},\phi_{n+2})=\frac{\langle i\ j\rangle^2}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}\Rightarrow B^{\langle\phi_{n+1}|\phi_{n+2}]}=\frac{\langle i\ j\rangle^2}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}$$

$$\langle\{g^+\},\phi_i,\phi_j|\,{\rm Tr}(\phi\phi)(q)|0\rangle=\frac{\langle i\ j\rangle^2}{\langle 1\ 2\rangle\cdots\langle n\ 1\rangle}$$

Example 2

$$L=L_{\rm SYM}-\frac{\kappa}{4}\,{\rm Tr}(\psi^{A'}_\alpha\bar\psi_{B'\dot\alpha})\,{\rm Tr}(\psi^{A\alpha}\bar\psi^{\dot\alpha}_B)$$

$$\mathcal{O}^{\langle\psi_{n+1}|\bar\psi_{n+2}]}=\widetilde\lambda_{n+1,\dot\alpha}\,{\rm Tr}(\psi^\alpha\bar\psi^{\dot\alpha})\lambda_{n+2,\alpha}$$

$$A_{n;2}(\{g^+\},\psi_i,\bar\psi_j;\psi_{n+1},\bar\psi_{n+2})=\frac{\langle i\ j\rangle\langle j,n+2\rangle\langle j|k_{n+1}+k_{n+2}|n+1]}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

$$B^{\langle\psi_{n+1}|\bar\psi_{n+2}]}=\frac{\langle i\ j\rangle\langle j,n+2\rangle\langle j|k_{n+1}+k_{n+2}|n+1]}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}$$

$$\langle\{g^+\},\psi_i,\bar\psi_j|\,{\rm Tr}(\psi^\alpha\bar\psi^{\dot\alpha})(q)|0\rangle=\frac{\langle i\ j\rangle}{\langle 1\ 2\rangle\langle 2\ 3\rangle\cdots\langle n\ 1\rangle}\lambda^\alpha_j(\lambda_{j\beta}q^{\beta\dot\alpha})$$

COMING SOON IN ARXIV...

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