The Soft Emissions Of Off-Shell Currents And Their On-Shell Limits In NLSM

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ABSTRACT

- We study the **single and double soft behaviors of tree level off-shell currents and on-shell amplitudes** in nonlinear sigma model by **Berends-Giele recursion**
- We first **propose and prove** the leading soft behavior of the **tree-level off-shell currents with a single soft particle**.
  In the **on-shell limit**, this single soft emission becomes the Adler's zero
- Then we **establish** the leading and sub-leading soft behaviors of **tree−level off-shell currents with two soft particles**.
  With a careful analysis of the **on-shell limit**, we obtain the double soft behaviors of **on-shell amplitudes**

THEORETIC FRAMEWORK AND METHOD

- Lagrangian for *U(N)* NLSM $\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$ and Cayley parameterization $U = 1 + 2\sum_{n=1}^{\infty}\left(\frac{1}{2F}\phi\right)^n$

  Vertices: $V_{2n+1} = 0$ → **Odd-point amplitude vanishes**

$$V_{2n+2} = \left(-\frac{1}{2F^2}\right)^n \left(\sum_{i=0}^{n} p_{2i+1}\right)^2 = \left(-\frac{1}{2F^2}\right)^n \left(\sum_{i=0}^{n} p_{2i+2}\right)^2$$

- Color-like (Flavor) Decomposition: $M(1^{a_1}, \ldots, n^{a_n}) = \sum_{\sigma \in S_{n-1}} \mathrm{Tr}(T^{a_1} T^{a_{\sigma_2}} \ldots T^{a_{\sigma_n}}) A(1, \sigma)$
- **Berends−Giele recursion**: **Insight into Feynman diagrams**

$$J(2, \ldots, 2n) = \frac{i}{P^2_{2,2n}} \sum_{m=2}^{n} \sum_{\text{Divisions}} iV_{2m}(p_1 = -P_{2,2n}, P_{A_1}, \cdots, P_{A_{2m-1}}) \times \prod_{k=1}^{2m-1} J(A_k)$$

Divisions: all possible divisions of on-shell particles $\{2, \ldots, 2n\} \to \{A_1\}, \ldots, \{A_{2m-1}\}$

BACKGROUND AND MOTIVATION

- The nonlinear model can be used to describe the behaviors of the Goldstone bosons from a global symmetry breaking
- **The scattering amplitudes** calculated at any point of the vacuum moduli **are identical**, thus the **vacuum structure after spontaneously global symmetry breaking** can be understood **from the scattering amplitude point of view**
- Strategy: Regulate Goldstone bosons of **zero-momentum**, which reflect the position of the vacuum, to fields of **tiny momenta** and **send them to zero eventually** with very careful analysis
- Expand the physical states in one vacuum around the states in another vacuum as

$$|\psi\rangle_\theta = |\psi\rangle + |\psi^{(1)}\rangle + |\psi^{(2)}\rangle + \ldots.$$

  the variation $|\psi^{(n)}\rangle$ contains information of *n* regulated soft Goldstone bosons
  - ✓ The first order variation: the **single soft** Goldstone boson emission presents the **"Adler zero"**
  - ✓ The second order variation: the **double soft** Goldstone boson emissions present **the invariance of the amplitude** at different vacuum positions via a **compensated rotation** of the generators for broken symmetries
- The soft behaviors studied recently in different frameworks and from different methods, for instance, the single- and double-soft limit structures in the NLSM are investigated from some compact amplitude constructions, i.e., BCFW recursion and CHY formula
- **Insight into Feynman diagrams** and the corresponding **soft-behaviors of off-shell currents** are worthy of understanding, via **Berends-Giele recursion** from the diagram structure point of view

SOFT BEHAVIORS OF THE OFF-SHELL CURRENTS AND THEIR ON-SHELL LIMITS

➢ Single Soft Behaviors Of Off-Shell Currents And On-Shell Limits (τ parameterizes the soft momentum)

$$J(2, \ldots, i-1, \tilde{i}, i+1, \ldots, 2n) = \left\{ \begin{array}{ll} 0 & (i \text{ is even}) \\ \left(\frac{1}{2F^2}\right) J(2, \ldots, i-1) J(i+2, \ldots, 2n) & (i \text{ is odd}) \end{array} \right\} + \mathcal{O}(\tau)$$

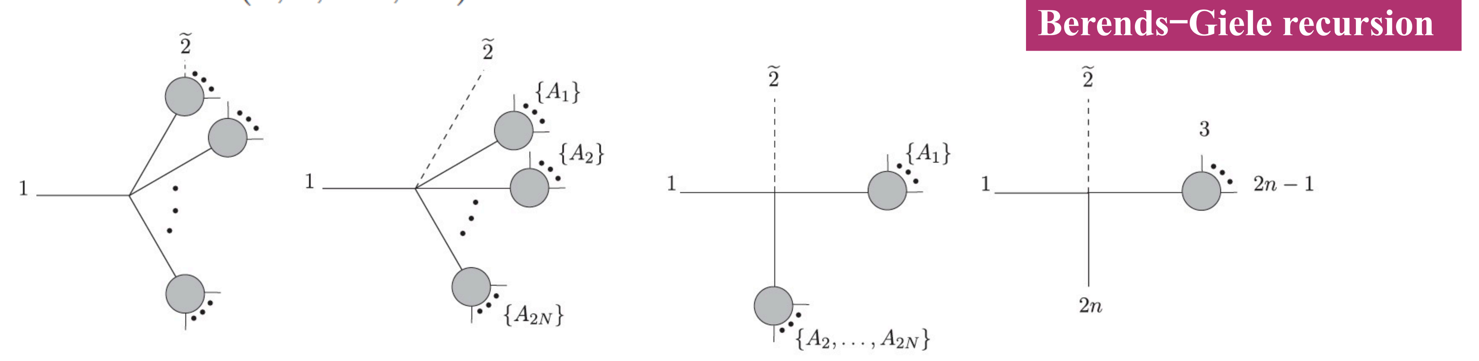
Taking On-shell Limit $P^2_{2,2n} \to 0$ → **"Adler Zero"!**

Soft Limit and On-shell Limit Can Be Exchanged

Case 1: the soft particle adjacent to the off-shell line

$J^{(0)}(\tilde{2}, 3, \ldots, 2n) = 0$ **"i is even" condition**

Illustration Strategy: Inductive proof from Berends−Giele recursion



Case 2: the even/odd soft particle non-adjacent to the off-shell line

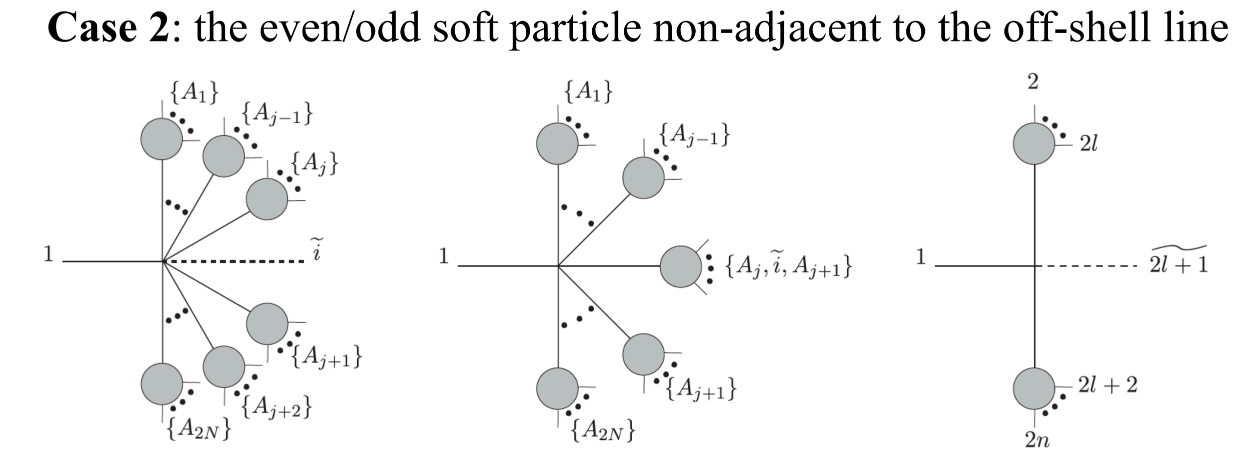


Illustration Strategy:
- Inductive assumption for lower-point off-shell currents
- Results of Case 1 should be used as well during the inductive proof
- In "i is even" condition, the first two diagrams cancel each other
- In "i is odd" condition, the first two diagrams cancel each other, the third one gives the non-zero term

➢ Double Soft Behaviors Of Off-Shell Currents Up To Sub-Leading Order

Case 1: one of the soft particles adjacent to the off-shell line

i.e., $J(\tilde{2}, \tilde{3}, 4, \ldots, 2n)$ with 2 and 3 as soft particles

$$J(\tilde{2}, \tilde{3}, 4, \ldots, 2n) = \tau^0 S^{(0)}_{2,3} J(4, \ldots, 2n) + \tau^1 S^{(1)}_{2,3} J(4, \ldots, 2n) + \mathcal{O}(\tau)$$

$$S^{(0)}_{2,3} = \left(\frac{1}{2F^2}\right) \frac{k_4 \cdot p}{k_4 \cdot (q+p)} \qquad S^{(1)}_{2,3} = \left(-\frac{1}{2F^2}\right) \left[(p \cdot q) \frac{k_4 \cdot p}{(k_4 \cdot (p+q))^2} + \frac{q_\mu p_\nu \mathcal{J}^{\mu\nu}_4}{k_4 \cdot (p+q)}\right]$$

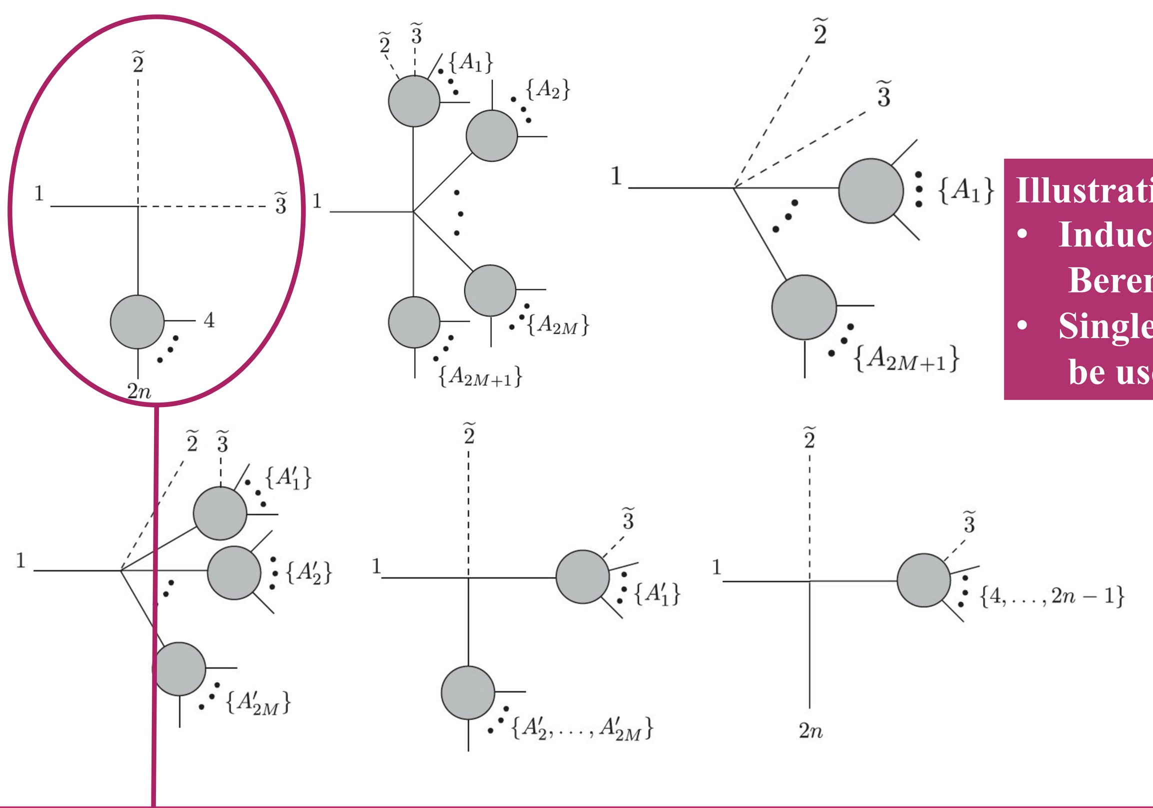
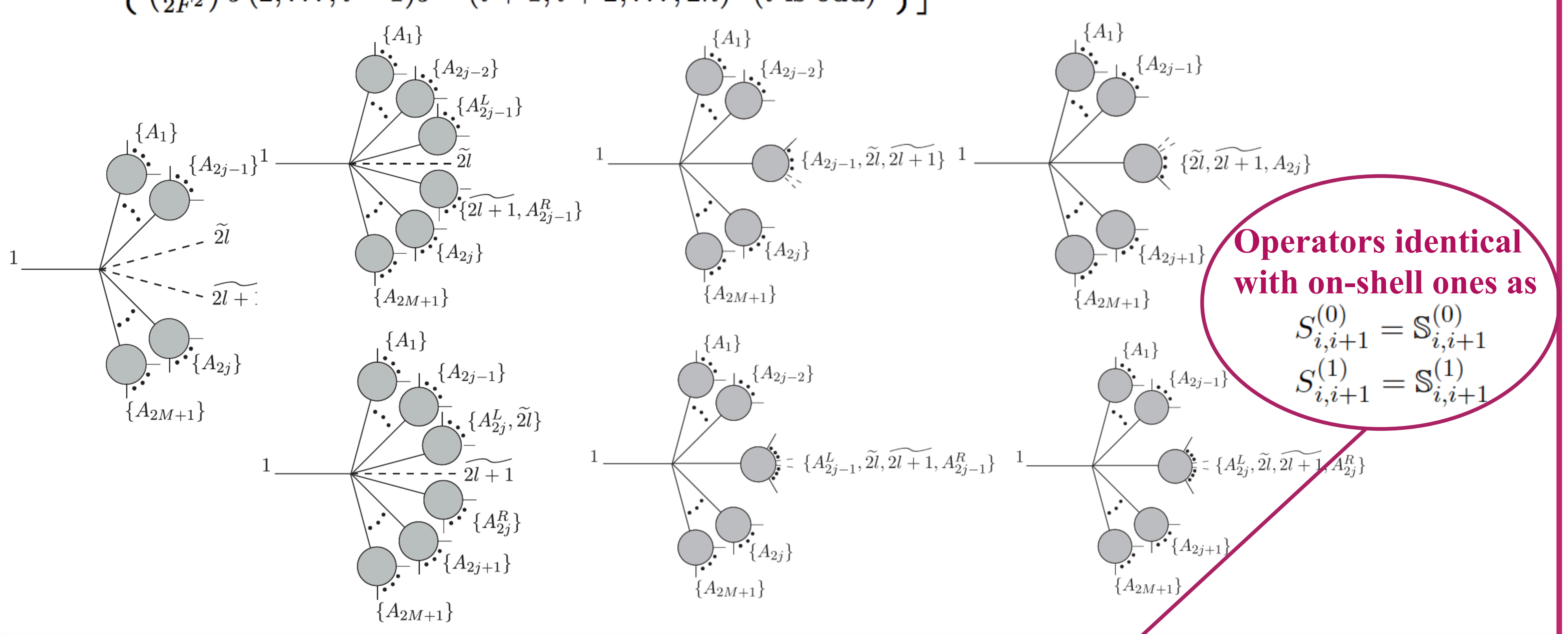


Illustration Strategy:
- Inductive proof via Berends−Giele recursion
- Single soft results should be used as well

Case 2: soft particles non-adjacent to the off-shell line

$$\begin{aligned} & J(2, \ldots, i-1, \tilde{i}, \widetilde{i+1}, i+2, \ldots, 2n) \\ & = \tau^0 S^{(0)}_{i,i+1} J(2, \ldots, i-1, i+2, \ldots, 2n) + \tau^1 \Bigg[S^{(1)}_{i,i+1} J(2, \ldots, i-1, i+2, \ldots, 2n) \\ & + \left\{ \begin{array}{ll} \left(\frac{1}{2F^2}\right) J^{(1)}(2, \ldots, i-1, \tilde{i}) J(i+2, \ldots, 2n) & (i \text{ is even}) \\ \left(\frac{1}{2F^2}\right) J(2, \ldots, i-1) J^{(1)}(\widetilde{i+1}, i+2, \ldots, 2n) & (i \text{ is odd}) \end{array} \right\} \Bigg] + \mathcal{O}(\tau^2) \end{aligned}$$

Illustration Strategy:
- Inductive proof via Berends−Giele recursion
- Results of Case 1 and the single-soft results should be used during the proof



Operators identical with on-shell ones as
$S^{(0)}_{i,i+1} = \mathbb{S}^{(0)}_{i,i+1}$
$S^{(1)}_{i,i+1} = \mathbb{S}^{(1)}_{i,i+1}$

➢ On-Shell Limits Of The Off-Shell Double-Soft Behaviors

- In the Case 2, soft limits and the on-shell limit can be exchanged
- A subtle diagram in Case 1, the on−shell limit should be imposed first
  - ✓ 0/0 appears if the soft limits are imposed first
- With a careful treatment, the double soft behaviors of the amplitudes in the NLSM are achieved as shown on the right side

$$A(1, \ldots, \tilde{i}, \widetilde{i+1}, \ldots, 2n) = \left(\tau^0 \mathbb{S}^{(0)}_{i,i+1} + \tau^1 \mathbb{S}^{(1)}_{i,i+1}\right) A(1, \ldots, i-1, i+2, \ldots, 2n) + \mathcal{O}(\tau^2)$$

$$\mathbb{S}^{(0)}_{i,i+1} = \left(-\frac{1}{2F^2}\right) \frac{1}{2} \left[\frac{k_{i-1} \cdot (p-q)}{k_{i-1} \cdot (p+q)} + \frac{k_{i+2} \cdot (q-p)}{k_{i+2} \cdot (q+p)}\right] \qquad \mathcal{J}^{\mu\nu}_a \equiv k^\mu_a \frac{\partial}{\partial k_{a,\nu}} - k^\nu_a \frac{\partial}{\partial k_{a,\mu}}$$

$$\mathbb{S}^{(1)}_{i,i+1} = \left(-\frac{1}{2F^2}\right) (p \cdot q) \left[\frac{k_{i-1} \cdot q}{(k_{i-1} \cdot (p+q))^2} + \frac{k_{i+2} \cdot p}{(k_{i+2} \cdot (p+q))^2}\right] + \left(-\frac{1}{2F^2}\right) \left[\frac{p_\mu q_\nu}{k_{i-1} \cdot (p+q)} \mathcal{J}^{\mu\nu}_{i-1} + \frac{q_\mu p_\nu}{k_{i+2} \cdot (p+q)} \mathcal{J}^{\mu\nu}_{i+2}\right]$$

CONCLUSION AND OUTLOOK

- ➢ This work provides an **off-shell diagrammatic understanding** on recent new progresses in soft behaviors of NLSM
- ➢ Further study on the **sub-leading order of a current** with **a single soft particle**
- ➢ The behaviors of currents with more soft Goldstones will be complicated and deserve study in future, to understand the finite rotation path in the vacuum
- ➢ Effective theories are expected to understood from the amplitude point of view

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