Master Integrals for Single Top in *t*-channel at $O(\alpha_s^2)$

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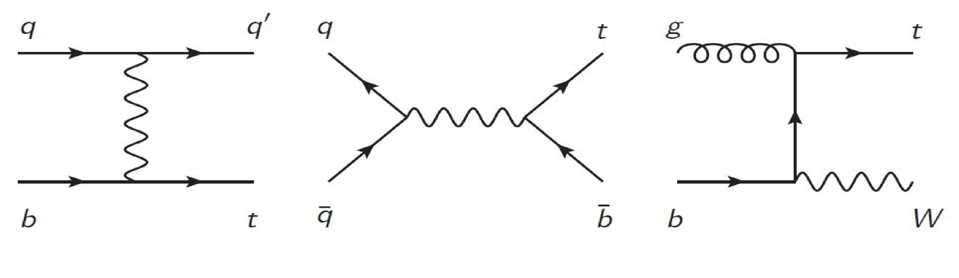
1) Electroweak production mechanism of single top (anti-top) quarks :



(a) t-channel (b) s-channel (c) tW production

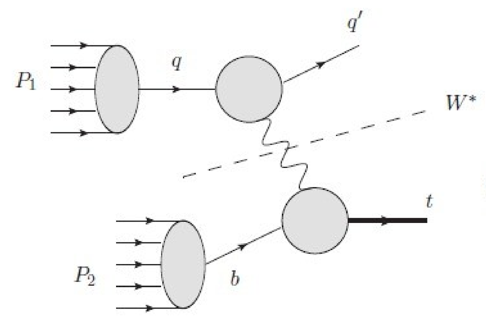
- *Electroweak* production is suppressed with respect to *strong* (top pair) production at hadron colliders
- Despite this fact, Single Top cross-section is enhanced by Phase Space, because of the presence of just one massive particle in the final state (in t- and s- channel)
- @LHC: t-channel cross-section equals roughly 1/3 of cross-section for top pair production and it dominates the other two weak production modes (t-ch > tW > s-ch)

State of Art of QCD corrections to Single Top

	@Tevatron(1.96TeV)	@LHC
Threshold resummation @NNLL	$+(2.06^{0.13}_{-0.13}pb)_t$ $+(1.03^{0.015}_{-0.015})_s$ (Kidonakis 2011)	7TeV:$(65.7^{1.9}_{-1.9}pb)_t + (4.5pb)_s$ $+(15.6pb)_{tW}$ 8TeV :$(87.1^{0.24}_{-0.24})_t + (5.55)_s$ $+(22.2)_{tW}$ (Kidonakis 2012)
***t*-channel NNLO inclusive and p_t-differential (numerical)**		8TeV* :$(54.2^{0.5}_{-0.2})_{top} + (29.7^{0.3}_{0.1})_{t\bar{o}p}$ * inclusive cross-sections (Caola,Melnikov, Brucherseifer 2014, 1404.7116)

2) T-channel Single Top in a CC-DIS framework :

- An analytical result for the inclusive cross-section σ(b + q → t + q') at NNLO-QCD (order α_s^2) is still lacking
- Our aim is thus to compute order α_s^2 corrections to σ(b + q → t + q') in a *structure function approach*.



We neglect *non-factorizable* corrections and thus write the hadronic cross-section as a product of hadronic tensors integrated over the variables Q^2, W_1^2, W_2^2 describing the phase space of the 2 →2 process.

$$d\sigma = \frac{1}{2S}4\left(\frac{g^2}{8}\right)^2 \frac{1}{(Q^2+m_W^2)^2} W^{\mu\nu}(x_1,Q^2)W_{\mu\nu}(x_2,Q^2,m_t^2)(2\pi)^2\frac{1}{4S}dQ^2dW_1^2dW_2^2,$$

- Q^2 is the virtuality of the W boson exchanged in t-channel
- $W_1^2=(P_1-q)^2$, $W_2^2=(P_2+q)^2$ are the hadronic remnants

- $W_{\mu\nu}$ is the generic hadronic tensor, which gets decomposed on a basis of projectors as

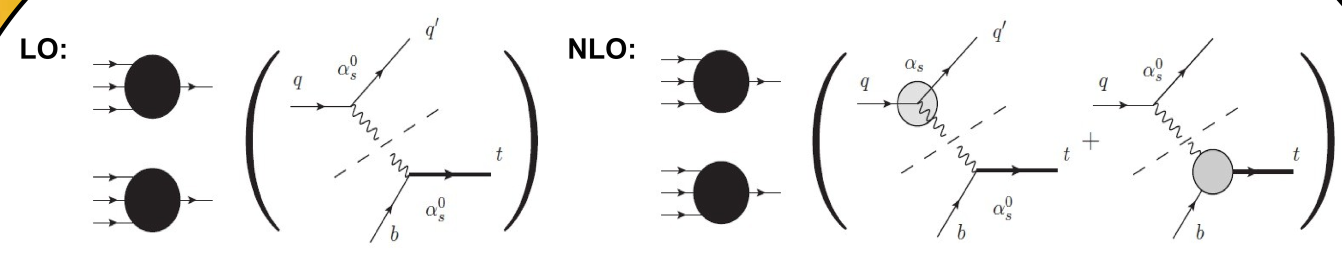
$$W_{\mu\nu} = F_1\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{F_2}{P\cdot q}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right) -i\frac{F_3}{2P\cdot q}\epsilon_{\mu\nu\rho\sigma}P^\rho q^\sigma + F_4 q_\mu q_\nu + F_5(P_\mu q_\nu + P_\nu q_\mu)$$

- $W_{\mu\nu}(x_1,Q^2)$, $W_{\mu\nu}(x_2,Q^2,m_t^2)$ describe the hadronic subprocesses p → *W* + q' (light)* and p+W* → t (*massive*).
- Structure Functions F_i are computed as convolutions between the PDFs (*parton distribution functions)* and the *coefficient functions* C_i , these latters being obtained by contracting the cross-section for the partonic subprocess with the ad-hoc projectors P_i selecting the desired tensorial structure.
  - Our final result will be built up as follows: the C_i are computed analytically and plugged in a stand-alone* code which then performs integrations over the global variables Q^2, W_1^2, W_2^2.

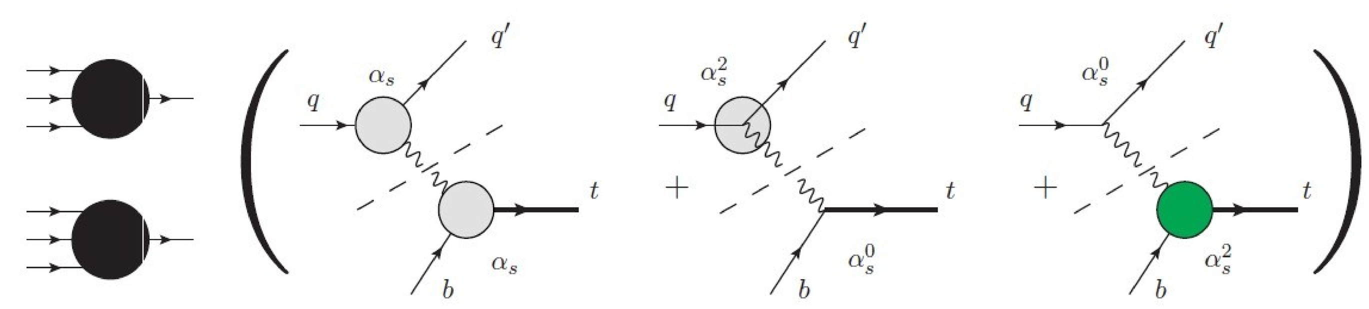
(In arXiv:1404.7116 all integrations, both over extra radiation (to obtain the C_i) and over global phase space are done numerically.)

3) Structure of contributions to σ(b + q → t + q' + X) up to $O(\alpha_s^2)$ in a DIS framework :

LO: NLO:



NNLO:

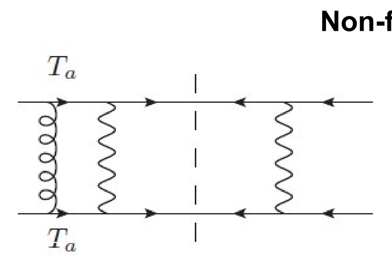


All corrections to the massless current $q \to W^* + q'$ are already known analytically up to order α_s^2, whereas corrections to the massive current b + W* → t only up to order α_s.

Thus, our goal is to compute σ(b + W* → t + X) at $O(\alpha_s^2)$! This is the only piece left, which is needed to complete the DIS-like picture of Single Top in *t*-channel (green blob in the above figure)!

Non-factorizable contributions:

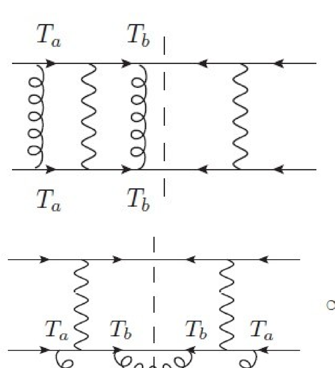
- NLO:



$\propto \mathrm{Tr}[T_a]\mathrm{Tr}[T_a] = 0$

At LO and NLO diagrams with gluon exchange between the two weak currents are zero thanks to *color,* so that the DIS-like approach to Single Top in *t*-channel is indeed correct!

- NNLO:



$\propto Tr[T_aT_b]Tr[T_aT_b] = \frac{C_aC_f}{2} = 2$

$\propto Tr[T_aT_bT_bT_a]N_c = C_f^2C_aN_c = 16$

At NNLO, cross-talks diagrams are not zero anymore, but non-factorizable corrections (Fig. above) are still suppressed thanks to color by a factor $1/N_c^2$ with respect to factorizable ones (Fig. Below).
Thus, at NNLO, the DIS-approach is not exact anymore, but the approximation we introduce is still reasonably good.

4) Organization of a NNLO computation: the case of σ(b + W* → t + X) at $O(\alpha_s^2)$

At NNLO 3 channels are open, corresponding to *bottom-, gluon-,* or *light quark*- initiated diagrams. Inside each channel, Double-Real (RR), Real-Virtual (RV), Double-Virtual (VV) diagrams can contribute, according to the table below.

	bottom	gluon	light quark
RR	$[b+W^*\to t+X]^{0l}_{X=gg,q\tilde{q},b\tilde{b}}$	$[g+W^*\to t+\tilde{b}+g]^{0l}$	$[q+W^*\to t+\tilde{b}+q]^{0l}$
RV	$[b+W^*\to t+X]^{1l}_{X=g}$	$[g+W^*\to t+\tilde{b}]^{1l}$	/
VV	$[b+W^*\to t]^{2l}$	/	/

The process is described by three independent dimensional scales:

- $s=(p_b+q)^2$ (energy in the partonic c.o.m. frame),
- Q^2 (virtuality of the *W* boson),
- m_t^2 (mass of the top quark).

We choose to reparametrize everything in terms of one dimensional and two adimensional Scales, so that our final result will be expressed by the set of variables

$$\{ z = m_t^2/s, \quad y = Q^2/s, \quad s \}.$$

5) Master Integrals technique:

RR, RV, VV are computed via a unique technique, namely Master Integrals (+ Reverse Unitarity to treat Phase Space integrals as Loop Integrals).

The steps which lead to final result are essentially three.

(a) Reduction of scalar matrix elements multiplied by the Phase Space measure to linear combinations of a finite (and possibly small!) set of scalar Feynman integrals, called *Master Integrals (Mis)*. This reduction is based on the solution of *Integration by Parts Identities (IBPs)* and can be performed automatically (we use Mathematica package FIRE).
(b) Explicit computations of Mis. This step is highly non-trivial, since it cannot be carried out automatically and its difficulty increases with the number of scales upon which the process depends.
(c) Plug-in of results for Mis into the matrix elements, renormalization and "cosmetics" of final expressions.

6) Solution of Master Integrals via Differential Equations

Given the number of Mis needed to describe our process and their complexity, we choose to solve them via the method of Differential Equations (DE).

I) Mis form a closed set under IBPs operation.

II) We can express the derivative of a Mi with respect to a certain external momentum as a linear combination of the other Mis, with coefficients which depend only on the external momenta.

III) The set of derivatives with respect to external momenta acting on the Mis can be remapped to the set of derivatives with respect to external invariants.

IV) A system of Partial Differential Equations for the set of Mis in the external Invariants is generated!
In the case of Single Top this reads

$$\begin{cases} \partial_z M_i(z,y,s,\varepsilon)= c_i^z(z,y,s,\varepsilon)\, M_i + \sum_{j\neq i} c_j^z(z,y,s,\varepsilon)\, M_j \\ \partial_y M_i(z,y,s,\varepsilon)= c_i^y(z,y,s,\varepsilon)\, M_i + \sum_{j\neq i} c_j^y(z,y,s,\varepsilon)\, M_j \\ \partial_s M_i(z,y,s,\varepsilon)= c_i^s(z,y,s,\varepsilon)\, M_i + \sum_{j\neq i} c_j^s(z,y,s,\varepsilon)\, M_j \end{cases}$$

Canonical form of Differential Equations

- Given the complexity of (partial) DEs obtained, we solve them order by order in ε.
- We apply the idea of J. Henn, arXiv:1304.1806: we find equivalent set of Mis $\{M_i'\}$ such that the ε dependence is factorized from the kinematics in the PDEs (*canonical form of DEs*). Given the vector $\boldsymbol{M}=\{M_i\}$ of masters and $\boldsymbol{x}=\{z,y,s\}$ of variables

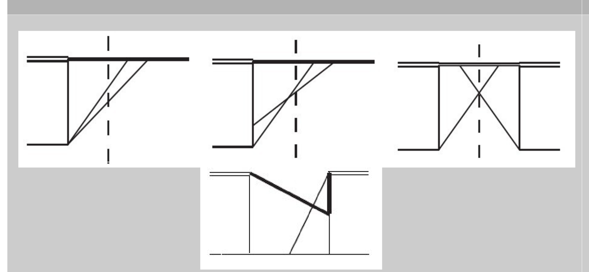
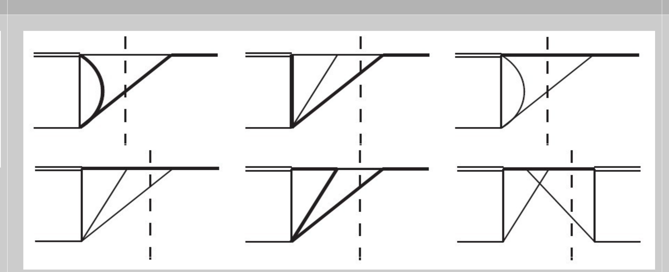
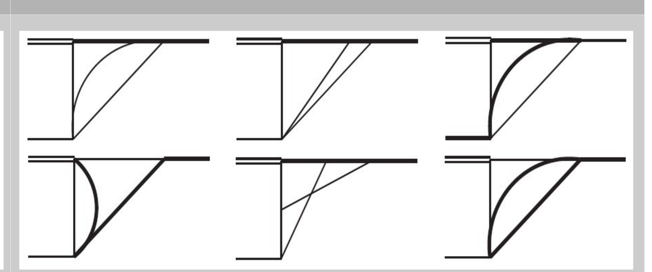
$$\partial_{\boldsymbol{x}} \boldsymbol{M}(\boldsymbol{x},\varepsilon)= \varepsilon\, [C(\boldsymbol{x})\, \boldsymbol{M}(\boldsymbol{x},\varepsilon)].$$

Systems of DEs in canonical form are integrated in one go recursively in ε.

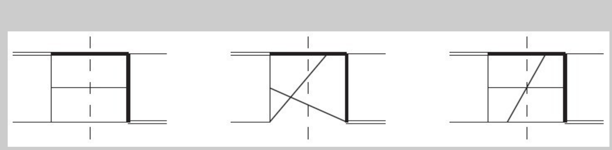
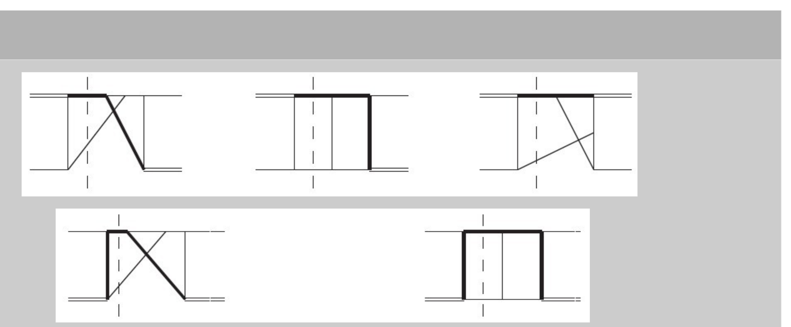
7) Master Integrals for σ(b + W* → t + X) at $O(\alpha_s^2)$

We report in the following drawings of a handful of Mis describing our process. Mis are listed according to the channel they describe and to the number of *cut propagators (*namely phase space *delta functions*) they contain. For each set of Mis, the corresponding *alphabet* needed to solve them is reported.

- Bottom:

RR (2 cuts)	RV (1 cut)	VV (0 cuts)
		
A = {z,1-z, y, 1+y, y+z, 1+y+z, 1+2y+z }	*A = {z, 1-z, y, 1+y, y+z, 1+y+z }*	*A = {y, 1+y }*

- Gluon:

RR (2 cuts)	RV (1 cut)
	
A_1 = {z, 1-z, y, 1+y, y+z, 1+y+z, 1+2y+z, $1-z-((1-z)^2+4y^2+4y)^{1/2}$, $1-z+4y^2+(1+2y)((1-z)^2+4y^2+4y)^{1/2}$, $(1-z)^2+4y^2+4y$} A_2 = {z, 1-z, y, 1+y, y+z, 1+y+z, 1+2y+z, $1+z+2y+((1+z)^2+4yz)^{1/2}$, $1+z+2yz+((1+z)^2+4yz)^{1/2}$, $(1+z)^2+4yz$}	A = {z, 1-z, y, 1+y, y+z, 1+y+z, 1+2y+z, $1+z+2y+((1+z)^2+4yz)^{1/2}$, $1+z+2yz+((1+z)^2+4yz)^{1/2}$, $(1+z)^2+4yz$}

8) Conclusions and Outlook

- The complete set of Master Integrals describing σ(b + W* → t + X) at $O(\alpha_s^2)$ has been determined and all Mis computed. The set contains ~*O(75)* Mis, depending on two dimensionless variables *z,y*. The most complicated part of the computation was represented by the determination of boundary conditions to be matched to the DEs and by the integration of Mis for the gluon channel whose alphabet exhibit quadratic dependence on both variables (in this case remappings were found to linearize the DEs). So, step number a) and b) (see blob n. 5)) are accomplished.
- We are now left with the last step c) to be done, namely plug-in of Mis into matrix elements, renormalization and cosmetics.
- As a final step, we will implement our analytic result for CC-DIS massive Coefficient Function into our stand-alone* code, which will eventually give us the value of t-channel Single Top cross-section. We will then be able to provide an analytical cross-check to the numerical results found in arXiv:1404.7116 and a fast and stable code to evaluate inclusive cross-section for Single Top in *t*-channel at NNLO.

* We kindly thanks the authors of the original version of the stand-alone code (SINGLE-TOP FORTRAN) S. Moch and M. Zaro.