Higher derivative SYM lagrangian in open superstring theory

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Introduction

In a nonabelian theory, tree level open string amplitudes of N gauge bosons are computed according to the formula

$$\mathcal{A}_N = i(2\pi)^D \delta^D(k_1+\ldots+k_N) \times \times \left[\text{tr}(\lambda^{a_1}\lambda^{a_2}\ldots\lambda^{a_N})\, A(1,2,\ldots,N) + \begin{pmatrix}\text{non-cyclic}\\ \text{permutations}\end{pmatrix} \right], \tag{1}$$

where D is the spacetime dimension and $A(1,\ldots,N)$ is the color ordered amplitude that string theory makes available using vertex operators.
For the **BOSONIC** open string:

$$A(1,\ldots,N) = C_N(\alpha')\, g^{N-2} \times \times \int_0^1 dx_{N-2} \int_0^{x_{N-2}} dx_{N-3} \ldots \int_0^{x_3} dx_2 \prod_{i<j}^{N} (x_j - x_i)^{2\alpha' k_i \cdot k_j} \times \times \exp\left(\sum_{i<j}^{N} \frac{2\alpha' \zeta_i \cdot \zeta_j}{(x_j - x_i)^2} - \sum_{i\neq j}^{N} \frac{2\alpha' k_j \cdot \zeta_i}{(x_j - x_i)} \right)\Bigg|_{linear}, \tag{2}$$

where $$x_1 = 0\,, \quad x_{N-1} = 1\,, \quad x_N = +\infty\,. \tag{3}$$

While for the **SUPERSYMMETRIC** open string (in the **RNS** formalism):

$$A(1,2,\ldots,N) = 2\frac{g^{N-2}}{(2\alpha')^2}(x_{N-1}-x_1)(x_N - x_1) \times \int_0^{x_{N-1}} dx_{N-2} \int_0^{x_{N-2}} dx_{N-3} \ldots \int_0^{x_3} dx_2 \times \times \int d\theta_1 \ldots d\theta_{N-2} \prod_{i<j}^{N} (x_j - x_i - \theta_j\theta_i)^{2\alpha' k_i \cdot k_j} \times \int d\phi_1 \ldots d\phi_N \times \exp\left(\sum_{i\neq j}^{N} \frac{(2\alpha')^1(\theta_j - \theta_i)\phi_j(\zeta_j \cdot k_i) - 1/2\,(2\alpha')^1 \phi_j\phi_i(\zeta_j\cdot\zeta_i)}{x_j - x_i - \theta_j\theta_i} \right). \tag{4}$$

Review of the low energy effective lagrangian in open superstring theory: the S-matrix method

1. Structure of the low energy effective lagrangian

In eqs. (5) and (6) we briefly review the structure of the low energy effective lagrangian (LEEL) up to α'^3 terms.
i) Low energy effective lagrangian up to α'^2 terms:

$$\mathcal{L}_{\text{eff}} = \frac{1}{g^2}\text{tr}\Big[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (2\alpha')^1 a_1 F_\mu{}^\lambda F_\lambda{}^\nu F_\nu{}^\mu + +(2\alpha')^2\Big(a_3 F^{\mu\lambda}F^\nu{}_\lambda F_\mu{}^\rho F_{\nu\rho} + a_4 F^\mu{}_\lambda F_\nu{}^\lambda F^{\nu\rho}F_{\mu\rho} + + a_5 F^{\mu\nu}F_{\mu\nu}F^{\lambda\rho}F_{\lambda\rho} + a_6 F^{\mu\nu}F^{\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\Big) + +O((2\alpha')^3) \Big]. \tag{5}$$

ii) Low energy effective lagrangian at α'^3 order:

$$\mathcal{L}_{\text{eff}}{}^{(3)} = \frac{(2\alpha')^3}{g^2}\text{tr}\Big[a_{10}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\rho{}^\sigma F_\sigma{}^\mu + a_{11}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\rho F_\sigma{}^\mu F_\rho{}^\sigma + + a_{12}\, F_\mu{}^\nu F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho F_\rho{}^\sigma + a_{13}\, F_\mu{}^\nu F_\rho{}^\sigma F_\nu{}^\lambda F_\sigma{}^\mu F_\lambda{}^\rho + + a_{14}\, F_\mu{}^\nu F_\nu{}^\lambda F_\lambda{}^\mu F_\rho{}^\sigma F_\sigma{}^\rho + a_{15}\, F_\mu{}^\nu F_\nu{}^\lambda F_\rho{}^\sigma F_\lambda{}^\mu F_\sigma{}^\rho + + a_{16}(D_\mu F_\nu{}^\lambda)(D^\mu F_\lambda{}^\rho)F_\sigma{}^\nu F_\rho{}^\sigma + a_{17}(D_\mu F_\nu{}^\lambda)F_\sigma{}^\nu(D^\mu F_\lambda{}^\rho)F_\rho{}^\sigma + + a_{18}(D_\mu F_\nu{}^\lambda)(D^\mu F_\lambda{}^\nu)F_\rho{}^\sigma F_\sigma{}^\rho + a_{19}(D_\mu F_\nu{}^\lambda)F_\rho{}^\sigma(D^\mu F_\lambda{}^\nu)F_\sigma{}^\rho + + a_{20}(D_\sigma F_\mu{}^\nu)F_\lambda{}^\rho(D^\mu F_\nu{}^\lambda)F_\rho{}^\sigma + a_{21}F_\mu{}^\nu(D^\mu F_\nu{}^\lambda)F_\rho{}^\sigma(D_\sigma F_\lambda{}^\rho) + + a_{22}F_\mu{}^\nu(D^\mu F_\lambda{}^\rho)(D_\sigma F_\nu{}^\lambda)F_\rho{}^\sigma \Big]. \tag{6}$$

2. Matching with string S-matrix calculations

Using the explicit expression of 3- and 4-point open string amplitudes that come from eqs. (2) and (4) it can be found that:

Coefficient	Bosonic open string theory	Supersymmetric open string theory
a_1	$-i/3$	0
a_3	$\pi^2/12$	$\pi^2/12$
a_4	$\pi^2/24$	$\pi^2/24$
a_5	$-\pi^2/48 - 1/8$	$-\pi^2/48$
a_6	$-\pi^2/96 + 1/8$	$-\pi^2/96$

The revisited S-matrix method

In ref. [1] we found that, in the case of open superstring theory,

$$\boxed{A(1,\ldots,N) \text{ does not contain } (\zeta\cdot k)^N \text{ terms}}\,. \tag{7}$$

3. Implications for the determination of the LEEL

i) Absence of $(\zeta\cdot k)^N$ terms in $A(1,\ldots,N)$ at α'^{N-2} order:

$$N = 3 \;\Rightarrow\; a_1 = 0\,. \tag{8}$$

$$N = 4 \;\Rightarrow\; a_3 = -8\,\mathbf{a_6}\,,\; a_4 = -4\,\mathbf{a_6}\,,\; a_5 = 2\,\mathbf{a_6}\,. \tag{9}$$

$$N = 5 \;\Rightarrow\; \begin{cases} -2a_{16} = -2a_{17} = 8a_{19} = -a_{20} = \mathbf{a_{22}}\,, \\ a_{18} = a_{21} = \mathbf{0}\,, \\ a_{11} = a_{13} = -2a_{15} = -i\,\mathbf{a_{22}}\,, \\ a_{10} = a_{12} = a_{14} = \mathbf{0}\,. \end{cases} \tag{10}$$

ii) α'^4 calculation ($N = 6$):

$$\mathcal{L}_{\text{eff}}{}^{(4)} = \frac{(2\alpha')^4\pi^4}{g^2}\left(\mathcal{L}_{F^6} + \mathcal{L}_{D^2F^5} + \mathcal{L}_{D^4F^4} \right), \tag{11}$$

$$\mathcal{L}_{F^6} = \frac{1}{46080}\, t_{(12)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5\mu_6\nu_6} \times \times\text{tr}\big(F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5}F_{\mu_6\nu_6}\big)\,, \tag{12}$$

$$\mathcal{L}_{D^2F^5} = \frac{56\,i}{46080} t_{(10)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5} \times \times\; \text{tr}\big(F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^\alpha F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5}\big) + + \frac{i}{46080}\,(\eta\cdot t_{(8)})^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4\mu_5\nu_5} \times \times\; \text{tr}\big(-169\, D^\alpha F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5} + + 68\, D^\alpha F_{\mu_1\nu_1}D_\alpha F_{\mu_2\nu_2}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5} + + 237\, F_{\mu_1\nu_1}D^\alpha F_{\mu_2\nu_2}D_\alpha F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5} + + 237\, F_{\mu_1\nu_1}D^\alpha F_{\mu_2\nu_2}F_{\mu_3\nu_3}D_\alpha F_{\mu_4\nu_4}F_{\mu_5\nu_5} + + 267\, F_{\mu_1\nu_1}F_{\mu_2\nu_2}D^\alpha F_{\mu_3\nu_3}D_\alpha F_{\mu_4\nu_4}F_{\mu_5\nu_5} + + 16\, F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^\alpha F_{\mu_4\nu_4}D_\alpha F_{\mu_5\nu_5} \big) - - \frac{i}{5760}\, t_{(8)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \times \times \Big\{ 17\,\text{tr}\big(D^{\mu_5}F_{\mu_1\nu_1}F_{\mu_2\nu_2}F_{\mu_3\nu_3}D^{\nu_5}F_{\mu_4\nu_4}F_{\mu_5\nu_5}\big) + + 2\,\text{tr}\big(F_{\mu_1\nu_1}D^{\mu_5}F_{\mu_2\nu_2}D^{\nu_5}F_{\mu_3\nu_3}F_{\mu_4\nu_4}F_{\mu_5\nu_5}\big) \Big\}\,, \tag{13}$$

$$\mathcal{L}_{D^4F^4} = -\frac{1}{11520}\, t_{(8)}^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \times \times\; \text{tr}\big(D^\alpha F_{\mu_1\nu_1}D_{(\alpha}D_{\beta)}F_{\mu_2\nu_2}D^\beta F_{\mu_3\nu_3}F_{\mu_4\nu_4} + + 8\, D^\alpha F_{\mu_1\nu_1}D_\alpha F_{\mu_2\nu_2}D^\beta F_{\mu_3\nu_3}D_\beta F_{\mu_4\nu_4}\big)\,. \tag{14}$$

4. Implications for the determination of a closed formula for (tree level) open superstring amplitudes

In refs. [2,3] it was found a closed expression for the open superstring tree level N-point amplitude, using the Pure Spinor formalism.
In ref. [4] we found that using the RNS formalism, based on the main observation of the *Revisited S-matrix method*, eq. (7), we can also arrive the same formula found in refs.[2,3] (at least for $N = 4, 5, 6$ and 7). For example, in the case of $N = 5$ we found that [4]:

$$A(1,2,3,4,5) = F^{\{23\}}(\alpha')\, A_{SYM}(1,2,3,4,5) + +F^{\{32\}}(\alpha')\, A_{SYM}(1,3,2,4,5)\,, \tag{15}$$

where $F^{\{23\}}(\alpha')$ and $F^{\{32\}}(\alpha')$ are known momentum factors whose expressions can be found in refs. [2,3,4].

Towards a generalization: the higher derivative SYM lagrangian

In ref. [5] we found that the main observation of the *Revisited S-matrix method*, eq.(7), is not something peculiar to open superstring theory: it is a common result to ANY high energy theory whose low energy limit is $D = 10$ SYM theory.

So the sort of constraints that we have found in eqs.(8), (9) and (10) are valid for ANY higher derivative SYM lagrangian.

In ref. [5] we have found the generalization of the constraint in eq.(7), but for interactions involving gauge bosons and gauginos as well.

So in ref. [5] we have found (at least for the first α' orders) the general structure that ANY higher derivative SYM lagrangian should obey.

Main references

[1] L. A. Barreiro, R. Medina, *Revisiting the S-matrix approach to the open superstring low energy effective lagrangian*, **JHEP 1210 (2012) 108**, [arXiv:1208.6066].

[2] C. Mafra, O. Schlotterer, S. Stieberger, *Complete N-point superstring disk amplitude I. Pure spinor computation*, **Nucl. Phys. B873 (2013) 419**, [arXiv:1106.2645].

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[4] L. A. Barreiro, R. Medina, *RNS derivation of N-point disk amplitudes from the revisited S-matrix approach*, **Nucl. Phys. B886 (2014) 870**, [arXiv:1310.5942].

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