Orientations of BCFW Charts on the Grassmannian

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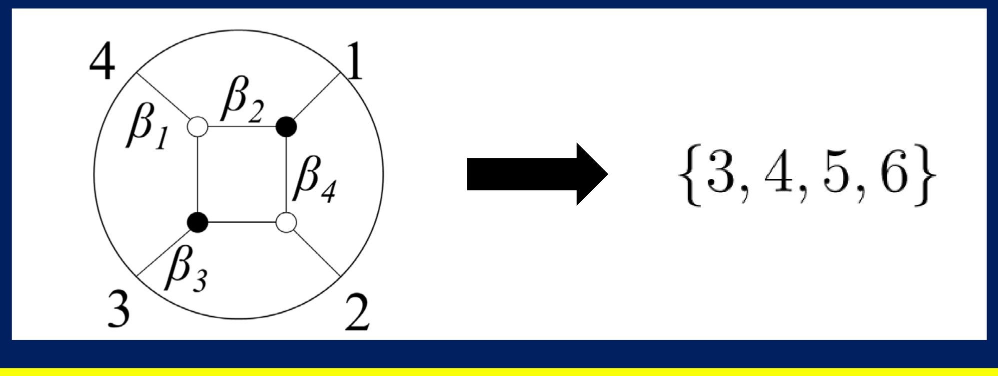
Setup

Planar N^{k}MHV tree amplitudes in *N*=4 super Yang-Mills theory (*N*=4 SYM) are obtained by evaluating certain contour integrals over the **Grassmannian Gr(*k*,*n*)**, which is the space of *k*×*n* matrices (mod GL(*k*)):

$$\mathcal{A}_n^{(k)} = \mathcal{A}_n^{\mathrm{MHV}} \oint_\Gamma \frac{d^{k\times n}C}{GL(k)\, M_1 M_2 \ldots M_n}\,\delta^{4k|4k}(C\cdot\mathcal{Z})$$
$$= \mathcal{A}_n^{\mathrm{MHV}} \times \sum \mathrm{Residues}$$

Residues can be represented as **on-shell diagrams** labeled by **decorated permutations**, e.g.



Diagrams are produced by sequences of **transpositions (*a b*)**, each of which adds a **BCFW bridge** to the diagram. Every bridge adds a coordinate α_i, so each diagram generates a **chart** on the Grassmannian:

$$\omega = \frac{d^{k\times n}C}{GL(k)\, M_1 M_2 \ldots M_n} \to \mathrm{dlog}\,\alpha_d \wedge \mathrm{dlog}\,\alpha_{d-1} \wedge \ldots \wedge \mathrm{dlog}\,\alpha_1$$

Problem

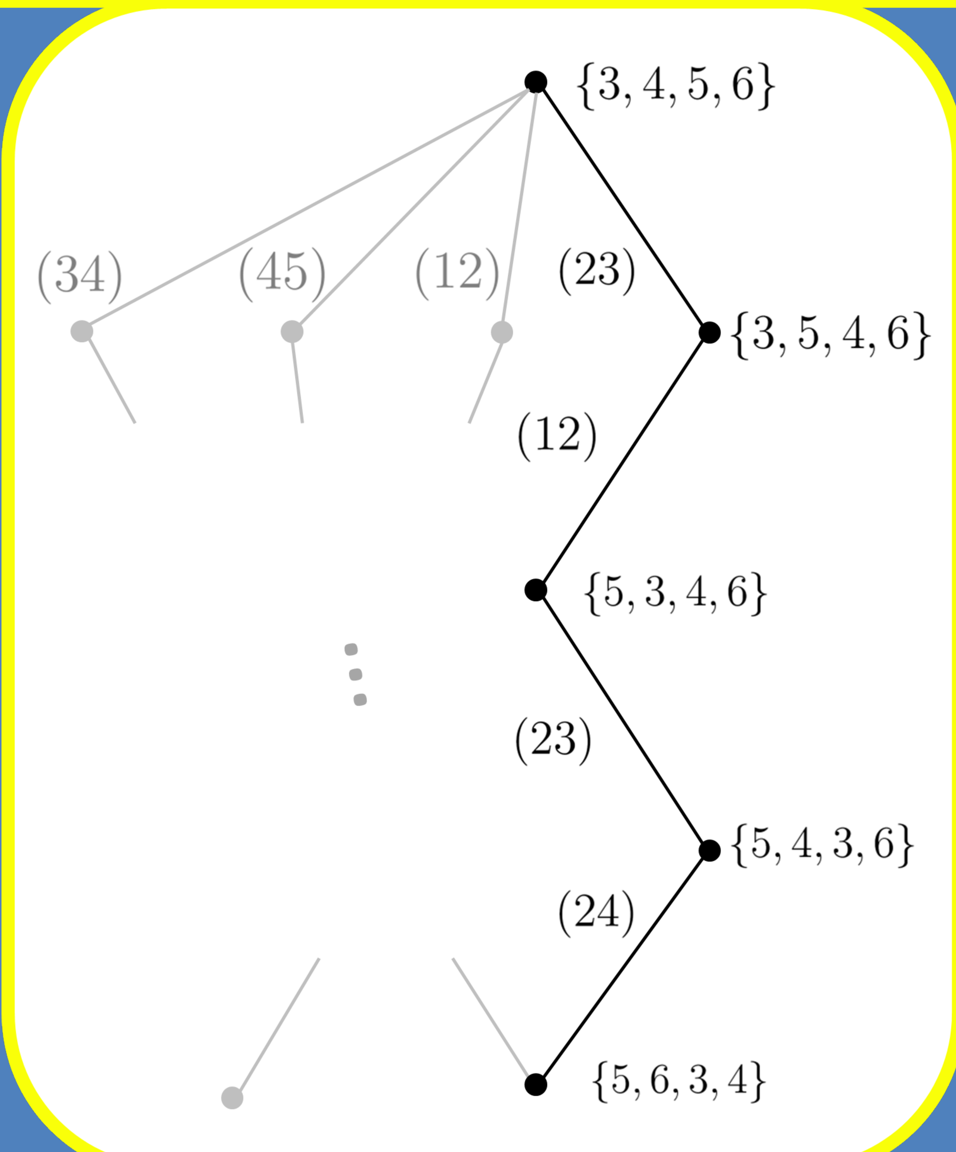
Chart orientation information lost!

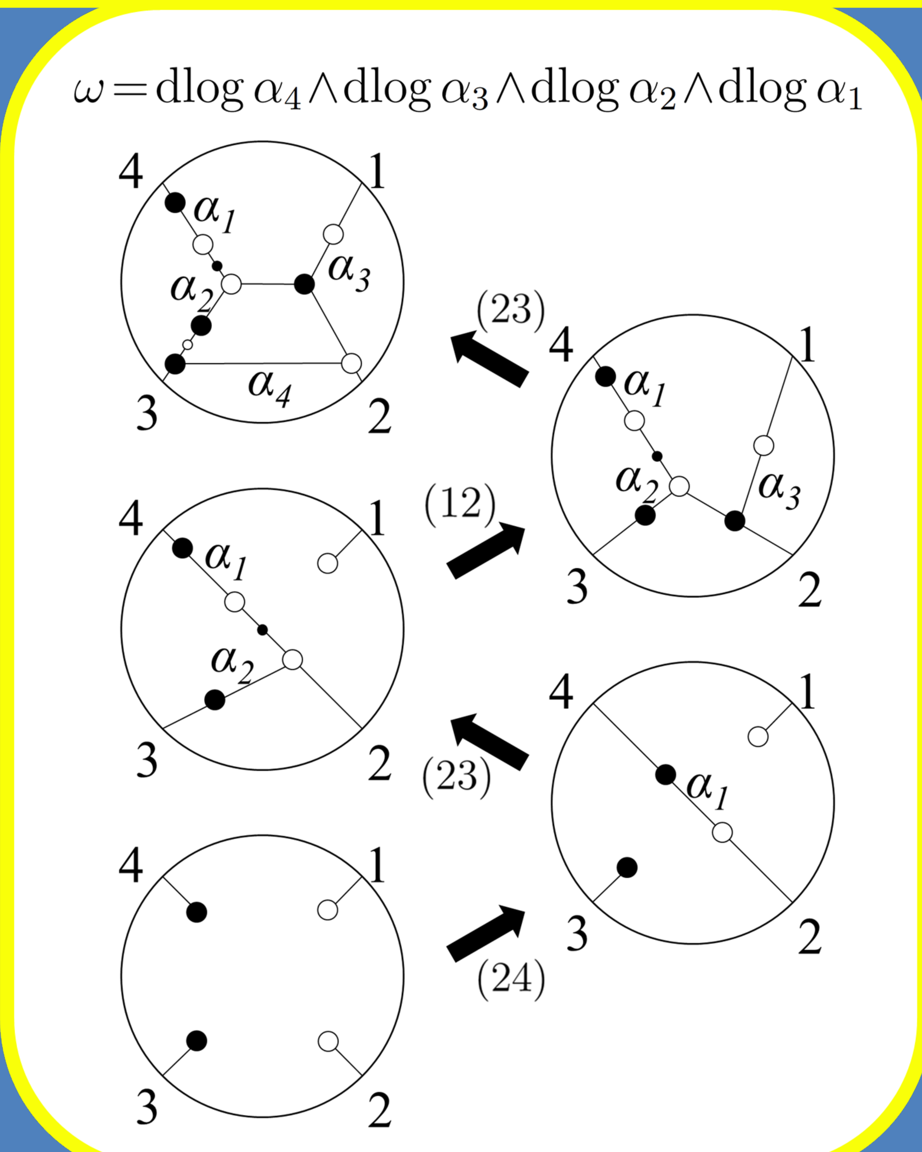
- ➡ Sign ambiguities when combining residues into amplitude
- ➡ Need consistent signs to cancel non-local divergences

Solution

- Charts correspond to paths in the partially ordered set (poset) with residues as vertices and edges representing transpositions.
- Weight edges with ±1 such that the product of signs around any quadrilateral in the poset is -1.
- *The relative orientation between two charts is the product of signs along each respective path times the signs along a path connecting the sequences.*
- This resolves the issue of sign ambiguities between residues in the amplitude sum and consistently cancels non-local poles.

Example Diagram Construction

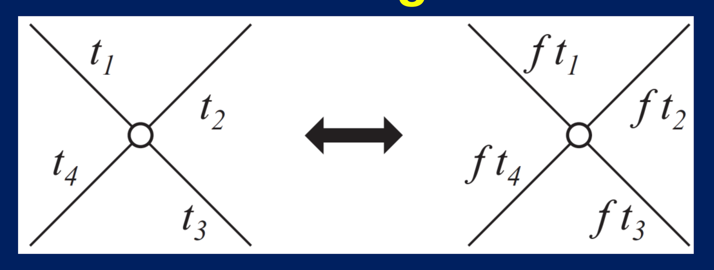




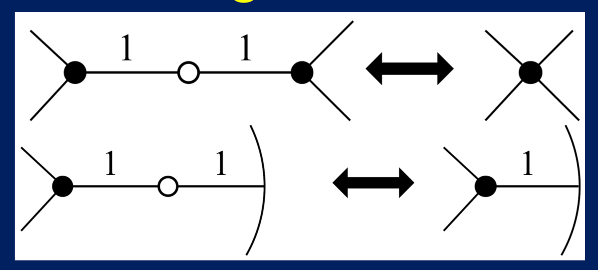
Equivalence Moves

- **Equivalence moves** translate between distinct diagrams representing a single residue.
- Each move has a well defined action on the coordinates, so one can explicitly relate different charts by a sequence of moves. The change of variables yields the relative orientation.
- By induction, it can be proven that the poset edge weights encode the same orientation information.

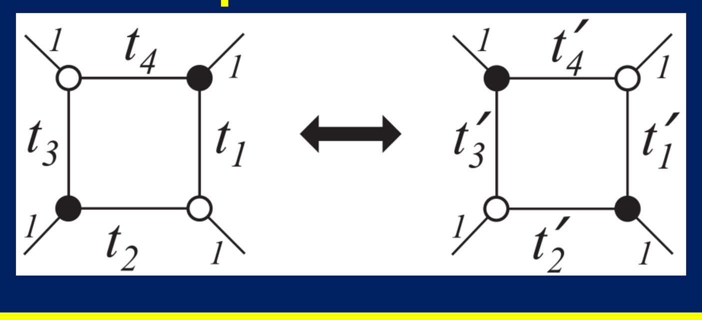
Scaling



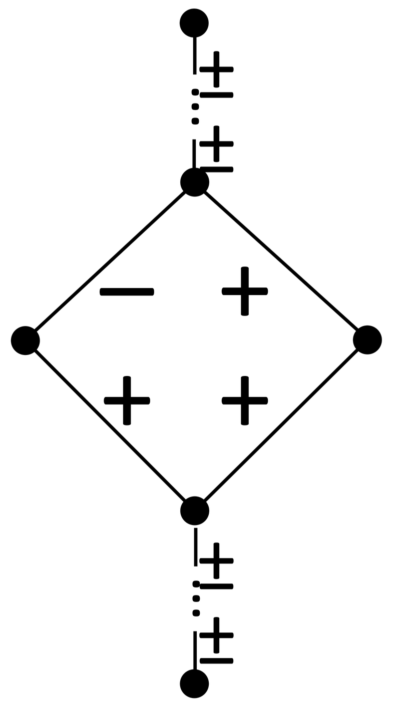
Merge/Delete



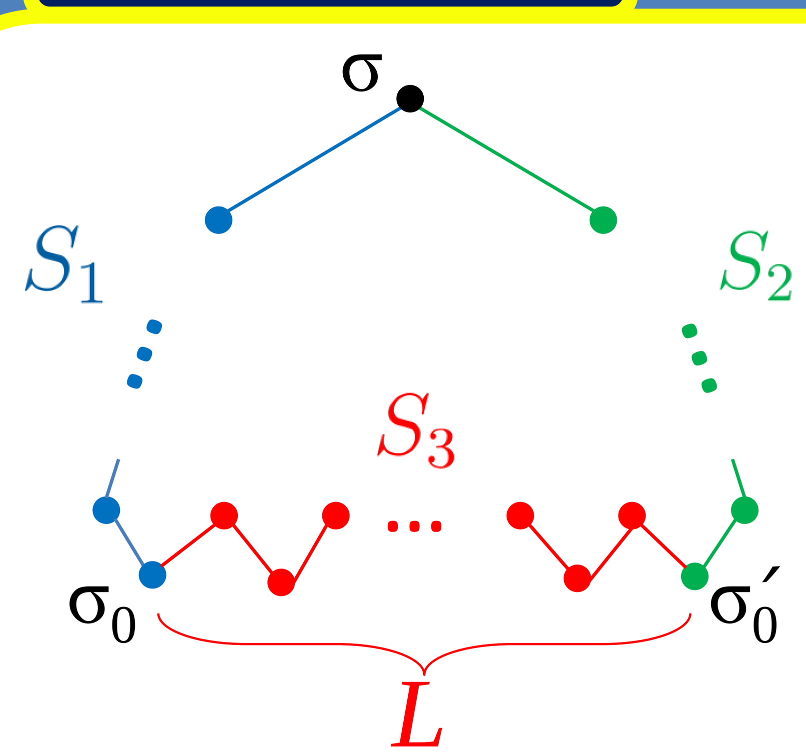
Square Move



Edge Weights



Result



Rel. sign= $S_1 \times S_2 \times S_3 \times (-1)^{L/2}$

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