Position space cuts for Wilson line correlators

A novel way to extract the imaginary part

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Introduction

Wilson line correlators → soft singularities, QCD amplitudes →
- Real-Virt cancellations
- Resummation
- Theory

$\mathcal{S} \equiv \langle 0|\Phi_{v_1} \otimes \cdots \otimes \Phi_{v_n}|0\rangle$

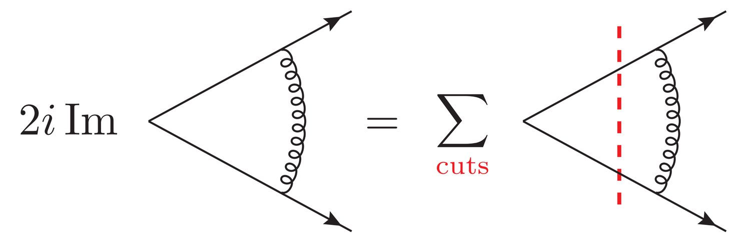
with $\Phi_{v_i} \equiv \mathcal{P}\exp\left(ig\int_0^\infty \mathrm{d}t\, v_i\cdot A(tv_i)\right)$

Im ↓

Imaginary part →
- Interquark potential
- Facorization breaking [1,2]
- Unitarity formalism

Interquark potential $\mathrm{Im}\,\Gamma_{\mathrm{cusp}} = -\frac{g^2 C_F}{4\pi\gamma} + \mathcal{O}(\gamma)$

Vanishes when the partons are not causally connected. Applies to...
- ✓ QED
- ✓ N=4 SYM [3]
- ≈ QCD [4]

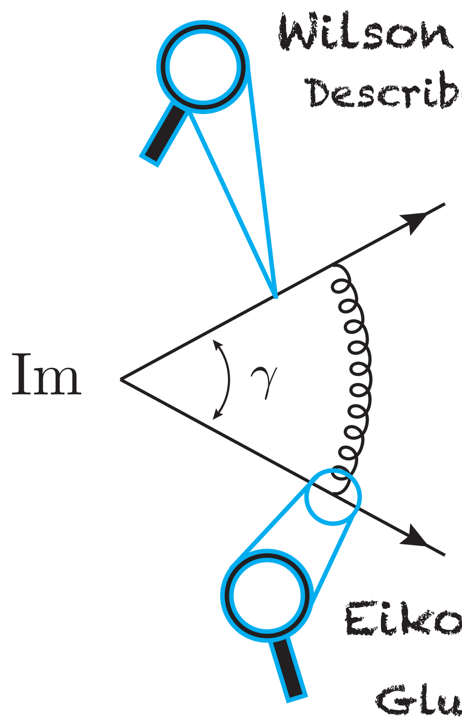


Analogous to unitarity cuts of ordinary Feynman diagrams in momentum space [5]

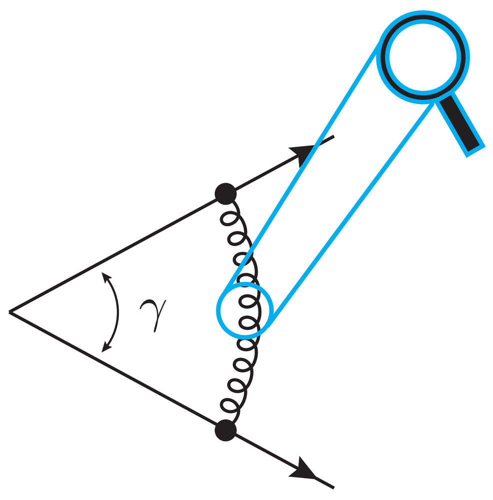
$\gamma \approx 0$ Non-relativistic limit

Imaginary part at one-loop

Wilson line
Describes highly energetic particle (q, e⁻, ...)



$= C(g,\epsilon)\coth\gamma =$



Eikonal vertex $\frac{i(\not p + \not k)}{(p+k)^2 + i\eta} \longrightarrow \frac{iv^\mu}{2v\cdot k + i\eta}$

Gluon "feels" direction of Wilson line, but not its energy or spin

Cut gluon propagator
Fixing the relative locations of the endpoints in position space (marked by BLACK DOTS)

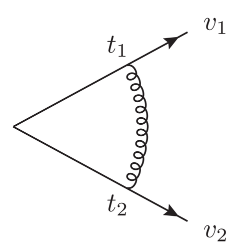
Gluon is fixed on shell
⇒ integral trivial

Imaginary part directly!

Method

Prescription for obtaining the imaginary part [6]

1) Extract leading divergence in 1/ε
from overall distance scale

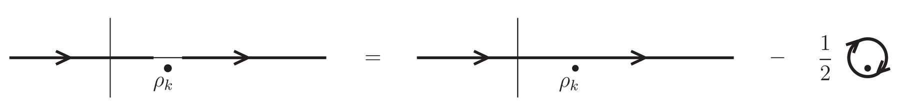


$$\propto \mu^{2\epsilon}\int_0^\infty\!\!\int_0^\infty \frac{\mathrm{d}t_1\,\mathrm{d}t_2\; v_1\cdot v_2}{\left[-(t_1v_1 - t_2v_2)^2 + i\eta\right]^{1-\epsilon}} \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \lambda\begin{pmatrix} x \\ 1-x \end{pmatrix}$$

$$= \frac{1}{2\epsilon}\left(\frac{\mu}{\Lambda}\right)^{2\epsilon}\int_0^1 \frac{\mathrm{d}x\; v_1\cdot v_2}{\left[-(xv_1-(1-x)v_2)^2 + i\eta\right]^{1-\epsilon}}$$

2) Cut residue of leading divergence (at ε = 0)
Put odd number of gluons on shell → $\delta\left((xv_i - (1-x)v_j)^2\right)$
and remaining gluons off shell (P.V. integral)

3) Integrate!
Write P.V. = Full − Im



Result in terms of MPL's (exploit Hopf algebra to perform the integrations along the Wilson lines)

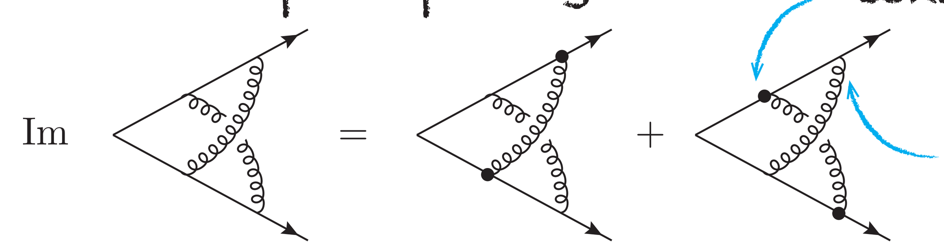
Literature

[1] Catani, de Florian and Rodrigo, JHEP 1207 (2012) 026
[2] Forshaw, Seymour and Siódmok, JHEP 1211 (2012) 066
[3] Chien, Schwarz, Simmons-Duffin and Stewart, Phys.Rev. D85 (2012) 045010
[4] Grozin, Henn, Korchemsky and Marquard, Phys.Rev.Lett. 114 (2015), no. 6 062006
[5] Korchemsky and Radyushkin, Nucl. Phys. B283 (1987) 342-364
[6] Laenen, Larsen and Rietkerk, Phys.Rev.Lett. 114 (2015), no. 18 181602
[7] Laenen, Larsen and Rietkerk, accepted by JHEP, [arXiv:1505.02555]

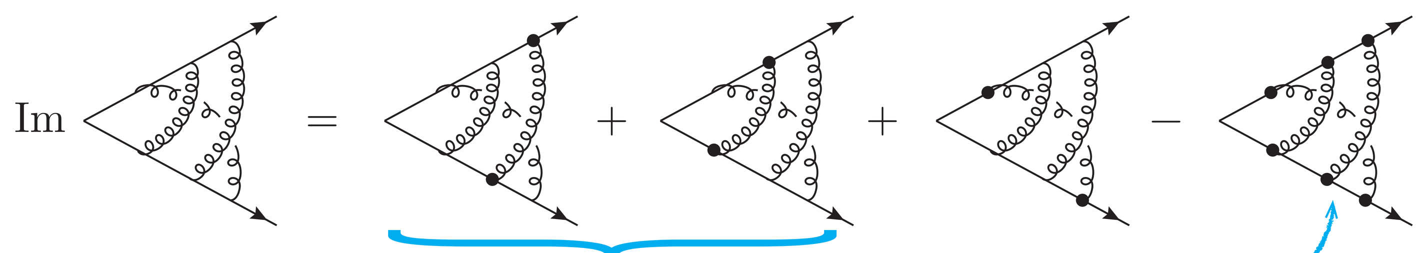
Results

Prescription demonstrated on various examples [7]

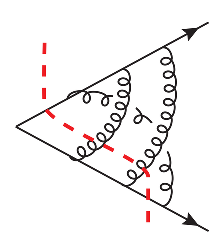
a) Two-loop cusp diagram



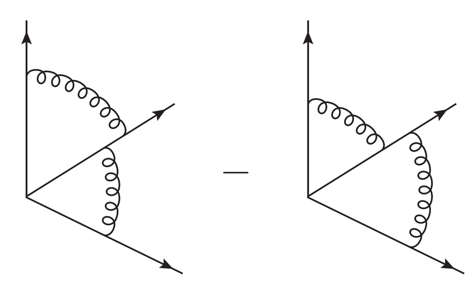
b) Three-loop cusp diagram



Avoids proliferation of complicated phase-space integrals, such as



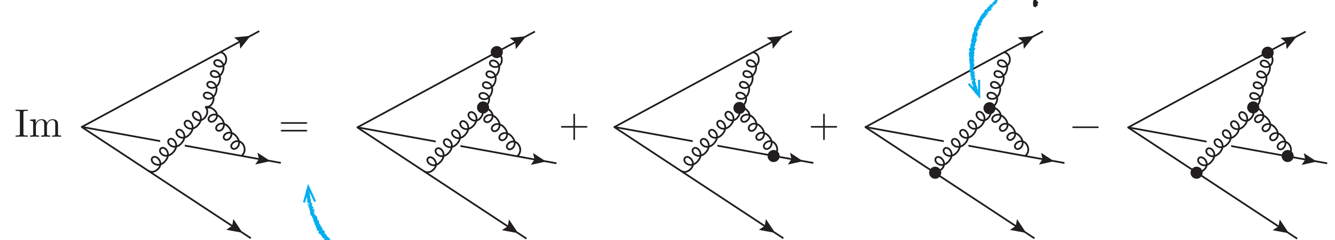
c) Two-loop web



Im part at subleading order in ε, due to cancellation of divergences

Works for all external kinematics:
$v_1\cdot v_2 \lessgtr 0,\ v_2\cdot v_3 \lessgtr 0$, or
$v_1\cdot v_2 \lessgtr 0,\ v_2\cdot v_3 \gtrless 0$

d) Three-gluon vertex diagram



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