On-shell recursion for off-shell amplitudes

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Motivation

Particle momentum in k_T-factorisation approach:

$$k_i^\mu = x_i\, p_i^\mu + k_{T\,i}^\mu$$

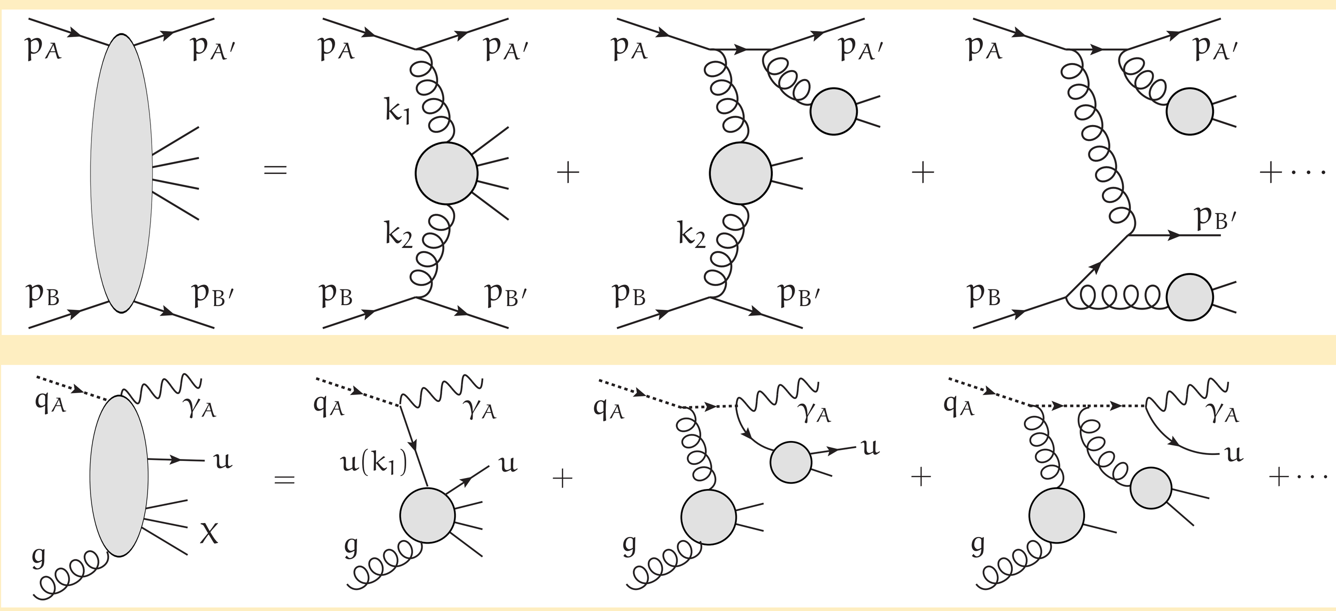
The off-shell component of the momentum requires gauge-invariant scattering amplitudes with off-shell legs.

Relevant for:

- Saturation (i.e. small-x) physics
- Hybrid factorisation (1 off-shell leg) for forward jets production
- Jet physics

The problem

Gauge invariance with off-shell particles is a highly non-trivial problem. One tricky methods were already devised and successfully tested [1, 2].
The idea: off-shell particles are emebedded in a gauge-invariant way through auxiliay pairs



The analytic expressions are strikingly similar to the on-shell case ⇒ is must be possible to reproduce these results by using some sort of BCFW recursion.

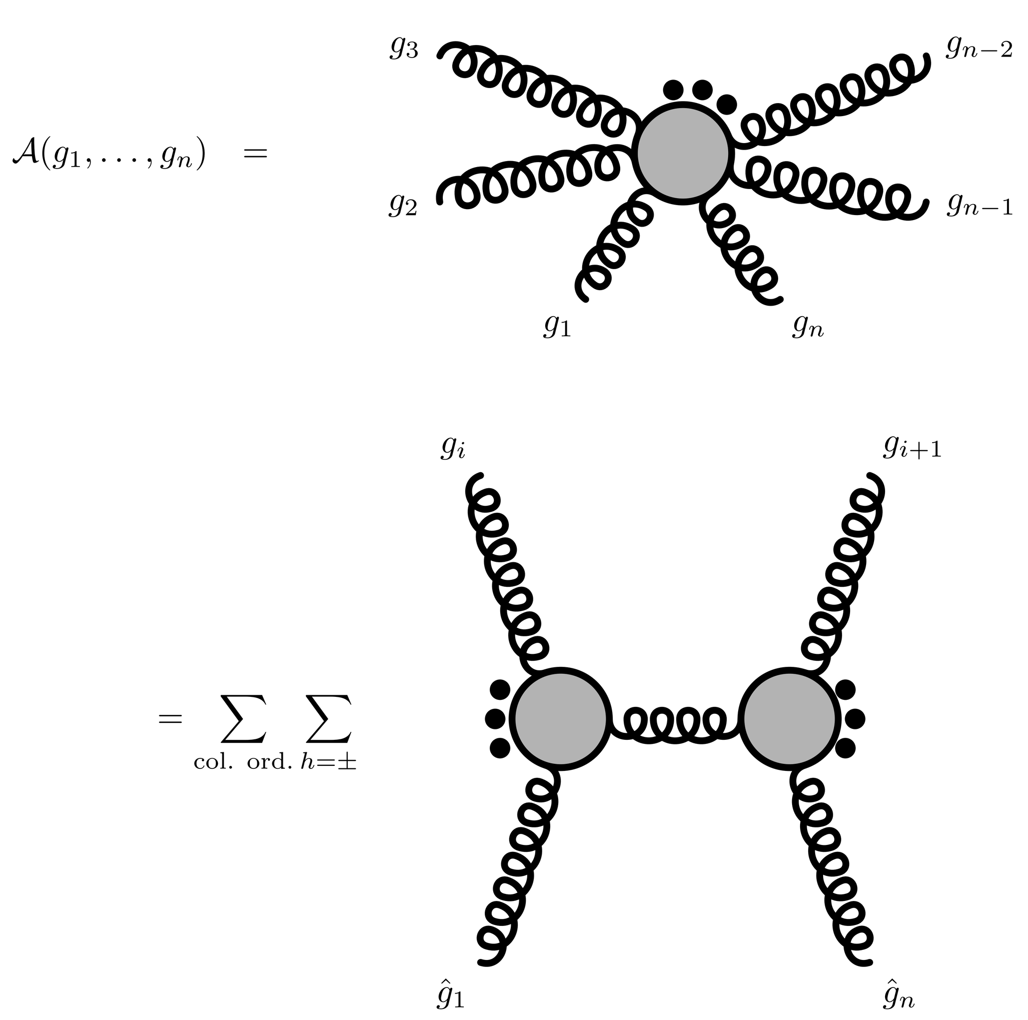
The on-shell BCFW construction

Recursive construction of on-shell tree-level amplitudes in Yang-Mills theory without and with fermion fully accomplished by 2005 and beautifully simple [3, 4].

From the residue theorem,

$$\lim_{z\to\infty} f(z) = 0 \Rightarrow f(0) = -\sum_i Res\left[\frac{f(z)}{z}\right]\Bigg|_{z=z_i}$$

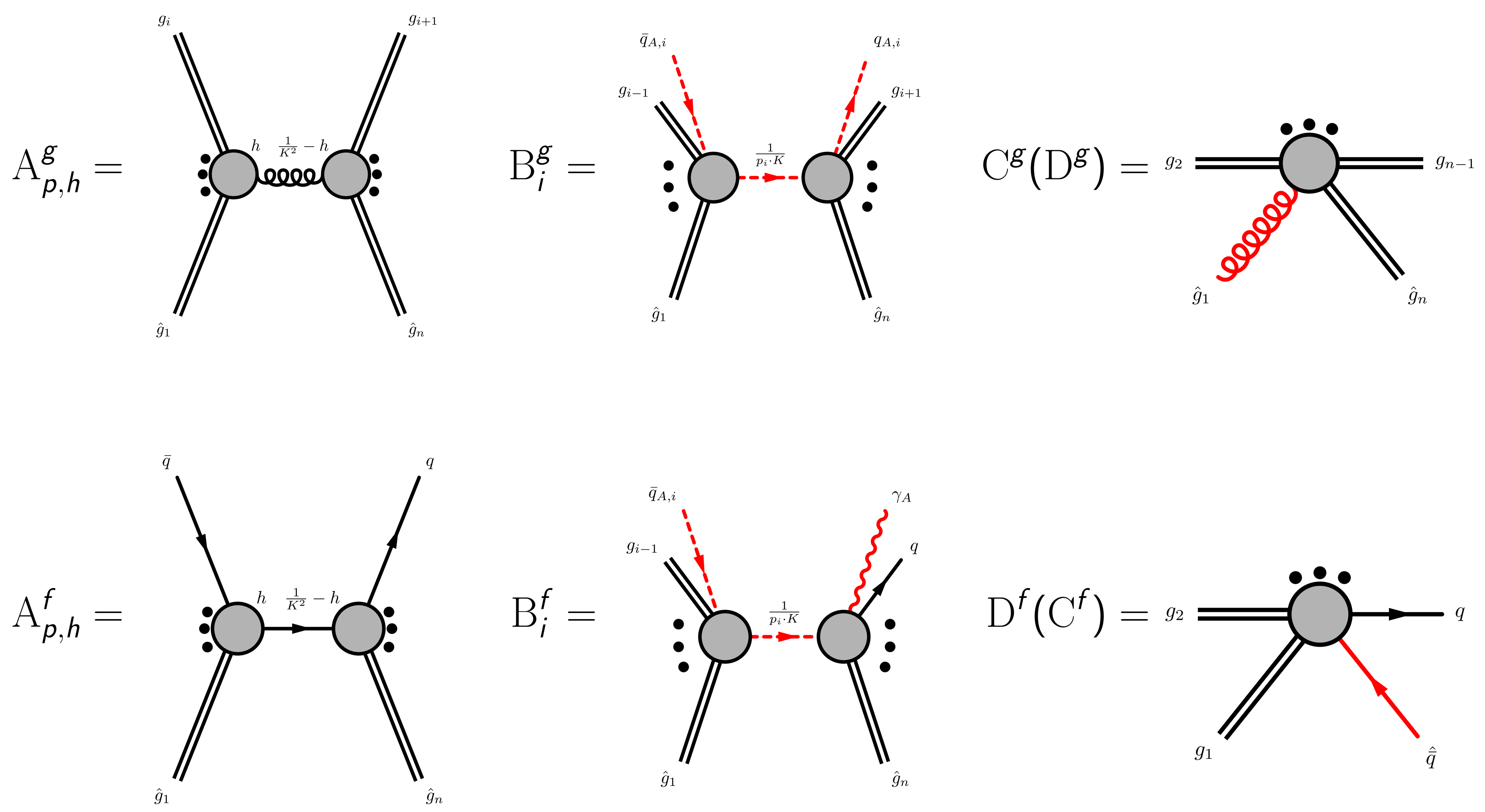
$$\lim_{z\to\infty} \mathcal{A}(p_1 + z\, e^\mu, \ldots, p_n - z\, e^\mu) = 0 \Rightarrow$$



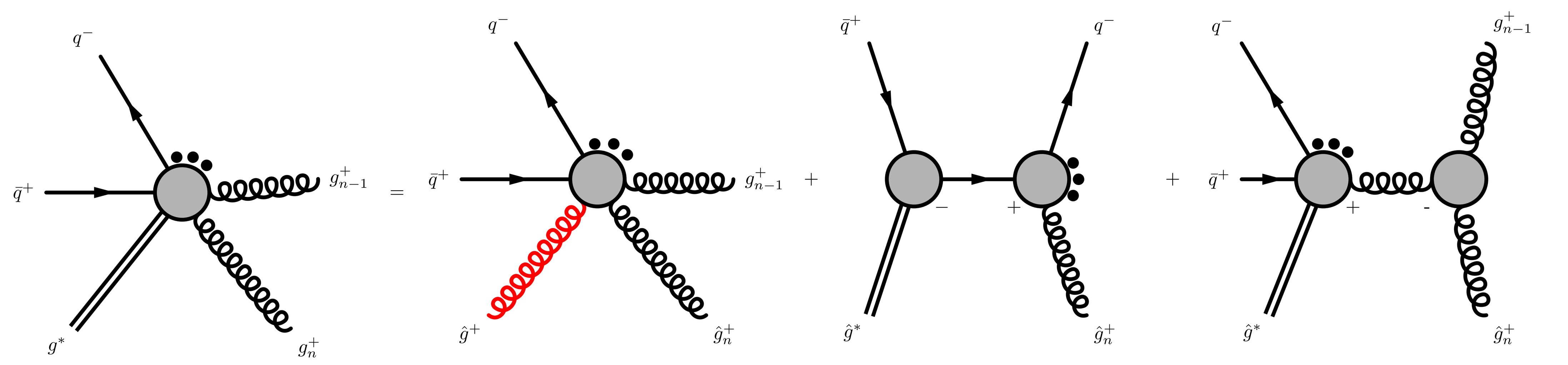
BCFW with off-shell legs in a nutshell

$$\mathcal{A} = \sum_{s=g,f}\left(\sum_p \sum_{h=+,-} \mathrm{A}^s_{p,h} + \sum_i \mathrm{B}^s_i + \mathrm{C}^s + \mathrm{D}^s\right)$$

Off-shell particles ≃ auxiliary eikonal particles pairs ⇔ new poles [5, 6]



Example: the recursion for MHV amplitudes with 1 off-shell gluon



Conclusions

- Conditions for BCFW recursion with off-shell legs fully determined
- All 4 and 5 points scattering amplitudes with 1-off-shell leg computed
- A notebook with all the analytical results is available with the arXiv release of [6]

Perspectives

- A first phenomenological study of 4 jet production in hybrid factorisation is under way
- Computing 4 and 5 legs amplitudes with 2-off-shell legs will be the next analytical step
- A package performing analytical results for up to 2 off-shell legs and any number of particles is on the agenda
- Loop corrections will come next...

Acknowledgements

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Traced energy-momentum tensor correlators in CFTs: solving an infinite hierarchy

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The motivation for a new tool

- The computation of Green functions of n energy-momentum tensors (EMTs) via Feynman diagrams is very demanding already for $n = 3$ [1]
- Traced correlators of the EMT are directly related to the low-energy effective action of CFTs, featuring a dilaton.
- Dilatons recently gathered a lot of interest because of the a-theorem in 4 dimensions [2].

The hierarchy

Every CFT on a curved background is affected by a trace anomaly. In $d = 4$ this is

$$\mathcal{A}[g] = \beta_a \left(F - \frac{2}{3} \Box R \right) + \beta_b \, G$$

The correlators are

$$\langle T^{\mu_1\nu_1}(x_1) \dots T^{\mu_n\nu_n}(x_n) \rangle \equiv 2^n \frac{\delta^n \mathcal{W}[g]}{\delta g_{\mu_1\nu_1} \dots \dots \delta g_{\mu_n\nu_n}} \bigg|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

The entire hierarchy of traced n-point functions is described by the Ward identities

$$\langle T(k_1) \, \dots \, T(k_{n+1}) \rangle = 2^n \frac{\delta^n \mathcal{A}[g]}{\delta g_{\mu_1\nu_1} \dots \dots \delta g_{\mu_n\nu_n}} \bigg|_{g_{\mu\nu}=\eta_{\mu\nu}} - 2 \sum_{i=1}^{n} \langle T(k_1) \dots T(k_{i-1}) T(k_{i+1}) \dots T(k_{n+1} + k_i) \rangle$$

Anomalies and counterterms in CFTs

Trace anomalies on curved backgrounds have a beautifully simple relation to the counterterms of the theory in dimensional regularization

$$\frac{2}{\sqrt{g}} g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} \int d^{4-\epsilon}x \, \sqrt{g} \, F = -\epsilon \left(F - \frac{2}{3} \Box R \right)$$

$$\frac{2}{\sqrt{g}} g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} \int d^{4-\epsilon}x \, \sqrt{g} \, G = -\epsilon \, G$$

The dilaton effective action from Weyl-gauging

The relations above recommend a deeper investigation.

Application of the Weyl-gauging technique [3] to the counterterms via a conformal compensator field (dilaton) yields the Wess-Zumino action for the conformal anomaly:

$$g_{\mu\nu}(x) \to e^{\tau/\Lambda} g_{\mu\nu}(x) \Rightarrow$$

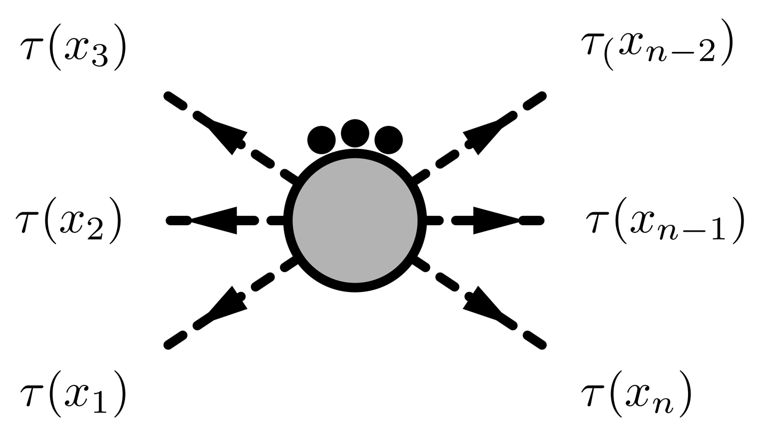
$$\mathcal{W}_{WZ}[\tau] = \int d^4x \left[\beta_a \frac{2}{\Lambda^2} (\Box\tau)^2 + (\beta_a + \beta_b) \left(-\frac{4}{\Lambda^3} (\partial\tau)^2 \Box\tau + \frac{2}{\Lambda^4} (\partial\tau)^4 \right) \right]$$

- **There are no n-dilaton interactions for $n > 4$**
- **For general CFTs in d dimensions n-dilaton interactions are 0 for $n > d$**

How to make the hierarchy completely trivial

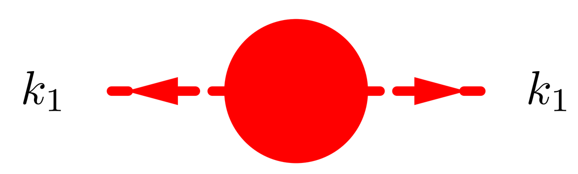
A very simple idea:

1. Expand formally the Wess-Zumino action in $1/\Lambda$ setting the metric to $\hat{g}_{\mu\nu} = \eta_{\mu\nu} \, e^{-\tau/\Lambda}$
2. Require the anomaly-induced effective action to match the perturbative expansion term by term

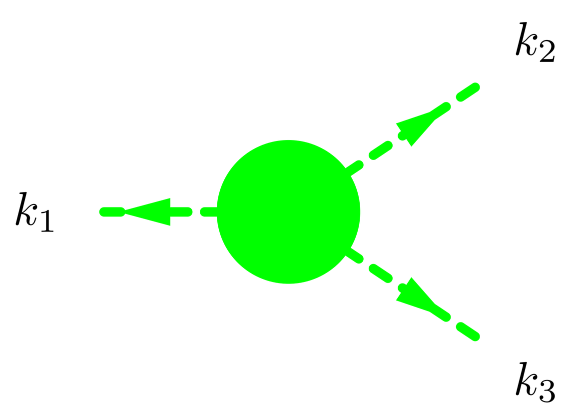


$$= -\frac{\delta \mathcal{W}_{WZ}[\tau]}{\delta\tau(x_1) \dots \delta\tau(x_n)}$$

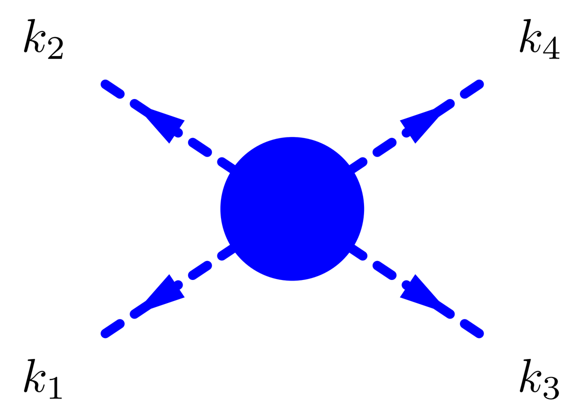
3. Solve recursively the resulting simple linear system



$$= \frac{1}{\Lambda^2} \langle T(k_1) T(-k_1) \rangle$$



$$= -\frac{1}{\Lambda^3} \left[\langle T(k_1) T(k_2) T(k_3) \rangle + 2 \sum_{i=1}^{3} \langle T(k_i) T(-k_i) \rangle \right]$$



$$= \frac{1}{\Lambda^4} \left[\langle T(k_1) T(k_2) T(k_3) T(k_4) \rangle + \dots \right.$$

$$0 = \frac{1}{\Lambda^5} \langle T(k_1) T(k_2) T(k_3) T(k_4) T(k_5) \rangle + \dots$$

The n-th correlator is found by trivially inverting the system up to the n-th equation...just a very small amount of algebra !

Conclusions

- A very efficient, recursive algorithm for the computation of traced correlators of EMTs tensor in CFTs has been developed and thoroughly tested
- Consistency between the anomaly-induced dilaton effective action and the perturbative expansion fixes the full hierarchy of correlators.
- Explicit results for CFTs are available in 2,4 and 6 dimensions for the most general renormalisation scheme [4, 5]

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