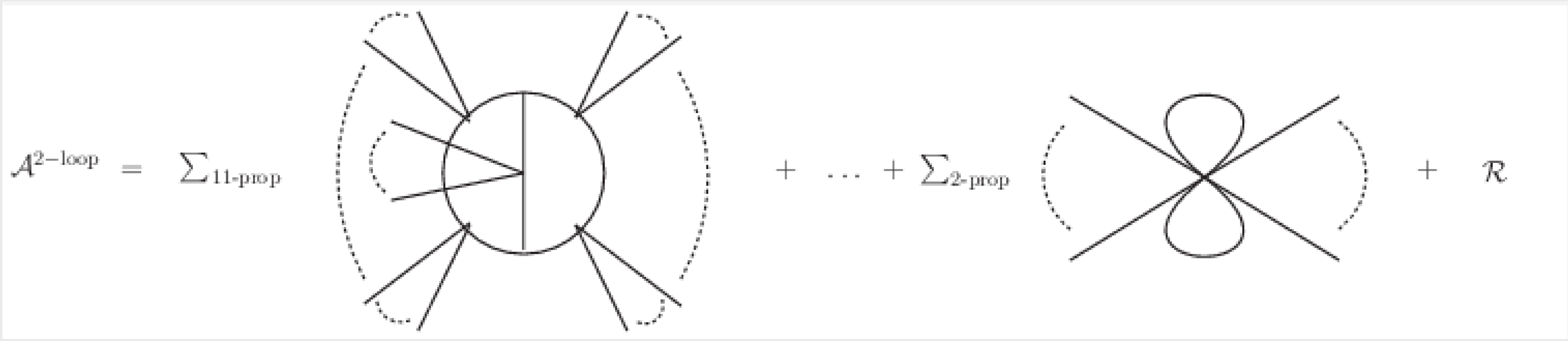
Developments on a massive planar pentabox with a differential equation method

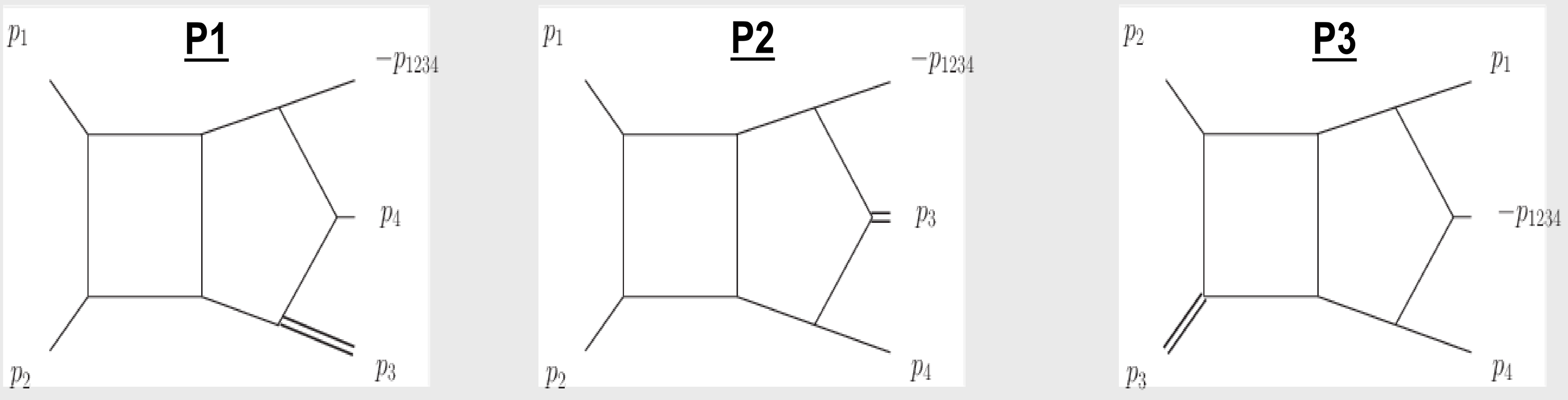
C. Wever, in collaboration with C. Papadopoulos and D. Tommasini
INPP, NCSR Demokritos

1. Motivation, introduction and goal

- Multi-loop calculations required for precision physics
  - NLO automation thanks to on-shell reduction methods
  - Next step: NNLO automation
- A finite basis of Master Integrals exists at two-loops:



  - Reduction to MI used for specific processes: *Integration by parts* (IBP) [1]
  - <u>Missing ingredient: library of Master integrals (MI)</u>
- Interested next in two-loop, five-point diagrams with <u>*one*</u> external mass and massless propagators
  - Relevant e.g. for virtual-virtual contribution to $2 \to 3$ LHC processes such as $H+2j, V+2j, Vb\bar{b}$ (Les Houches Wishlist) at NNLO QCD
  - Three planar families:



<u>Goal</u>: compute all planar five-point MI with one external mass and massless internal propagators

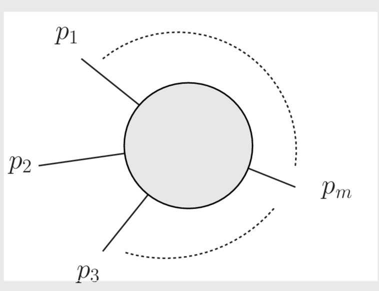
2. Theoretical basis: Simplified Differential Equations (SDE)

- Introduce auxiliary x in the denominators of loop integral [2]
- x-parameter describes off-shellness of (some) external legs:

<u>Massive</u> legs: p_1 —— x-parametrize ——→ $p_1 + (1-x)q$

<u>Massless</u> legs: p_2 ←—— p_2 or xp_2 ($x = 1$)

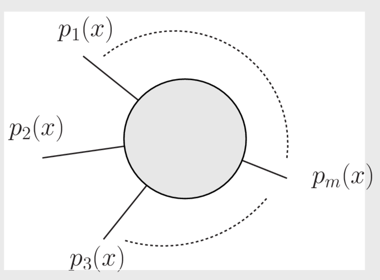
<u>General case:</u>



$$G_{a_1\cdots a_n}(s,\epsilon) = \int \Big(\prod_i d^d k_i\Big) \frac{1}{D_1^{2a_1}(k,p)\cdots D_n^{2a_n}(k,p)}$$

$x = 1$ ↑ ↓ x-parametrize

$$p_i(x) = p_i + (1-x)q_i \qquad \sum_i q_i = 0$$



$$G_{a_1\cdots a_n}(x,s,\epsilon) = \int \Big(\prod_i d^d k_i\Big) \frac{1}{D_1^{2a_1}(k,p(x))\cdots D_n^{2a_n}(k,p(x))}$$

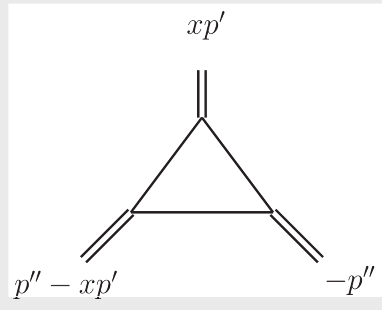
$$D_i(k,p) = c_{ij}k_j + d_{ij}p_j, \qquad s = \{p_i.p_j\}|_{i,j}$$

$$\frac{\partial}{\partial x}\vec{G}^{MI}(x,s,\epsilon) \overset{IBP}{=} \overline{\overline{M}}(x,s,\epsilon).\vec{G}^{MI}(x,s,\epsilon), \qquad s = \{p_i.p_j\}|_{i,j}$$

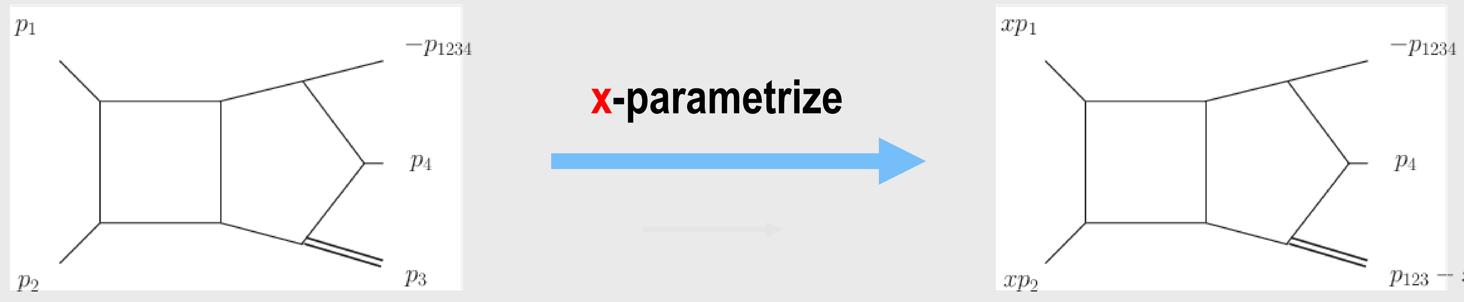
3. SDE method for the planar pentabox

Main criteria for choice of x-parametrization: *require Goncharov Polylog (GP) solution for DE*

In practice enough to choose the external legs such that the corresponding massive MI triangles (found by pinching external legs) are as follows:



x-parametrization for P1 family (74 MI in total):



$p_1^2 = p_2^2 = p_4^2 = p_{1234}^2 = 0, \quad p_3^2 \neq 0$ $\qquad$ $p_1^2 = p_2^2 = p_3^2 = p_4^2 = p_{1234}^2 = 0$

$$G^{(P1)}_{a_1\cdots a_{11}}(x) := \int \frac{d^dk_1}{i\pi^{d/2}}\frac{d^dk_2}{i\pi^{d/2}} \frac{1}{k_1^{2a_1}(k_1+xp_1)^{2a_2}(k_1+xp_{12})^{2a_3}(k_1+p_{123})^{2a_4}(k_1+p_{1234})^{2a_5}}$$
$$\times \frac{1}{k_2^{2a_6}(k_2-xp_1)^{2a_7}(k_2-xp_{12})^{2a_8}(k_2-p_{123})^{2a_9}(k_2-p_{1234})^{2a_{10}}(k_1+k_2)^{2a_{11}}}$$

- DE for P1 are known and integration underway in terms of GP's
- Reduction for P2 done (75 MI in total), P3 underway (bottleneck)

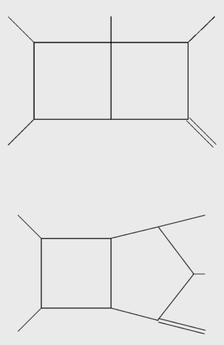
<u>Boundary term:</u> Integrands contain *branch points* or *poles* at $x = \{x_1, x_2, \ldots, \infty\}$ of form $(x - x_i)^{m+n\epsilon}$

<u>Observation:</u> **Boundary term always captured by integration from $x = 0$ or appropriate x_i**

- Massless $x = 1$ limit captured by resumming logs of $(1 - x)$

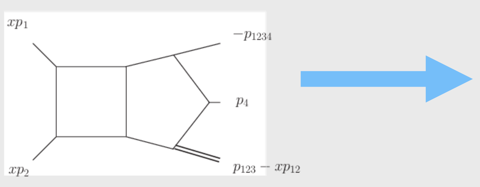
4. Preliminary results for P1 and Outlook

Remainders:



P1	ε^{-4}	ε^{-3}	ε^{-2}	ε^{-1}	ε^{0}
$G_{111000-11111}$	✓	✓	✓	✓	✓
$G_{11100001111}$	✓	✓	✓	✓	✓
$G_{111-10001111}$	✓	✓	✓	✓	
$G_{111001-11111}$	✓	✓	✓		
$G_{11100101111}$	✓	✓	✓		
$G_{111-10101111}$	✓	✓			

- Solutions expressed in terms of GPs with argument x:



$$G^{(P1)}_{11100101111}(x,s,\epsilon) = \frac{C_0(s,\epsilon)}{x^4(s_{12}(x-1)(x(-s_{23}+s_{45}+s_{51})-s_{45})+s_{34}(s_{45}(x-1)-s_{23}x)-s_{45}s_{51}(x-1))}$$
$$\times \Big\{x^{-4\epsilon}C_1(x,s,\epsilon)+x^{-3\epsilon}C_2(x,s,\epsilon)+\frac{C_3(x,s)}{\epsilon^4}+ \frac{1}{\epsilon^3}\left(GP(0;x)C_4(x,s)+GP(1;x)C_5(x,s)+GP\left(s_{45}/s_{12};x\right)C_6(x,s)\cdots\right)+\cdots\Big\}$$

In *Euclidean region* agreement with SecDec [3]:

$$x = 1/13, \; s_{12} = -2, \; s_{23} = -3, \; s_{34} = -5, \; s_{45} = -7, \; s_{51} = -11$$

Analytical: $G^{(P1)}_{11100101111} = \frac{1307.56}{\epsilon^4} + \frac{7834.53}{\epsilon^3} + \frac{22985.4}{\epsilon^2} + \frac{\cdots}{\epsilon} + \cdots + \mathcal{O}(\epsilon)$

SecDec: $G^{(P1)}_{11100101111} = \frac{1307.56}{\epsilon^4} + \frac{7833.34}{\epsilon^3} + \frac{22972.4}{\epsilon^2} + \frac{59772.6}{\epsilon} + 186628 + \mathcal{O}(\epsilon)$

<u>Outlook and Summary:</u>

- In progress: two-loop pentaboxes with one massive leg
- SDE method captures boundary terms by choosing the boundary at an appropriate branch point or pole

5. References

1. Tkachov '81, Chetyrkin & Tkachov '81
2. Papadopoulos '14, Papadopoulos, Tommasini, CW '14
3. Borowka, Heinrich et al '11-'15