On-shell Diagrams, Graßmannians and Integrability for Form Factors

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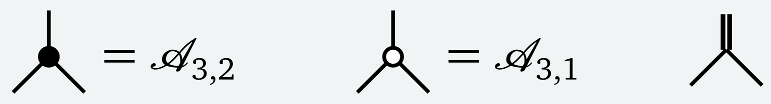
We show that tree level form factors in planar $\mathcal{N}=4$ SYM exhibit many of the interesting structures discovered during the last years for amplitudes.

Form factors of the chiral stress tensor multiplet:

$T(x,\theta^+) = \mathrm{tr}(\phi^{++}\phi^{++}) + \cdots + \frac{1}{3}(\theta^+)^4 \mathscr{L}$

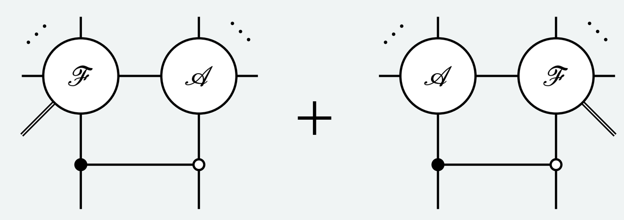
Building blocks:



Minimal form factor [1]

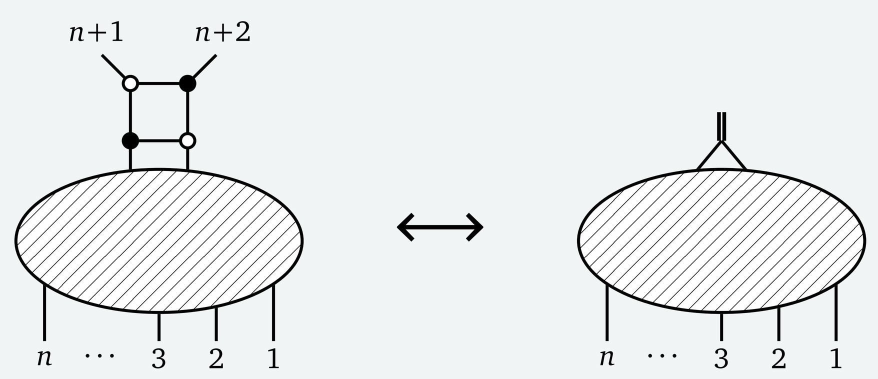
$$\mathcal{F}_{2,2} = \frac{1}{\langle 12\rangle\langle 21\rangle}\delta^4(\lambda_1\tilde\lambda_1 + \lambda_2\tilde\lambda_2 - q)\,\delta^4(\lambda_1\tilde\eta_1^+ + \lambda_2\tilde\eta_2^+)\,\delta^4(\lambda_1\tilde\eta_1^- + \lambda_2\tilde\eta_2^- - \gamma^-)$$

BCFW recursion relations [1]:



On-shell diagrams

From amplitude to form factor diagrams by replacing box with minimal form factor



– works for BCFW terms
– all examples we studied: works for the top-cell [2]
We need to sum over cyclic permutations

Top-cell diagrams

Kinematics: off-shell (super) momentum encoded in two on-shell momenta:

$$\lambda_{n+1}\tilde\lambda_{n+1} + \lambda_{n+2}\tilde\lambda_{n+2} = -q$$
$$\lambda_{n+1}\tilde\eta_{n+1} + \lambda_{n+2}\tilde\eta_{n+2} = -\gamma$$

Graßmannian $G(k,n+2)$ [2]

$$\sim \int \frac{d^{k\times(n+2)}C}{\mathrm{Vol}[GL(k)]}\,\Omega_{n,k}\;\delta^{2\times k}(C\cdot\tilde\lambda)\,\delta^{4\times k}(C\cdot\tilde\eta)\,\delta^{2\times(n+2-k)}(C^\perp\cdot\lambda)$$

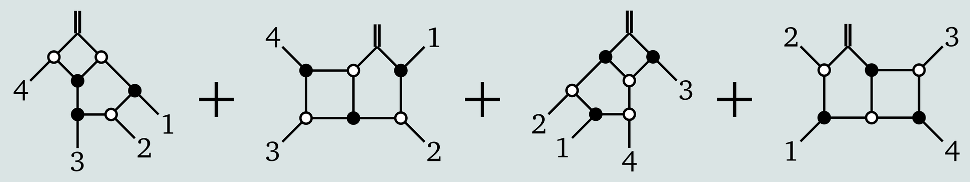
Form:

$$\Omega_{n,k} = \frac{Y\,(1-Y)^{-1}}{(1\cdots k)\cdots(n\cdots k-3)(n+1\cdots k-2)(n+2\cdots k-1)} + \text{cyclic}$$

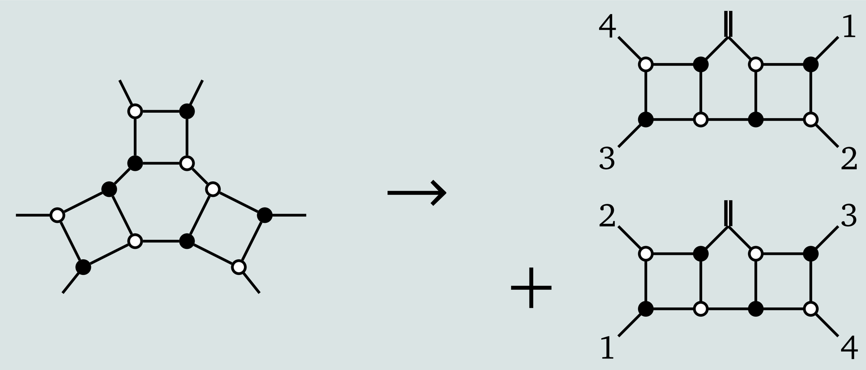
$$Y = \frac{(n-k+2\cdots n\,n+1)(n+2\,1\cdots k-1)}{(n-k+2\cdots n\,n+2)(n+1\,1\cdots k-1)}$$

Graßmannian integrals

$$\mathcal{F}_{4,3} =$$



→



→

$$\int \frac{d^{3\times(4+2)}C}{\mathrm{Vol}[GL(3)]}\,\delta^6(C\cdot\tilde\lambda)\,\delta^{12}(C\cdot\tilde\eta)\,\delta^6(C^\perp\cdot\lambda)$$
$$\times\frac{\frac{(345)(612)}{(346)(512)}\left(1-\frac{(345)(612)}{(346)(512)}\right)^{-1}}{(123)(234)(345)(456)(561)(612)}$$

↓

$$\int \frac{d^{1\times(4+2)}D}{\mathrm{Vol}[GL(1)]}\,\frac{\frac{(5)(6)}{(1)(4)}\left(1-\frac{(5)(6)}{(1)(4)}\right)^{-1}}{(1)(2)(3)(4)(5)(6)}\;\delta^{4|4}(D\cdot\mathscr{Z})$$

←

$$\frac{\mathcal{F}_{4,3}}{\mathcal{F}_{4,2}} = [12345] - \frac{1}{1+\frac{\langle1346\rangle\langle1345\rangle}{\langle3456\rangle\langle1356\rangle}}[13456]$$
$$+[12345]^{\text{shifted by 2}} - \left[\frac{1}{1+\frac{\langle1346\rangle\langle1345\rangle}{\langle3456\rangle\langle1356\rangle}}[13456]\right]^{\text{shifted by 2}}$$

Residues

Residues of the Graßmannian integral form give BCFW terms

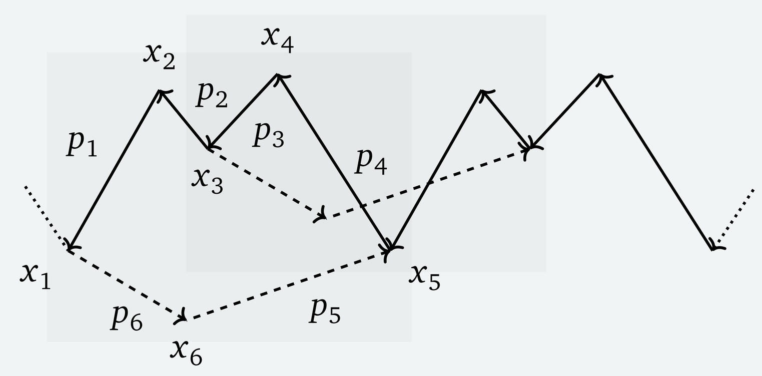
Checks:
– all MHV form factors
– NMHV: 3, 4 and 5 points
– NNMHV: 4 points

Momentum twistor space

Graßmannian integral can be transformed to momentum twistor space [3]

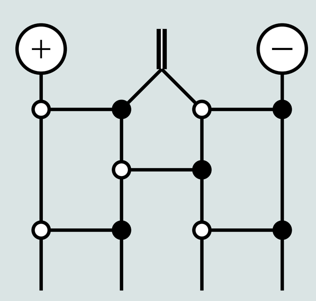
Graßmannian $G(k-2,n+2)$

Instead of summing over shifted forms, shift kinematics along periodic configuration:



Integrability

$\sigma = (4231) =$
$(12)(34)(23)(12)(34)$



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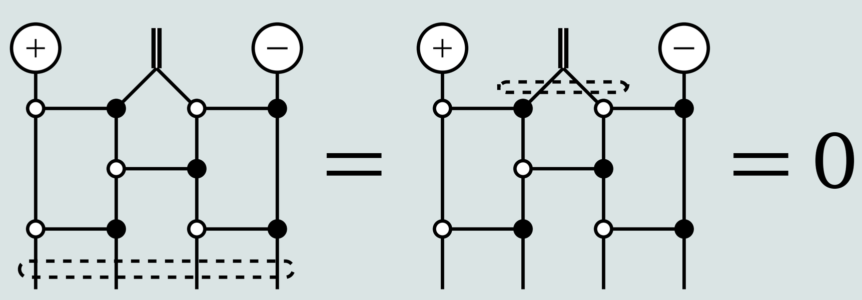

R operators & deformations

Construction via R operators [4] allows to introduce deformations

$$\mathrm{R}_{ij}(u) = \left(\mathscr{W}_j\cdot\frac{\partial}{\partial\mathscr{W}_i}\right)^u \;\sim\; \text{deformed BCFW bridge}$$

Minimal form factor acts as a vacuum state

Example: MHV three-point

$$\mathrm{R}_{23}(u_{32})\,\mathrm{R}_{12}(u_{31})\,\delta_1^+\,\mathcal{F}_{2,2}(2,3) = \frac{\delta^4(P)\,\delta^4(Q^+)\,\delta^4(Q^-)}{\langle12\rangle^{1-u_{23}}\langle23\rangle^{1-u_{31}}\langle31\rangle^{1-u_{12}}}$$

Transfer matrix identities

Amplitudes are Yangian invariant [5]:

$$\mathscr{M}(u)\;\mathscr{A} = \mathscr{A}$$

All form factors of the chiral stress tensor multiplet are annihilated by the transfer matrix $\mathscr{T} = \mathrm{str}\,\mathscr{M}$:

$$\mathscr{T}(u)\;\mathcal{F} = 0$$

All planar on-shell diagrams glued together with the minimal form factor of an arbitrary operator $\mathcal{O}$ (→ leading singularities) are eigenstates of the transfer matrix, if the operator is an eigenstate of the integrable model:

$$\mathscr{T}(u)\;\mathcal{F}_{\mathcal{O}} = \mathcal{F}_{\mathscr{T}(u)\,\mathcal{O}} = \tau(u)\;\mathcal{F}_{\mathcal{O}}$$

Selected references

[1] Brandhuber, Gurdogan, Mooney, Travaglini, Yang, 1107.5067
[2] Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka, 1212.5605
[3] Mason, Skinner, 0909.0250
[4] Chicherin, Derkachov, Kirschner, 1309.5748; Kanning, Lukowski, Staudacher, 1403.3382; Broedel, de Leeuw, Rosso, 1403.3670
[5] Drummond, Henn, Plefka, 0902.2987; Frassek, Kanning, Ko, Staudacher, 1312.1693

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