Numerical evaluation of multi-scale integrals with SecDec 3

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MC@NNLO

Project in collaboration with
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1502.06595 [hep-ph] (CPC, in press)

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http://secdec.hepforge.org/

Precise theoretical predictions in the LHC era

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- Numerical methods are in general easier to automate, multiple scales do not pose a problem
  $\rightarrow$ highly interesting in light of the current need for predictions involving massive particles!

Numerical evaluation of Feynman integrals

Many people are/have been working on PURELY numerical methods, e.g. Anastasiou/Beerli/Kunszt et al., Becker/Reuschle/Weinzierl et al., Binoth/Heinrich et al., Boughezal/Melnikov/Petriello et al., Czakon et al., Freitas et al., Kurihara et al., Nagy/Soper et al., Passarino et al., ...

- Problems beyond the one-loop level mainly are
  - Extraction of IR and UV singularities
  - Numerical convergence in the presence of integrable singularities (e.g. thresholds)
  - Speed / accuracy

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- Problems beyond the one-loop level mainly are
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- Problems beyond the one-loop level mainly are
  - Extraction of IR and UV singularities (solved with SecDec 1)
  - Numerical convergence in the presence of integrable singularities (e.g. thresholds) (solved with SecDec 2)
  - Speed / accuracy (further improved in SecDec 3)

IR and UV singularity extraction beyond 1-loop

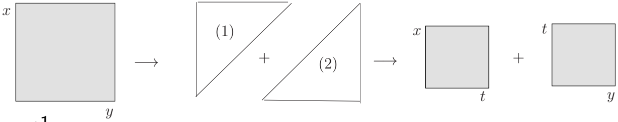
Diverse methods have been worked out

- R*-operation Chetyrkin, Tkachov, V.A.Smirnov '70s, '80s
- Polynomial exponent raising Tkachov '96, Passarino '00
- Sector decomposition Binoth & Heinrich '00
- Computation of residues within Mellin-Barnes representation Anastasiou, Daleo '06; Czakon '06
- Subtraction terms Freitas '12; Becker, Weinzierl '12
- Quasi-finite basis Panzer '14; Manteuffel, Schabinger, Panzer '14

+ important other works on UV renormalization Bogoliubov, Parasiuk, Hepp, Zimmermann, Broadhurst, Kreimer, Connes,...

The method of sector decomposition

- Idea and method of sector decomposition pioneered by
  Hepp '66, Denner & Roth '96, Binoth & Heinrich '00



$$\int_0^1 dx_1 \int_0^1 dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}}$$
$$= \int_0^1 dx_1 \int_0^1 dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}} \left(\theta(x_1-x_2)+\theta(x_2-x_1)\right)$$
$$= \int_0^1 dx_1 \int_0^{x_1} dx_2 \; \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^{x_2} dx_1 \; \frac{1}{(x_1+x_2)^{2+\epsilon}}$$
$$= \int_0^1 dx_1 \int_0^1 dt \; \frac{x_1}{(x_1+x_1 t)^{2+\epsilon}} + \int_0^1 dx_2 \int_0^1 d\tilde{t} \; \frac{1}{x_2^{1+\epsilon}(\tilde{t}+1)^{2+\epsilon}}$$

- iterative sector decomposition is highly automatable

Public codes using the sector decomposition method

Public codes:

- sector_decomposition (uses GiNaC) Bogner & Weinzierl '07
  supplemented with `CSectors` Gluza, Kajda, Riemann, Yundin '10
  for construction of integrand in terms of Feynman parameters
- FIESTA* (uses Mathematica, C) A.V. Smirnov, V.A. Smirnov, Tentyukov '08 '09, A.V. Smirnov '13
- SecDec* (uses Mathematica, Fortran/C++)
  Carter & Heinrich '10; SB, Carter, Heinrich '12; SB & Heinrich '13;
  SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

*Multi-scale integrals not limited to the Euclidean region
SB, J. Carter & G. Heinrich '12; A.V. Smirnov '13

SecDec 3 can tackle ...

Feynman graph | or | parametric function

SecDec is a tool to numerically compute

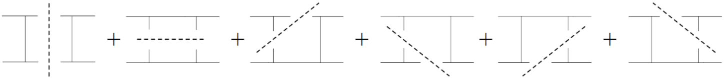
- General **Feynman** integrals for **arbitrary** kinematics and with numerators
- Integrals **matching** a Feynman integral **structure**
- More general **parametric** functions

Feynman loop integrals

- Scalar multi-loop integral in Feynman parametrization

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N} x_l)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x},s_{ij})}$$

  with $N_\nu = \sum_{j=1}^{N}\nu_j$ in D dimensions with L loops, N propagators to power ν_j
- Feynman integrals with (contracted) numerators of rank R
- $\mathcal{U}$ and $\mathcal{F}$ can be constructed via **topological cuts** or by specifying the individual propagators in momentum space



$$\mathcal{F} = -s_{12}\,x_1x_3 - s_{23}\,x_2x_4 - p_1^2\,x_1x_2 - p_2^2\,x_2x_3 - p_3^2\,x_3x_4 - p_4^2\,x_4x_1\,.$$

Modified Feynman loop integrals

More general **user-defined polynomial integrals** matching the Feynman loop integral structure

$$G_{user} = P(\varepsilon) \int_0^1 \prod_{j=1}^{N} \mathrm{d}x_j \; {x_j}^{a_j(\varepsilon)} \, \mathcal{N}(\vec{x}, s_{ij}, \varepsilon) \, \mathcal{U}^{\mathrm{expoU}(\varepsilon)}(\vec{x}, s_{ij}) \, \mathcal{F}^{\mathrm{expoF}(\varepsilon)}(\vec{x}, s_{ij})$$

with a prefactor P and a numerator function $\mathcal{N}$ and exponents a_j

- $\mathcal{U}$ and $\mathcal{F}$ can have negative exponents, also $a_j < 0$ allowed
- integrals without δ-constraint
- $\mathcal{F}$ is used for construction of deformation of the integration contour in the physical region
- user has more responsibility when using deformation of integration contour

Multi-dimensional parameter integrals

A general parametric function can be

- a phase space integral where IR divergences are regulated dimensionally, e.g.

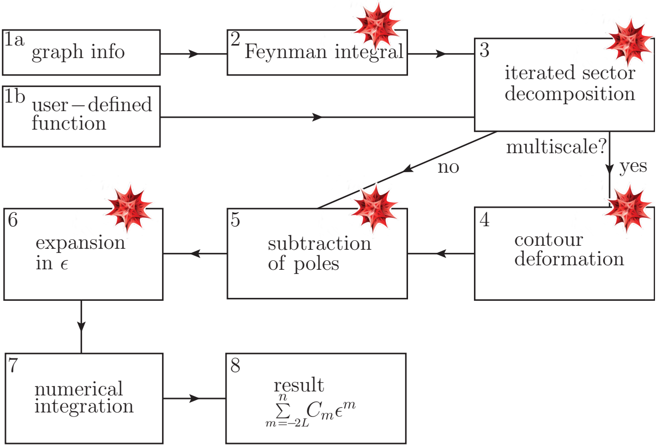
$$\int d\Phi^D |\mathrm{ME}|^2 \propto \int ds_{13}\, ds_{23}\, s_{13}^{-1-\varepsilon}\, \frac{\mathcal{F}(s_{13}, s_{23})}{s_{13}+s_{23}}$$
$$\to \int_0^1 \mathrm{d}x\, \mathrm{d}y\, x^{-1-\varepsilon}\, \frac{\mathcal{F}(x,y)}{x+y}$$

- functions of the type of hypergeometric functions, e.g.

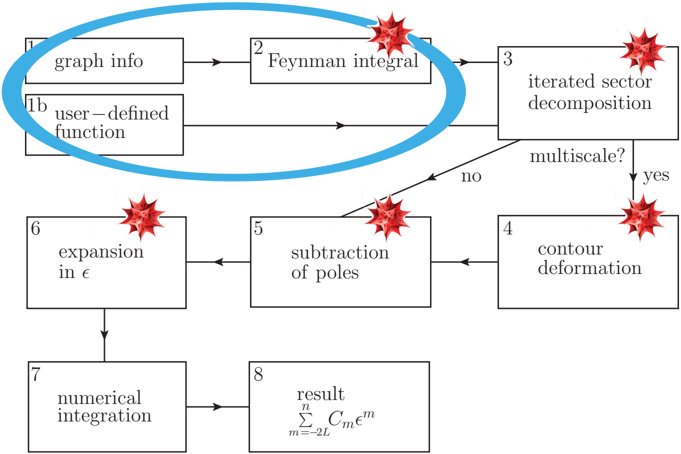
$${}_3F_2(a_1,..., a_3; b_1, b_2; \beta) \propto$$
$$\iint_0^1 \mathrm{d}x\mathrm{d}y\; x^{a_1-1}(1-x)^{b_1-a_1-1} y^{a_2-1}(1-y)^{b_2-a_2-1}(1-\beta xy)^{-a_3}$$

- NEW in SecDec 3: additional ε-dependent functions $g(\varepsilon, \vec{x})$ can be included (no iterated sector decomposition applied)

The program - Outline



Operational sequence of the SecDec 3 program



Improved user interface in SecDec 3

SecDec needs 3 input files:

- param.input: minimal info needed to run SecDec

```
graph=Box2L
epsord=0
```

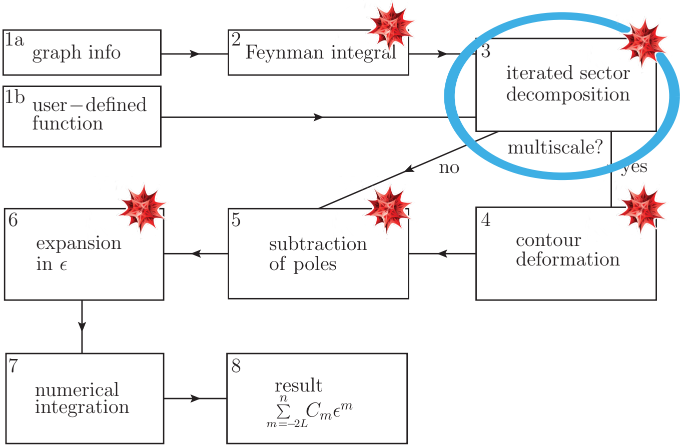
- kinem.input: contains point name and numerical values for kinematics

```
p1 -3 -2
p2 1 4
```

- math.m: graph definition, enhanced flexibility

```
momlist={k1,k2};
proplist={ k1^2, (k1+p2)^2,
           (k1-p1)^2, (k1-k2)^2,
           (k2+p2)^2, (k2-p1)^2,
           (k2+p2+p3)^2, (k1+p3)^2 };
powerlist={1,1,1,1,1,1,1,-1};
ExternalMomenta={p1,p2,p3,p4};
externallegs=4;
prefactor=Gamma[1+eps]^2;
KinematicInvariants = {s,t};
Masses={};
ScalarProductRules = {
  SP[p1,p1]->0,
  SP[p2,p2]->0,
  SP[p3,p3]->0,
  SP[p4,p4]->0,
  SP[p1,p2]->s/2,
  SP[p2,p3]->t/2,
  SP[p1,p3]->-s/2-t/2
};
Dim=4-2*eps;
```

Operational sequence of the SecDec 3 program



Sector decomposition algorithms

- Iteration of the sector decomposition leads to extraction of IR and UV divergences
- Heuristic algorithm (Binoth & Heinrich '00, strategy `X`) so far most efficient one, included since SecDec-1.0
  - Infinite recursion may appear
- Other strategies avoid infinite recursion
  Bogner, Weinzierl '07 '08, A. Smirnov, Tentyukov '08, Kaneko, Ueda '09 '10
  - For complicated examples: lead to more decomposed sectors or functions of higher complexity

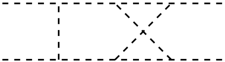
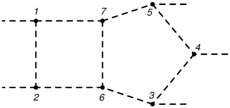
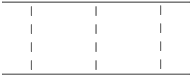
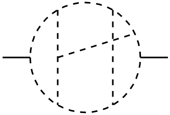
NEW in SecDec-3

- Geometric strategy by Kaneko and Ueda (`G1`)
- Geometric strategy Kaneko and Ueda combined with Cheng-Wu theorem (`G2`)

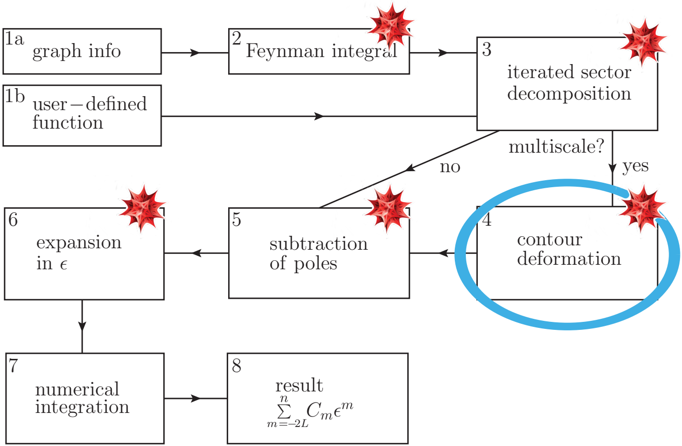
Sector decomposition strategy `G2`

- Exploits Cheng-Wu theorem Cheng, Wu '87: resolve δ-constraint of Feynman integral for one variable, integrate all other Feynman parameters to infinity
- Then perform the sector decomposition using computational geometry Kaneko, Ueda '09 '10
  - Calculate Newton polytope of $\mathcal{F} \times \mathcal{U} \times \mathcal{N}$
  - If vertex lies in more than $N-1$ facets of the polytope, a triangulation is performed (with NORMALIZ Bruns, Ichim, Roemer, Soeger)
  - Perform a change of variables to map upper bound of $N-1$ Feynman parameters to 1

Comparison of decomposition strategies

Diagram	Strategy X	Strategy G1	Strategy G2
	282 sectors 1 s	266 sectors 8 s	166 sectors 4 s
	368 sectors 1 s	360 sectors 9 s	235 sectors 5 s
	548 sectors 3 s	506 sectors 15 s	304 sectors 4 s
	infinite recursion	72 sectors 5 s	76 sectors 1 s
	27336 sectrs 5510 s	32063 sectrs 11856 s	27137 sectrs 443 s

Operational sequence of the SecDec 3 program

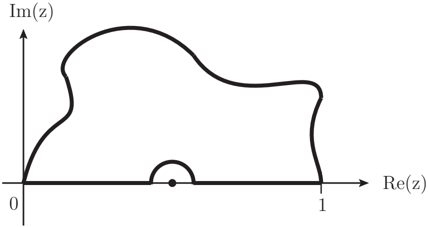


Extension to physical kinematics

- For kinematics in the physical region, $\mathcal{F}$ can still vanish after sector decomposition

$$\mathcal{F}_{Bubble} = -s\, t_1(1 - t_1) + m^2 - i\delta$$

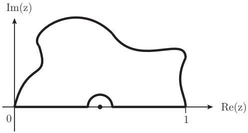
but a deformation of the integration contour



and Cauchy's theorem can help

$$\oint_c f(t)\mathrm{d}t = \int_0^1 f(t)\mathrm{d}t + \int_1^0 \frac{\partial z(t)}{\partial t} f(z(t))\mathrm{d}t = 0$$

Deformation of the integration contour to integrate mass thresholds



- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i}\sum_j y_j(\vec{t})\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

- The integration contour is deformed by

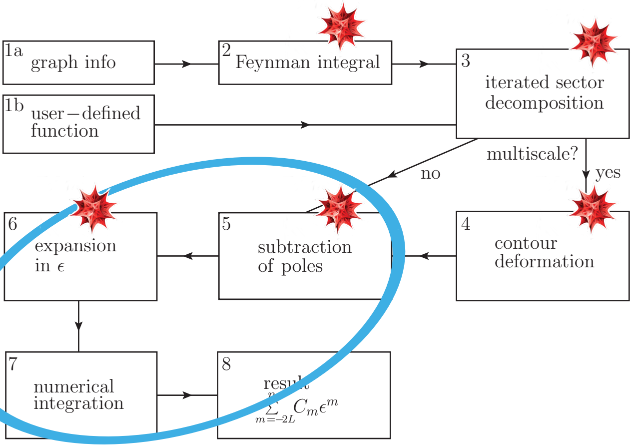
$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y} \ ,$$

$$y_j(\vec{t}) = -\lambda t_j(1 - t_j)\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

Soper '99

Soper, Nagy; Binoth; Anastasiou/Beerli/Kunszt et al., Kurihara et al., Freitas et al., Becker/Reuschle/Weinzierl et al.

Operational sequence of the SecDec 3 program



Subtraction, Expansion, Numerical Integration

Subtraction

- The factorized poles in a subsector integrand $\mathcal{I} \propto \mathcal{U}, \mathcal{F}$ are extracted by subtraction (e.g. logarithmic divergence)

$$\int_0^1 dt_j\, t_j^{-1-b_j\epsilon}\mathcal{I}(t_j,\epsilon) = -\frac{\mathcal{I}(0,\epsilon)}{b_j\epsilon} + \int_0^1 dt_j\, t_j^{-1-b_j\epsilon}(\mathcal{I}(t_j,\epsilon) - \mathcal{I}(0,\epsilon))$$

Expansion

- After the extraction of poles, an expansion in the regulator ϵ is done

Numerical Integration with

- Integrators in CUBA-4 library Hahn et al. '04 - '15
- BASES Kawabata '95

NEW in SecDec 3:

CQUAD Gonnet '10 (fastest for 1-dim), NINTEGRATE Wolfram Research

Summary of new features in SecDec version 3

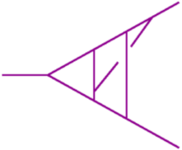
SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

- Two additional **decomposition algorithms** based on computational geometry (avoid infinite recursion)
- Numerators can be given in terms of **inverse propagators**
- **Linear** propagators can be treated
- ε-dependent symbolic functions allowed in parametric integrals
- Restructured **user input** helps interfacing with reduction programs
- 2 new **integrators** included CQuad, NIntegrate
- Usage of batch systems facilitated, scans over parameter ranges accelerated
- Internal **structure** largely **rewritten**

SecDec is ready for large-scale applications!

Download SecDec 3

http://secdec.hepforge.org/



SecDec

Sophia Borowka, Gudrun Heinrich, Stephen Jones, Matthias Kerner, Johannes Schlenk, Tom Zirke

A program to evaluate dimensionally regulated parameter integrals numerically

home   download program   user manual   faq   changelog

NEW: Version 3.0 of the program can be downloaded as SecDec-3.0.7.tar.gz.

Install SecDec 3

- **Install**:
  tar xzvf SecDec-3.0.7.tar.gz
  cd SecDec-3.0.7
  make
  (make check)

- **Prerequisites**:
  Mathematica (version 7 or above), Perl, Fortran and/or C++ compiler, NORMALIZ Bruns, Ichim, Roemer, Soeger for usage of geometric decomposition strategies

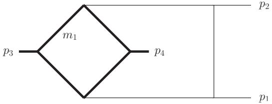
Selection of applications - Outline

1) Master integrals
2) Large(r)-scale applications
3) Miscellaneous

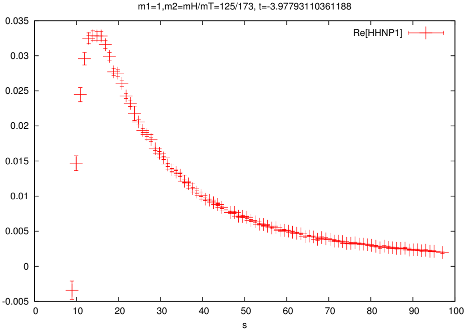
Master Integrals

Non-planar 2L box with 2 mass scales

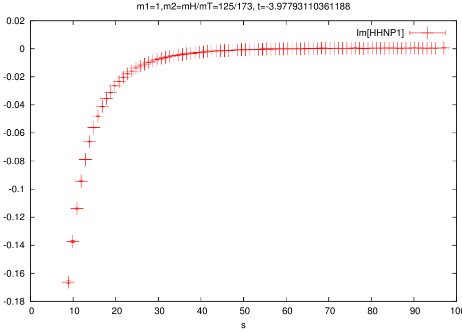
$$s = (p_1 + p_2)^2$$



$m_1^2 = 1,\ m_2^2 = 0.522,\ p_3^2 = p_4^2 = m_2^2,\ p_1^2 = p_2^2 = 0,\ t = -3.978$



Real part



Imaginary part

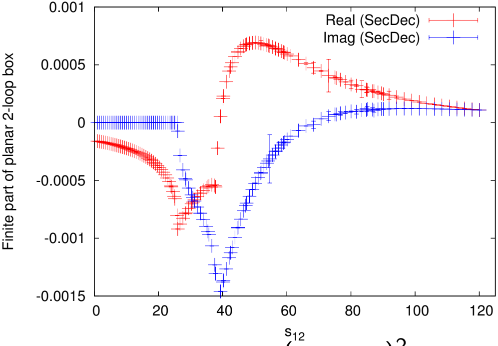
timings: 16-42 secs (CPU time),
rel. accuracy: 10^{-3}, abs. accuracy: 10^{-5}

SB & Heinrich Nov '14

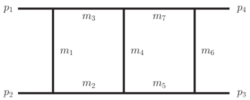
All-massive planar 7-propagator 2L box

- 13 independent mass scales, full numerical approach
  $\Rightarrow$ Many scales are not a bottleneck

Finite part



$s_{12} = (p_1 + p_2)^2$



$m_1^2 = 2,\ m_2^2 = 6,$
$m_3^2 = 7,\ m_4^2 = 8,$
$m_5^2 = 9, m_6^2 = 10,$
$m_7^2 = 12,\ p_1^2 = 1,$
$p_2^2 = 3,\ \ p_3^2 = 4,$
$p_4^2 = 5, s_{23} = -0.25$

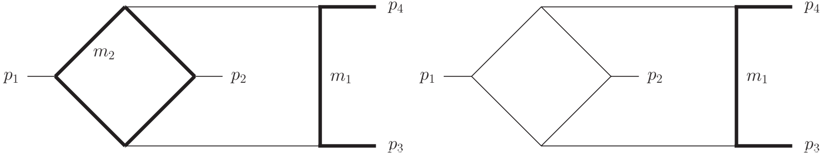
- timings: 10-80 secs (SecDec 2),
  rel. accuracy 10^{-3}, abs. accuracy: 10^{-8}

SB Jun '14

$s_{12} + s_{23} + s_{13} = (\sum_{i=1}^{4} p_i)^2$

Top-quark pair production @ NNLO

Two of most complicated 2-loop diagrams:

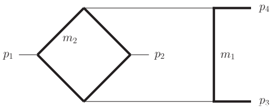


(a) ggtt1 (b) ggtt2

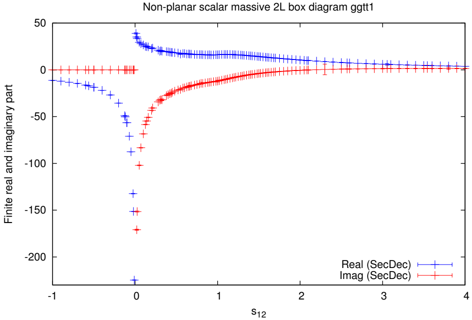
- *ggtt*1: enters **heavy** fermionic corrections:
  no analytical result available ⇒ fully numerical approach easy
- *ggtt*2: enters **light** fermionic corrections:
  more complicated infrared singularity structure, spurious divergences, numerical cancellations
  → pure numerical approach difficult
  ⇒ mixed approach: analytical preparation SB & Heinrich Mar '13

Results for the non-planar massive 2L-diagram ggtt1

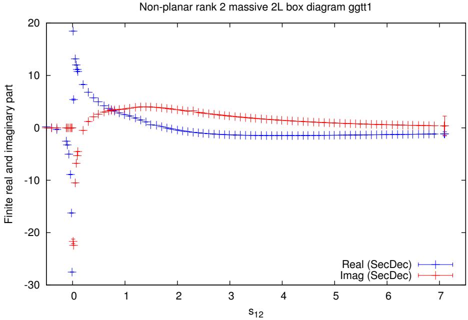
$$s_{12} = (p_1 + p_2)^2$$



$m_1^2 = m_2^2 = 1,\ p_3^2 = p_4^2 = m_1^2,\ p_1^2 = p_2^2 = 0,\ s_{23} = -1.25$



Scalar integral



Rank 2 integral

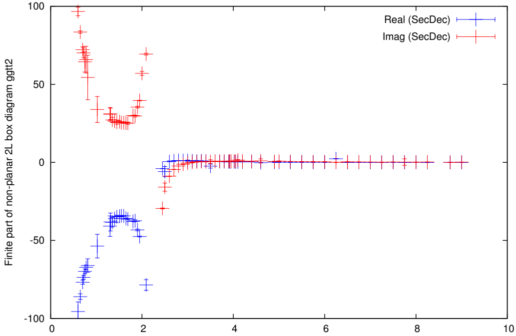
timings (SecDec 2): 11-1600 secs (scalar), 5-700 secs (rank 2), rel. accuracy: 10^{-3}, abs. accuracy: 10^{-5}

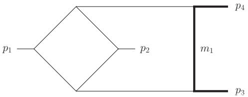
SB & Heinrich Mar '13

Results for the non-planar massive 2L-diagram ggtt2

- mixed analytical & numerical approach

Finite part





$m_1^2 = 1,$
$p_1^2 = p_2^2 = 0,$
$p_3^2 = p_4^2 = m_1^2,$
$s_{23} = -1.25$

$s_{12} = (p_1 + p_2)^2$

- timings (SecDec 2): 250-4000 secs, rel. accuracy $5 \cdot 10^{-3}$, abs. accuracy: 10^{-5}
- analytic results: Manteuffel & Studerus Sep '13

SB & Heinrich Mar '13

Massive 2-loop master integrals

Two-loop master integrals for non-leptonic heavy-to-heavy decays

Tobias Huber and Susanne Kränkl

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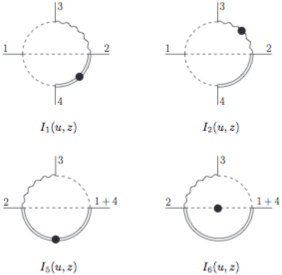
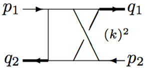
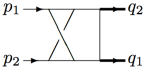


Abstract: We compute the two-loop master integrals for non-leptonic heavy-to-heavy decays analytically in a recently-proposed canonical basis. For this genuine two-loop, two-scale problem we first derive a basis for the master integrals that disentangles the kinematics from the space-time dimension in the differential equations, and subsequently solve the latter in terms of iterated integrals up to weight four. The solution constitutes another valuable example of the finding of a canonical basis for two-loop master integrals that have two different internal masses, and assumes a form that is ideally suited for a subsequent convolution with the light-cone distribution amplitude in the framework of QCD factorisation.

Off-shell 2-loop box master integrals

The Two-Loop Master Integrals for $q\bar{q} \to VV$

$f_{72}^{\mathrm{C254}} =$ 

$f_{73}^{\mathrm{C382}} =$ 

Thomas Gehrmann,[a] Andreas von Manteuffel,[b] Lorenzo Tancredi,[a] Erich Weihs[a]

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Abstract: We compute the full set of two-loop Feynman integrals appearing in massless two-loop four-point functions with two off-shell legs with the same invariant mass. These integrals allow to determine the two-loop corrections to the amplitudes for vector boson pair production at hadron colliders, $q\bar{q} \to VV$, and thus to compute this process to next-to-next-to-leading order accuracy in QCD. The master integrals are derived using the method of differential equations, employing a canonical basis for the integrals. We obtain analytical results for all integrals, expressed in terms of multiple polylogarithms. We optimize our results for numerical evaluation by employing functions which are real valued for physical scattering kinematics and allow for an immediate power series expansion.

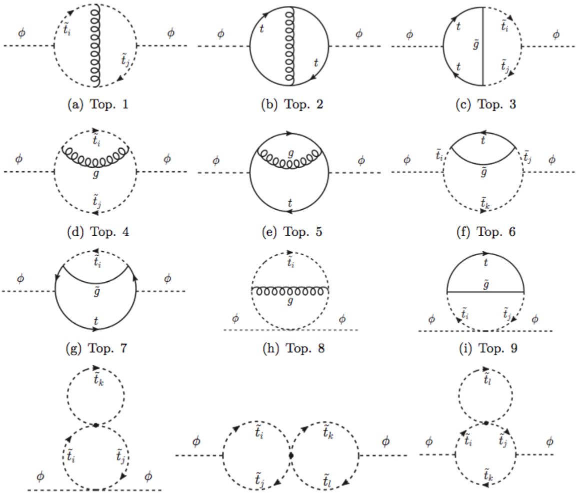
Large(r)-scale projects

Higgs-boson self-energy diagrams for $\mathcal{O}(\alpha_s \alpha_t)$

$p^2 = 0$ result: Heinemeyer, Hollik, G. Weiglein '98
$p^2 \neq 0$ result: SB, Hahn, Heinemeyer, Heinrich, Hollik Apr '14; Degrassi, Di Vita, Slavich Oct '14

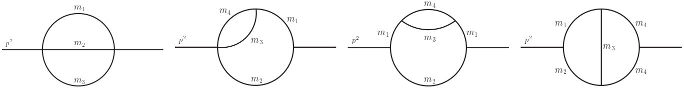
- Tensor reduction with TwoCalc Weiglein et al. '93 & FormCalc Hahn et al. '99 '08
- Numerical evaluation of momentum-dependent integrals with a preliminary version of SecDec 3

$$\phi = h, H, A$$

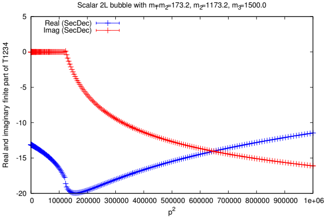


(a) Top. 1 (b) Top. 2 (c) Top. 3 (d) Top. 4 (e) Top. 5 (f) Top. 6 (g) Top. 7 (h) Top. 8 (i) Top. 9 (j) Top. 10 (k) Top. 11 (l) Top. 12

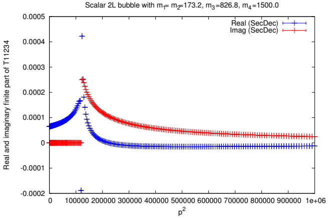
Numerical evaluation of momentum-dependent integrals



- 34 mass configurations run with SecDec, e.g.



T_{1234}, finite part



T_{11234}, finite part

- differences of kinematic invariants of up to 14 orders of magnitude
- rel. accuracy better than 10^{-5}, timings range from $0.01 - 100\,\mathrm{secs}$

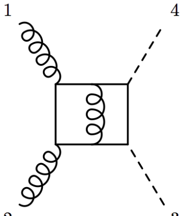
Full 2-loop process: gg $\rightarrow$ HH

- Higgs-boson pair production in gluon fusion interesting for measurement of Higgs-boson self-coupling
- LO (1-loop) known Glover, van der Bij '88
- NLO in $m_t \rightarrow \infty$ limit Plehn, Spira, Zerwas '96; Dawson, Dittmaier, Spira '98
- NLO with $m_t \rightarrow \infty$ but supplemented with $1/m_t$ expansion Grigo, Hoff, Melnikov, Steinhauser '13
- NNLO in $m_t \rightarrow \infty$ limit De Florian, Mazzitelli '13
- NNLO $m_t \rightarrow \infty$ with all matching coefficients Grigo, Melnikov, Steinhauser '14
- NNLO $m_t \rightarrow \infty$ + NNLL threshold resummation De Florian, Mazzitelli '15
- Full mass dependence in real radiation part + matching to parton shower Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14; Maltoni, Vryonidou, Zaro '14
- Full top-mass dependence at NLO missing so far!

gg → HH - Two-loop integrals

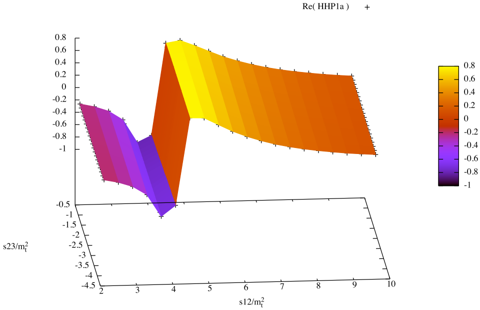
SB, Heinrich, Greiner, Jones, Kerner, Luisoni, Mastrolia,
Schlenk, Schubert, Stoyanov, Di Vita, Zirke

- Requires computation of unknown two-loop integrals
- 4 independent scales: s_{12}, s_{23}, m_H, m_t
- numerical evaluation with SecDec



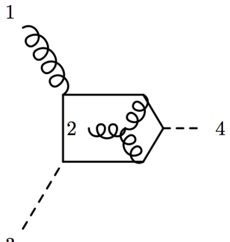
(a) HHP1A

$m_H = 125$ GeV
$m_t = 173$ GeV

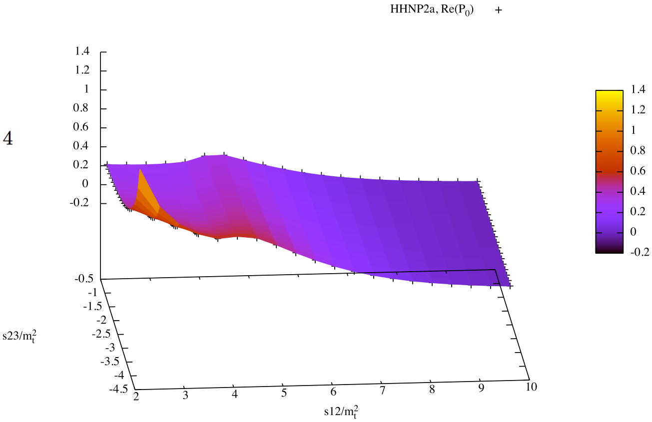


Plot by Gudrun Heinrich

gg → HH - Two-loop integral examples



(a) HHNP2A

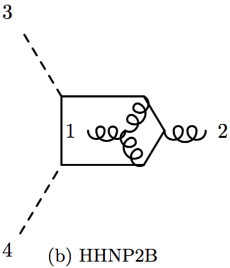


$$I = \frac{P_{-1}}{\varepsilon} + P_0$$

$$m_H = 125\,\text{GeV}$$
$$m_t = 173\,\text{GeV}$$

Plot by Gudrun Heinrich

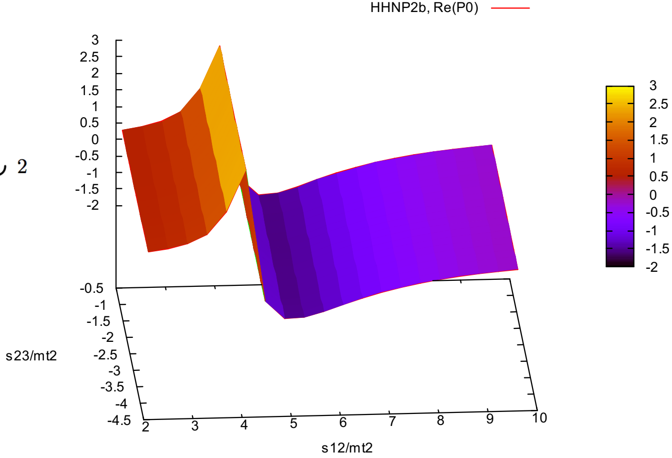
gg → HH - Two-loop integral examples



(b) HHNP2B

$$I = \frac{P_{-1}}{\varepsilon} + P_0$$

$m_H = 125$ GeV
$m_t = 173$ GeV



Plot by Gudrun Heinrich

Automated Calculations of Dijet Soft Functions in SCET

Guido Bell || Rudi Rahn || **Jim Talbert**

18 June 2015 || *Radcor/Loopfest 2015, UCLA, CA USA*

Oxford Physics

- We utilize the 'general' mode of the program. Simple interface to our NLO and NNLO master formulas (✔), multiple numerical integrators for crosschecks (✔)
- We use *SecDec* to calculate the double emission contribution. To obtain the renormalized soft function we have to add the counterterms, which are known analytically at the required order.

from Jim Talbert's talk at RadcorLoopfest 2015

- Can make use of the new ε dependent dummy functions feature in SecDec 3
- We work together to implement further new features needed for their project

Miscellaneous

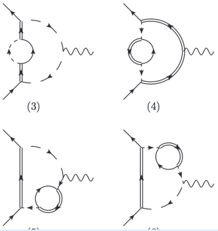
WIMP el.-magnetic form factors, 2-loop

Magnetic dipole moment of neutral particles from quantum corrections at two-loop order

Carlos Tamarit[1,*] and Itay Yavin[2,1,†]

[1] *Perimeter Institute for Theoretical Physics 31 Caroline St. N, Waterloo, Ontario, Canada N2L 2Y5.*
[2] *Department of Physics & Astronomy, McMaster University 1280 Main St. W. Hamilton, Ontario, Canada, L8S 4L8.*

The tentative gamma-ray line in the Fermi data at $\sim$ 135 GeV motivates a dark matter candidate that couples to photons through loops of charged messengers. It was recently shown that this model can explain the observed line, but achieving the correct phenomenology requires a fairly sizable coupling between the WIMP and the charged messengers. While strong coupling by itself is not a problem, it is natural to wonder whether the phenomenological success is not spoiled by higher order quantum corrections. In this work we compute the dominant two-loop contributions to the electromagnetic form-factors of the WIMP and show that over a large portion of the relevant parameter space these corrections are under control and the phenomenology is not adversely affected. We also discuss more generally the effects of these form-factors on signals in direct-detection experiments as well as on the production of the WIMP candidate in colliders. In particular, for low masses of the charged messengers the production rate at the LHC enjoys an enhancement from the threshold singularity associated with these charged states.



Model for neutrino mass generation, 3-loop

Predictive Model for Radiatively Induced Neutrino Masses and Mixings with Dark Matter

Michael Gustafsson,[1] Jose M. No,[2] and Maximiliano A. Rivera[3]

[1] *Service de Physique Théorique, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium*
[2] *Department of Physics and Astronomy, University of Sussex, BN1 9QH Brighton, United Kingdom and*
[3] *Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaiso, Chile*

(Dated: May 16, 2013)

A minimal extension of the standard model to naturally generate small neutrino masses and provide a dark matter candidate is proposed. The dark matter particle is part of a new scalar doublet field that plays a crucial role in radiatively generating neutrino masses. The symmetry that stabilizes the dark matter also suppresses neutrino masses to appear first at three-loop level. Without the need of right-handed neutrinos or other very heavy new fields, this offers an attractive explanation of the hierarchy between the electroweak and neutrino mass scales. The model has distinct verifiable predictions for the neutrino masses, flavor mixing angles, colliders and dark matter signals.

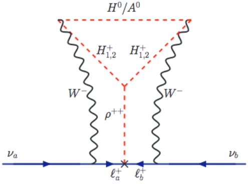


FIG. 1: The "cocktail diagram."

Summary and Outlook

Summary

- SecDec allows for computation of diverse integrals contributing to scattering amplitudes
- SecDec 3: new decomposition strategies, improved user interface, negative propagator powers and linear propagators allowed, integrators added, efficiency increased
- New features in SecDec 3 prepare for large(r) scale applications

Outlook

- Push limits of the method further (minimize spurious singularities)
- Interface to other programs, e.g. FIRE, LiteRed, Reduze
- Application to phenomenologically relevant processes

Backup

Change of variables in decomposition strategy G2

$$x_i = \prod_{F \in S_j} y_F^{\langle \vec{e}_i, \vec{n}_F \rangle}$$

F: facet
S_j: Set of facets a vertex lies in
y_F: new coordinate for each facet F
$\vec{e}_i$: unit vector in $\mathbb{R}^{N-1}$
$\vec{n}_F$: primitive normal vector of the facet F

Kaneko, Ueda '09 '10