Computing Higgs production at three loops in QCD

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on behalf of the N3LO team:

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Higgs production in...

Higgs production in...

$\mathcal{N} = 0$

Higgs production in...

$\mathcal{N} = 0$

non-supersymmetric

Higgs production in...

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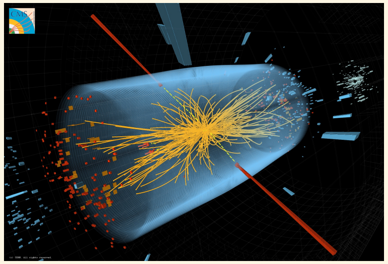
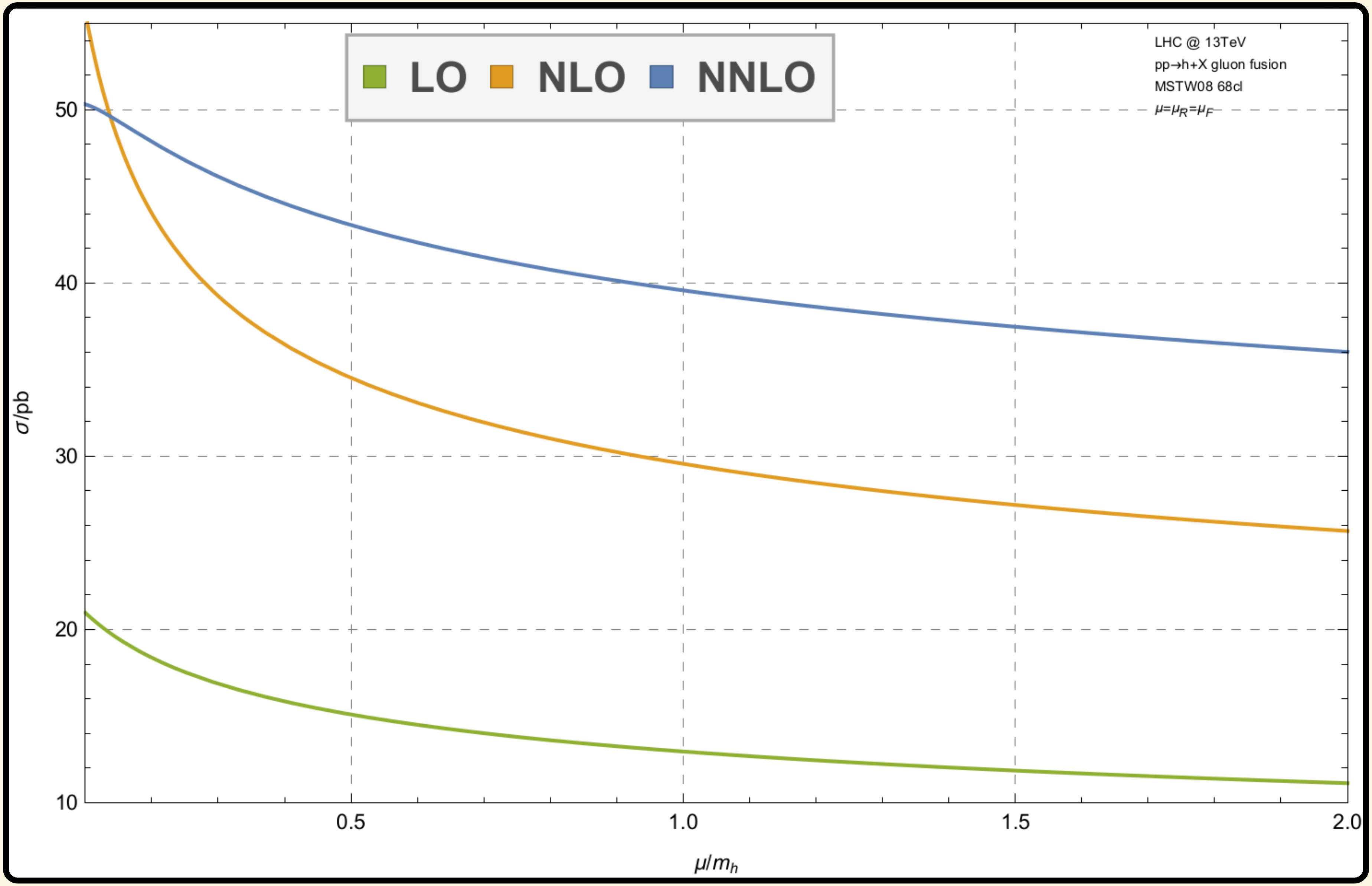
QCD

Cross sections

- We are computing cross sections to make predictions for the LHC.
- Problem: Divergent integrals → regularization and renormalization.
- Results depend on an unphysical parameter 'scale'.
- Artifact of perturbation theory, all loop result should not depend on this scale.
- Scale dependence is reduced when increasing the order in perturbation theory.
- Reduced scale sensitivity makes calculations more predictive.

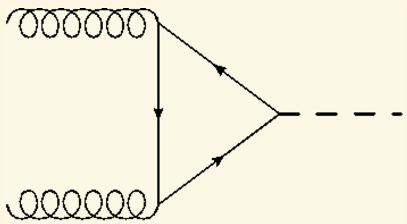


Cross sections



The gluon fusion cross section

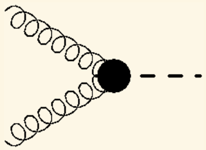
- The dominant Higgs production mode at the LHC is gluon fusion
- Loop induced process with massive particles (top-quark) in the loop



- Leading order amplitude already starts at one loop ☹
- Integrals with internal masses are an open problem starting from two loops (elliptic integrals) ☹
- Better to get rid of the massive loop!

The gluon fusion cross section

- Let us just compute



- Dimension five operator in an effective theory for gluon fusion in the limit of infinitely heavy particles in the loop

$$\mathcal{L} = \mathcal{L}_{QCD} - \frac{1}{4v} C H G_a^{\mu\nu} G^a_{\mu\nu}$$

- Subleading corrections depending on the top mass are known at NNLO.

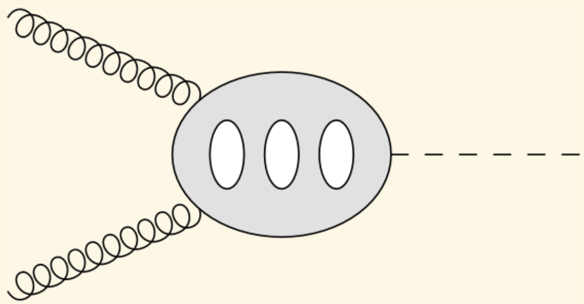
[Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]

- Next important term is the leading contribution at N3LO.

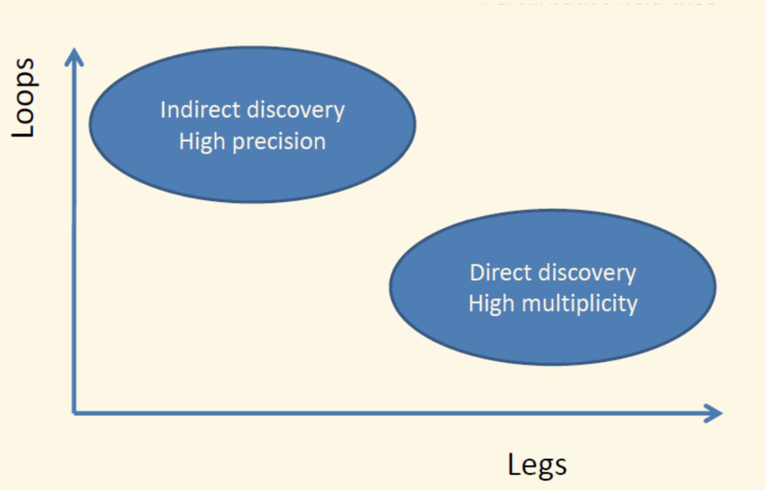
- We are calculating in pure massless QCD coupled to a massive scalar.

The gluon fusion cross section

- We want to compute

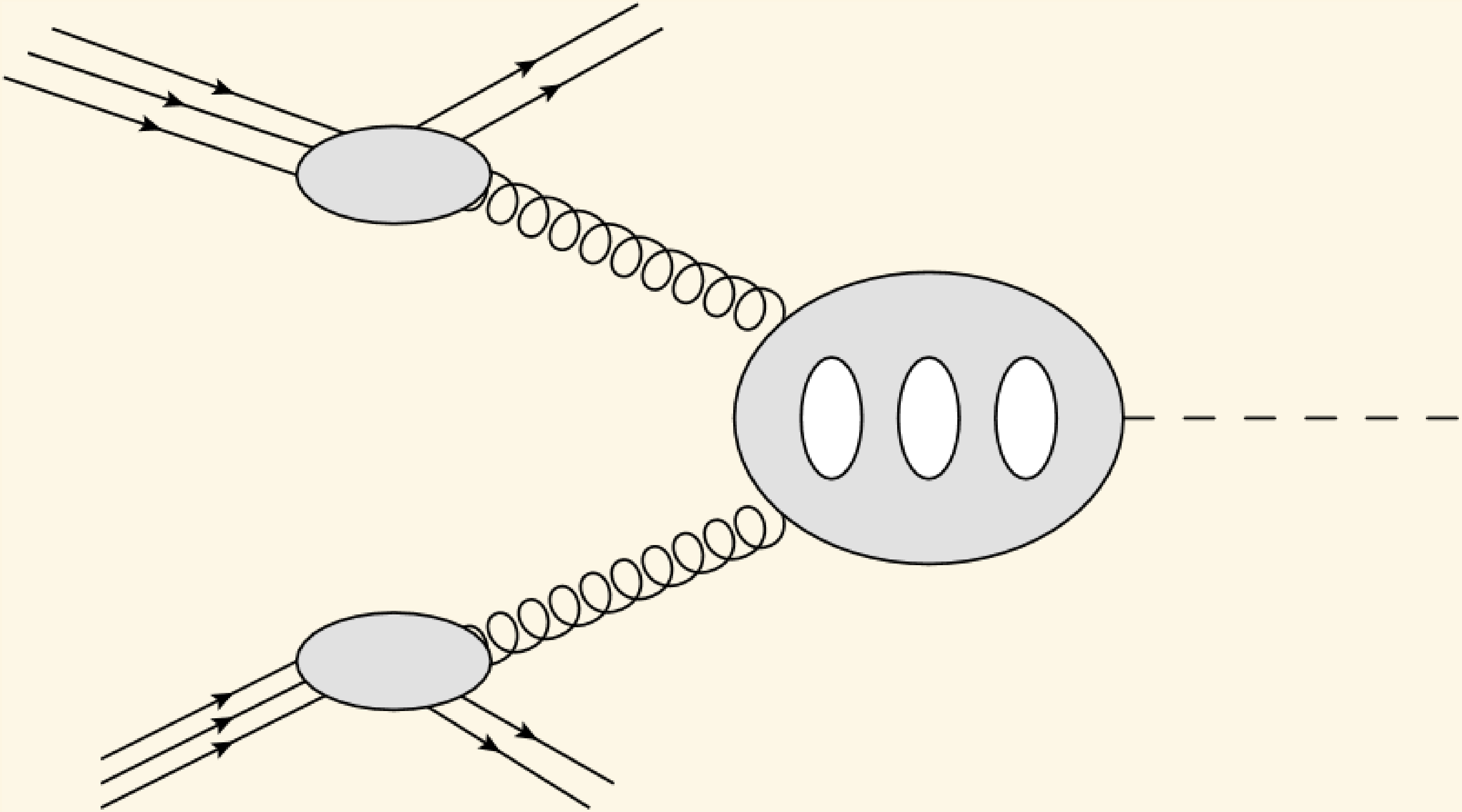


- This puts us firmly on the 'high precision' side



The gluon fusion cross section

- The LHC does not collide gluons though



The gluon fusion cross section

- To connect to actual physics we compute the hadronic cross section in perturbation theory

$$\sigma(pp \to H + X) = \tau \sum_{ij} \int_\tau^1 \mathrm{d}z \mathcal{L}_{ij}(z) \hat{\sigma}_{ij}\left(\frac{\tau}{z}\right)$$

- The partonic cross section $\hat{\sigma}$ is a function of the ratios

$$\tau = \frac{m_H^2}{E_{\mathrm{col}}} \qquad z = \frac{m_H^2}{s}$$

- τ and $\mathcal{L}$ parametrize the experiment.

- Focus on the computation of $\hat{\sigma}(z)$ in perturbation theory

$$\hat{\sigma}(z) = \alpha_s^2 \sigma_{\mathrm{LO}} + \alpha_s^3 \sigma_{\mathrm{NLO}} + \alpha_s^4 \sigma_{\mathrm{NNLO}} + \alpha_s^5 \sigma_{\mathrm{N3LO}} + \dots$$

The gluon fusion cross section

- The partonic cross section was know through NNLO

  [Dawson; Djouadi, Spira, Zerwas; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

- At N3LO only approximations were known.

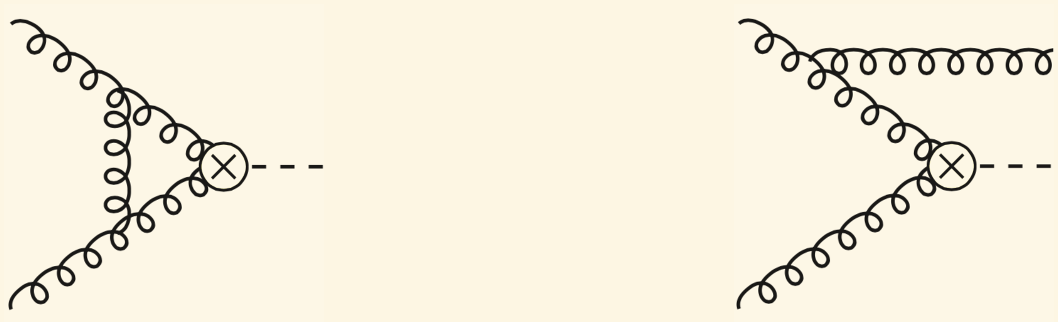
  [Moch, Vogt; Ball, Bonvini, Forte, Marzani, Ridolfi; Bühler, Lazopoulos]

- Can we push the state of the art in QCD to N3LO?
- Improve predictions for the LHC.
- Will we find something new and unexpected?
- Is it even possible to compute in QCD at this order?
- Uncharted territory in perturbation theory.
- Culmination of many developments from amplitudes.

	σ [8 TeV]	$\delta\sigma$ [%]
LO	9.6pb	$\sim 25\%$
NLO	16.7pb	$\sim 20\%$
NNLO	19.6pb	$\sim 8\%$
N3LO	???	$\sim 3\%$

From amplitudes to cross sections...

- We want to compute finite physical cross sections.
- Not enough to just consider loop (virtual) corrections.
- Also need the corresponding real corrections.



- Both are individually divergent in dimensional regularisation.
- Infrared poles need to cancel between real and virtual corrections.
- E.g. at NLO we have

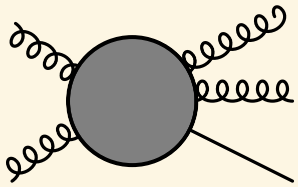
$$\sigma_{gg\to H}^{NLO} \propto \langle \mathcal{A}_0^{(1)} | \mathcal{A}_0^{(0)} \rangle + \langle \mathcal{A}_1^{(0)} | \mathcal{A}_1^{(0)} \rangle$$

From amplitudes to cross sections...

- We compute the inclusive cross section from two ingredients

$$\hat{\sigma} = \int \mathrm{d}\Phi\, |\mathcal{A}|^2$$

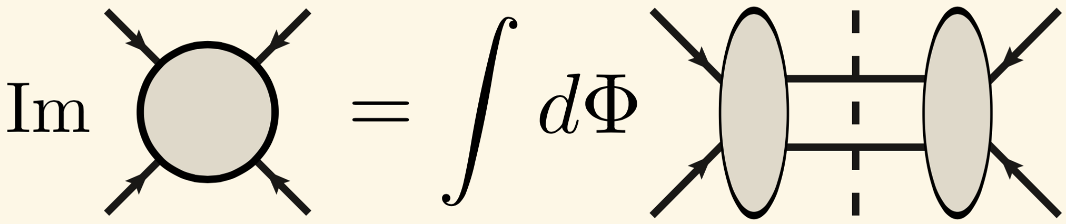
- Amplitude



- Phase space integral
  Integrate over final state momenta of the amplitude

... and back ...

- Optical theorem



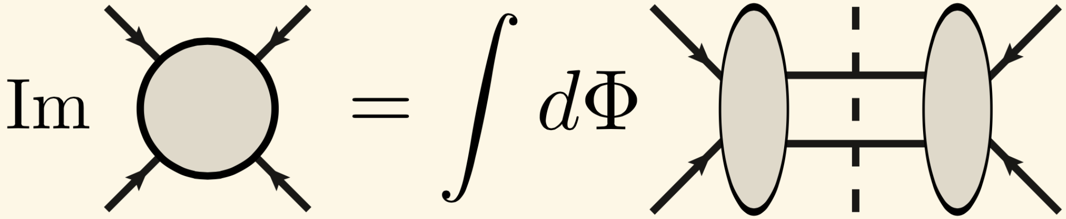
- Discontinuities of loop amplitudes are phase space integrals.

- Discontinuities of loop integrals computed from Cutkosky rule

$$\frac{1}{p^2 - m^2 + i\epsilon} \to \delta^+(p^2 - m^2) = \delta(p^2 - m^2)\theta(p^0)$$

... and back ...

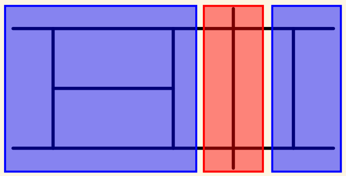
- Optical theorem



- Optical theorem can be read 'backwards'. Use it to write phase space integrals as unitarity cuts of loop integrals → **Reverse Unitarity**

[Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

- Compute loop integrals with cuts instead of phase-space integrals
- This duality unifies the two ingredients of cross sections

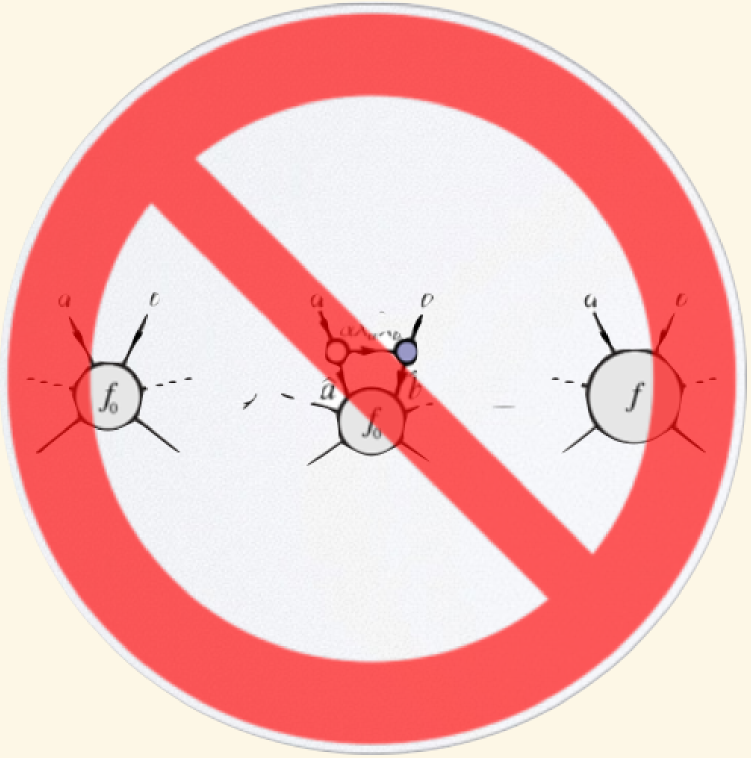


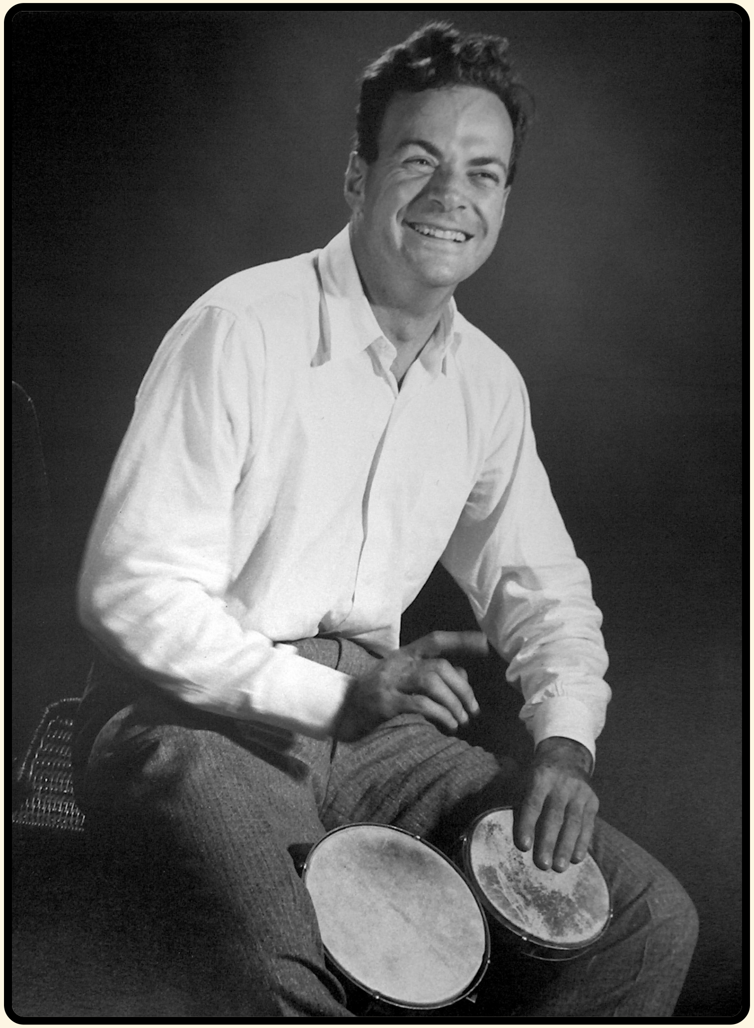
… with reverse unitarity

- Reverse unitarity allows us to not distinguish between loop integrals and phase space integrals
- We just compute forward scattering amplitudes with cuts
- Enables the use of the rich technology developed for loop integrals
  - Integration-by-parts (IBP) reductions
  - Master integrals
  - Differential equations for master integrals
- Unifies the treatment of different contributions to the cross section

Good ol' Feynman diagrams

- Our calculation is beyond any modern unitarity or on-shell based techniques.



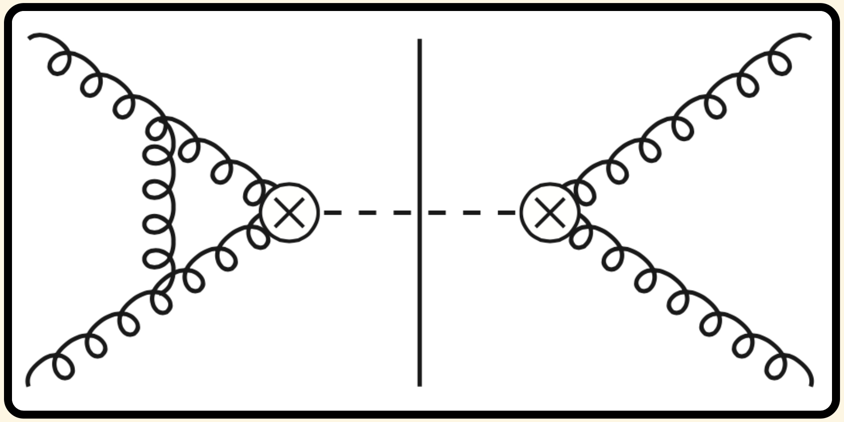


~ 100000 Feynman diagrams

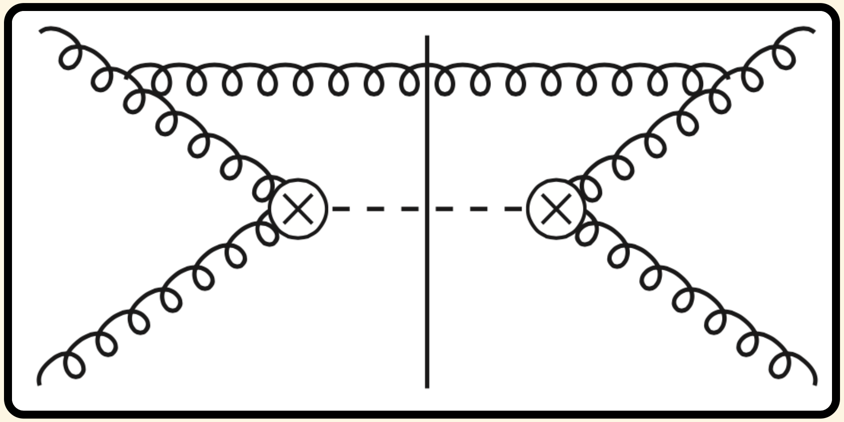
The gluon fusion cross section

- Contributions at next-to-leading order

[Dawson; Djouadi, Spira, Zerwas]



Virtual corrections (loops)



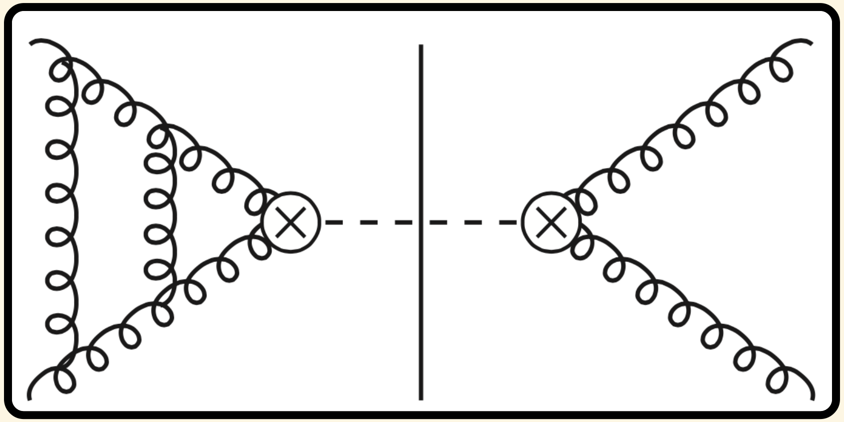
Real corrections (phase space)

- Both combinations are individually and in combination divergent
- UV divergences are taken care of by renormalization
- Initial state IR singularities are cancelled by PDF counter terms

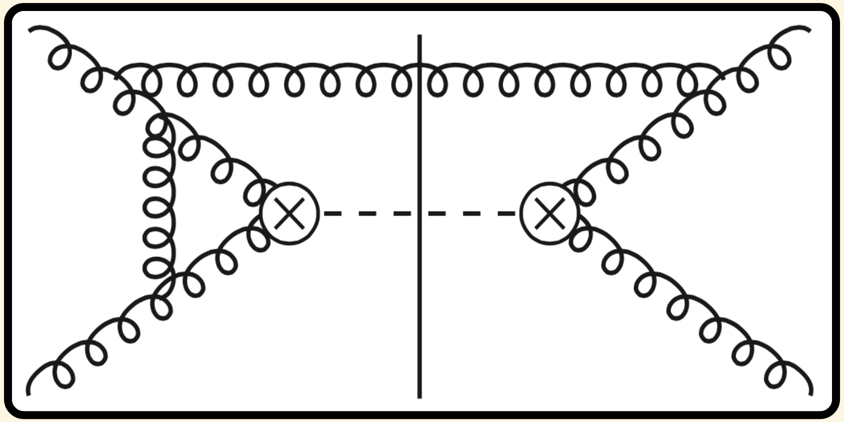
The gluon fusion cross section

- Contributions at next-to-next-to-leading order

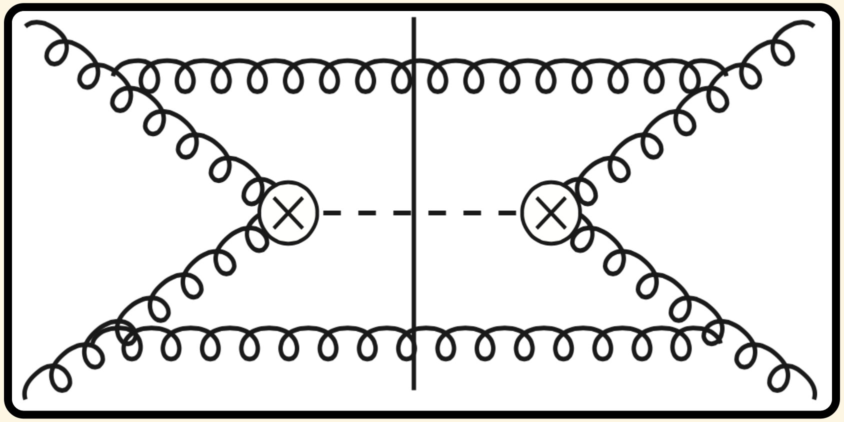
[Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]



Double virtual



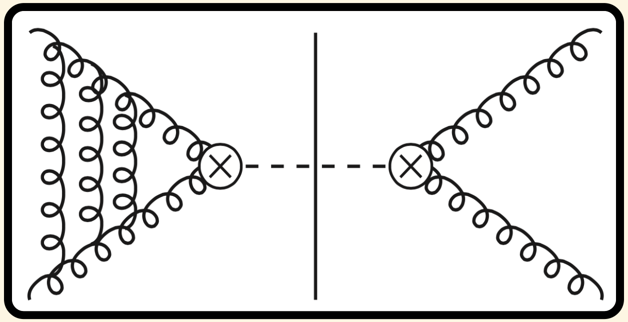
Real-virtual



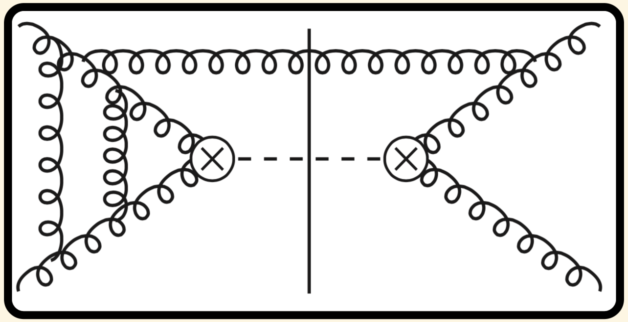
Double real

The gluon fusion cross section

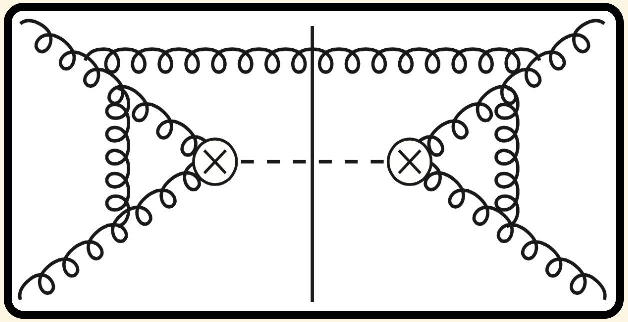
- Contributions at next-to-next-to-next-to-leading order



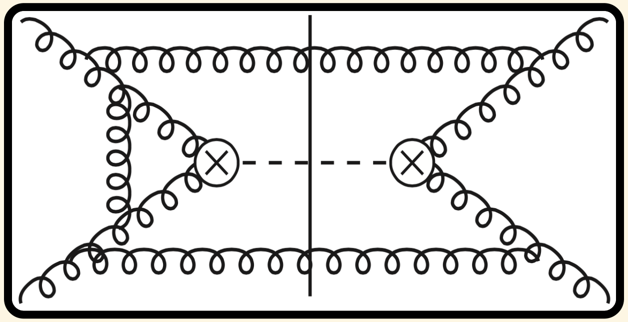
Triple virtual



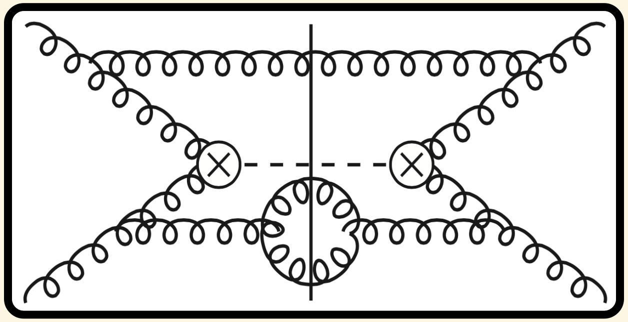
Double virtual real



Real-virtual^2



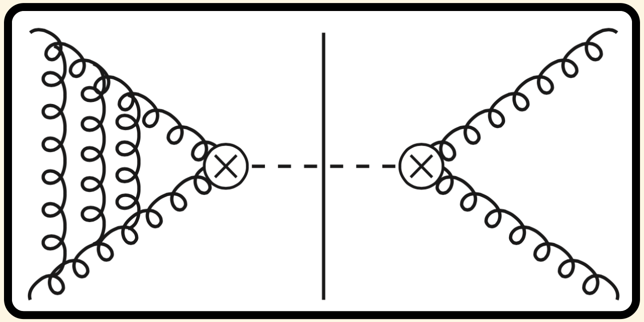
Double real virtual



Triple real

Triple virtual corrections

- Purely virtual corrections are related to the QCD form factor



- QCD form factors have been computed
  - at one loop
  - at two loops

[Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maitre]

  - at three loops

[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

- Pure loop corrections were known before

Real contributions

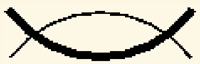
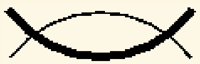
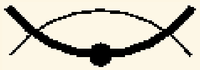
- All remaining contributions involve phase space integrals.
- Due to reverse unitarity dual to loop integrals, but still more complicated.
- Loop integrals are integrals over
$$\mathbb{R}^{\ell}_{d-1,1}$$
punctured by the Landau singularities of the integrand.
- Phase space integrals are intgrals over punctured algebraic varieties
$$\{(p_1, \dots, p_n) \in \mathbb{R}^n_{d-1,1} | p_1^2 = 0, \dots, p_n^2 = 0, p_1 + \dots + p_n = 0\}$$
- To actually do the phase space integral we need to find local coordinates on these varities.
- We would like to do as few as possible of these integrals.
- Reverse unitarity can help us here.

Integral reductions

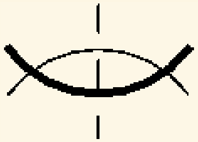
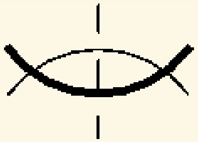
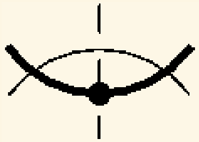
- In dimensional regularisation we have by construction

$$\int \mathrm{d}^d k \frac{\partial}{\partial k^\mu} f(k) = 0$$

- Loop integrals are not independent.
- Due to reverse unitarity also phase space integrals.
- Trivial example:

$\frac{\partial}{\partial k_\mu} k_\mu$  $= -(d-3)$  $+(p^2 - m^2)$  $-$ 

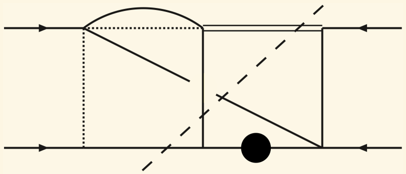
- With a phase space cut:

$\frac{\partial}{\partial k_\mu} k_\mu$  $= -(d-3)$  $+(p^2 - m^2)$ 

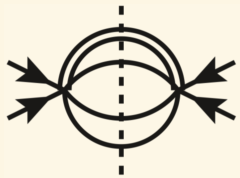
- We can find integration-by-parts (IBP) identities between different integrals.

Integral reductions

- The IBP identities for all integrals in a family form a linear system.
- Linear systems become very large.
- Systems can be solved with efficient computer algebra `AIR, FIRE, Reduze`
- Solution is a basis for all integrals in a family $\rightarrow$ master integrals.
- All integrals can be reduced to a small set of master integrals
- Reduction from $\sim 10^9$ integrals to ≤ 1000 master integrals.
- Example:

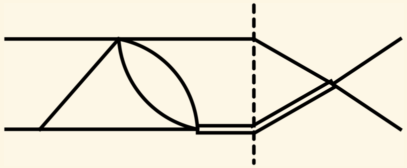


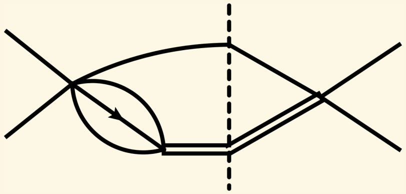
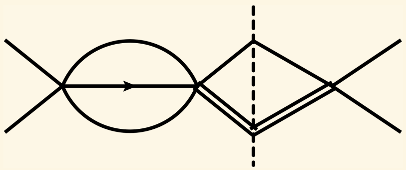
$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)} \times$$



Differential equations

- We can also take derivatives w.r.t external kinematic parameters of the integrals.
- In our case the relative mass z of the scalar.
- The derivative of a master integral will be some linear combination of integrals.
- Using IBP reductions the derivative can be expressed in terms of the master integrals. In particular also in terms of the integral itself → Differential equation.
- The derivative of a master integral will be usually expressible in terms of the integral itself and in terms of simpler master integrals.

$$\left[\partial_{\bar{z}} - 3\epsilon \frac{1}{1-\bar{z}}\right]$$ 

$$= \epsilon \frac{1}{1-\bar{z}}$$  $$-3\epsilon \frac{1}{1-\bar{z}}$$ 

Differential equations

- The differential equations for all our master integrals form a coupled system of first order differential equations.

$$\partial_{\bar{z}} f_i(\bar{z}) = \mathcal{A}_{ij}(\bar{z}, \epsilon) f_j(\bar{z})$$

- Formal solution of the system is

$$f_i(z) = \mathcal{P} e^{\int d\bar{z} \mathcal{A}_{ij}(\bar{z})} f_j(\bar{z}_0)$$

- To solve the system we should decouple it.
- We are ultimately only interested in solutions that are expansions in ϵ.
- It suffices if the system decouples in the limit $\epsilon \to 0$.
- Methods to solve such systems have been studied extensively in recent years in the context of canonical bases.

[Henn]

Differential equations

- The idea is to find a basis transformation
$$g_i(\bar{z}) = T_{ij}(\bar{z}) f_j(\bar{z})$$
$$\mathcal{A}_{ij} \to \frac{\partial T_{ik}}{\partial \bar{z}} T^{-1}_{kj} + T_{ik} \mathcal{A}_{kl} T^{-1}_{lj}$$
which puts the system into the form
$$\partial_{\bar{z}} g_i(\bar{z}) = \epsilon \sum_{\sigma} \frac{A^{\sigma}_{ij}}{\bar{z} - \sigma} g_j(\bar{z})$$
- R.h.s is proportional to ϵ. System decouples in the limit $\epsilon \to 0$.
- Explicit dependence on $\bar{z}$ is only through $d\log$ forms.
- See talks by **Johannes** and **Lorenzo**.

Differential equations

- The system can be solved by explicitly expanding the path ordered exponential

$$\mathcal{P}e^{\epsilon \int d\bar{z} \sum_\sigma \frac{A^\sigma}{\bar{z}-\sigma}} = 1 + \epsilon \int d\bar{z} \sum_\sigma \frac{A^\sigma}{\bar{z}-\sigma} + \dots$$

- Compare with the definition of the multiple polylogarithms

$$G(a_1, \dots, a_n, z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \dots, a_n, t)$$

- Solution of the system is expressible as a linear combination of multiple polylogarithms with alphabet

$$\{\bar{z} - \sigma_i\}$$

- Well studied class of functions with useful analyical properties
- See e.g. **Erik's talk**

Differential equations

- Finding a transformation to the canonical basis is not always possible.
- Even if it is possible, finding the transformation is not necessarily straightforward.
- In our case we find some square root singularities that need to be transformed away to go to a canonical basis
- Faster way for us: We just expand the differential equations around $\bar{z} = 0$.

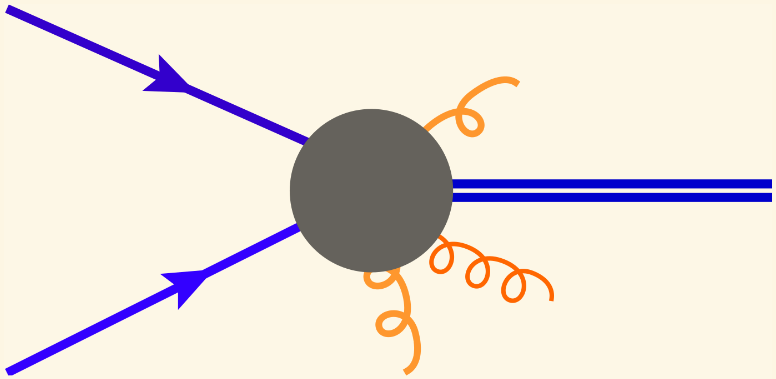
$$\partial_{\bar{z}} f_i(\bar{z}) = \left(\frac{A_{ij}^{(0)}(\epsilon)}{\bar{z}} + \sum_k \bar{z}^k A_{ij}^{(k)}(\epsilon) \right) f_j(\bar{z})$$

- Solved by Laurent series in $\bar{z}$

$$f(\bar{z}) = \sum_k \bar{z}^{a_k \epsilon} \sum_{l=-1}^{\infty} \bar{z}^i c_{kl}$$

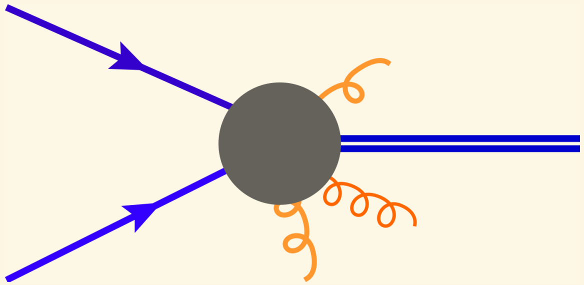
Boundary conditions

- Finding the general solutions of the differential equations is not enough.
- General solutions need to be specialised by fixing a boundary condition at some point $\bar{z}_0$.
- We need to evaluate Feynman integrals directly in some limit $\bar{z} \to \bar{z}_0$.
- We choose $\bar{z} = 0$ which is the so-called soft-limit.



- $\bar{z} = 0 \Leftrightarrow s = m$, all energy is used to create the Higgs at rest
- No energy for hard gluonic radiation.

Boundary conditions



- Simplified kinematic constraints around the soft limit.
- In the strict soft limit there is a duality to Wilson line scattering.
- Some boundary conditions can be obtained by computing the scattering of Wilson lines
- General strategy:
- We need to evaluate master integrals explicitly in the soft-limit.
- Need to do an explicit Feynman integral calculation for every boundary condition.
- Can we further reduce the ammount of explicit integrals we have to calculate?

Boundary conditions

- Boundary conditions fix the coefficients of the branch cuts at $\bar{z} = 0$

$$\begin{aligned} \partial_{\bar{z}} f(\bar{z}) &= \epsilon \left(\frac{\alpha}{\bar{z}} + \ldots \right) f(\bar{z}) \\ f(\bar{z}) &= \bar{z}^{\alpha\epsilon} f_0 (1 + \ldots) \end{aligned}$$

- In the case of a system of differential equations

$$\begin{aligned} \partial_{\bar{z}} f_i(\bar{z}) &= \epsilon \left(\frac{A_{ij}}{\bar{z}} + \ldots \right) f_j(\bar{z}) \\ f_i(\bar{z}) &= \bar{z}^{\epsilon A_{ij}} f_j^{(0)} + \ldots \end{aligned}$$

- Solutions will have branch cuts of the form $\bar{z}^{\epsilon \lambda_i}$, λ_i are the eigenvalues of A_{ij}.

- In our case only some branch cuts are allowed

$$\bar{z}^{-2\epsilon},\ \bar{z}^{-3\epsilon},\ \bar{z}^{-4\epsilon},\ \bar{z}^{-5\epsilon},\ \bar{z}^{-6\epsilon}$$

- Some eigenvalues of A_{ij} are prohibited by physics, coefficient must be zero.

Boundary conditions

- In the case of a system of differential equations

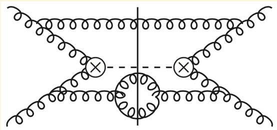
$$\begin{aligned} \partial_{\bar{z}} f_i(\bar{z}) &= \epsilon \left(\frac{A_{ij}}{\bar{z}} + \ldots \right) f_j(\bar{z}) \\ f_i(\bar{z}) &= \bar{z}^{\epsilon A_{ij}} f_j^{(0)} + \ldots \end{aligned}$$

- A boundary condition f_i is associated to an eigenvalue λ_i, 'eigenfunctions'.
- In general A_{ij} will not have full rank, less eigenfuctions than the dimension of the system.
- We do not find one indepedent boundary condition per master integral.
- We can find relations between different boundary conditions by going to a Jordan basis.
- Reduces the ammount of boundary conditions that actually need to be computed.

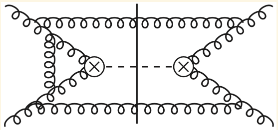
[Henn, Smirnov^2 ; Mistlberger, FD]

Boundary conditions

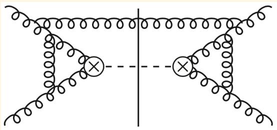
- The boundary conditions that remain after this reduction need to be computed explicitly.
- Need to do explicit phase space integrals for:



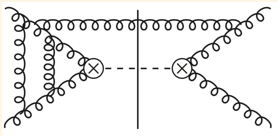
$H + 3g$ phase space integrals over tree level amplitudes



$H + 2g$ phase space integrals over one-loop amplitudes



$H + 1g$ phase space integrals over squares of one-loop amplitudes



$H + 1g$ phase space integrals over two-loop amplitudes

Boundary conditions

- General phase space integrals over

$$\{(p_1, \ldots, p_n) \in \mathcal{M}^n_{d-1,1} | p_1^2 = 0, \ldots, p_n^2 = 0, p_1 + \cdots + p_n = 0\}$$

- In general very complicated integrals.
- In particular, usually not possible to find linear parametrisations for phase space integrals beyond $H + 1g$.
- No direct analog to the Feynman parameters
- Not straightforward to obtain 'parameter integrals'.

Boundary conditions

- One possible parametrisation is in terms of the energies and angles of the massless momenta.
- Not very useful in general, but ...
- ...in the soft limit the energy integrals factor from the angular integrals.
- Possible to derive Mellin-Barnes representations for angular integrals for arbitrary number of legs

[Somogyi]

- Canonical way to derive Mellin-Barnes representations for soft phase space integrals.
- Also works for phase space integrals over loop amplitudes, provided we can find a Mellin-Barnes representation of the loop integral.

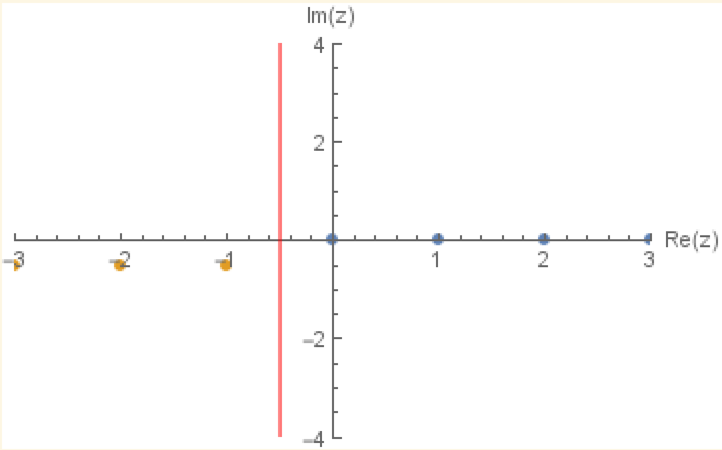
Mellin-Barnes integrals

- Binomial series

$$(1+x)^{\lambda} = \sum_{n=0}^{\infty} \binom{\lambda}{n} x^{n}$$

- Mellin-Barnes integral

$$(1+x)^{\lambda} = \int_{c-i\infty}^{c+i\infty} dz \frac{\Gamma(-z)\Gamma(z-\lambda)}{\Gamma(-\lambda)} x^{\lambda-z}$$



Mellin-Barnes integrals

- Repeated use of the basic Mellin-Barnes integral enables us to integrate arbitrarily complicated rational functions.
- At the price of introducing Mellin-Barnes integrals with complicated pole structures.
- Mellin-Barnes integrals are conventionally solved by taking residues and summing.
- Need to perform nested Euler-Zagier sums and generalisations.
- In our case one obtains after summation the result for the boundary condition as linear combination of multiple zeta-values.

Mellin-Barnes integrals

- Pole structures can become very complicated and lead to very difficult nested sums.
- We map contour integrals on $\mathbb{C}^n$ to a parametric integrals over the real line.
- Remove poles by introducing auxilliary integrals.

$$\Gamma(a)\Gamma(b) = B(a,b)\Gamma(a+b)$$
$$B(a,b) = x^{a-1}(1-x)^{b-1}$$

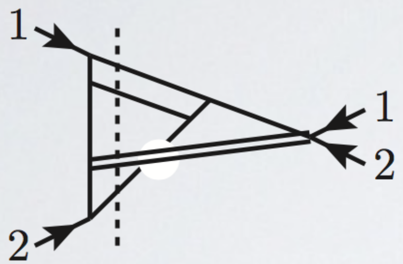
- Mellin-Barnes integral can be rewritten as a nested parametric integral.
- Nested parametric integrals can be performed in terms of iterated integrals over multiple polylogarithms.

[Brown; Anastasiou, Duhr, FD, Herzog, Mistlberger]

- Linear reducibility criterion needs to be fulfilled.

Mellin-Barnes integrals

- Using these techniques all ~ 90 boundary conditions can be computed
- All boundary conditions are linear combinations of mutliple zeta-values up to weight 6.



$$= \frac{160}{\epsilon^5} - \frac{1712}{\epsilon^4} + \frac{1}{\epsilon^3}\Big(-120\,\zeta_2 + 2784\Big) + \frac{1}{\epsilon^2}\Big(-120\,\zeta_3 + 1284\,\zeta_2 + 31968\Big)$$
$$+ \frac{1}{\epsilon}\Big(2520\,\zeta_4 + 1284\,\zeta_3 - 2088\,\zeta_2 - 216864\Big) + 15720\,\zeta_5 + 1920\,\zeta_2\,\zeta_3$$
$$- 26964\,\zeta_4 - 2088\,\zeta_3 - 23976\,\zeta_2 + 795744 + \epsilon\Big(82520\,\zeta_6 + 9600\,\zeta_3^2$$
$$- 168204\,\zeta_5 - 20544\,\zeta_2\,\zeta_3 + 43848\,\zeta_4 - 23976\,\zeta_3 + 162648\,\zeta_2 - 2449440\Big)$$
$$+ \mathcal{O}(\epsilon^2)\,.$$

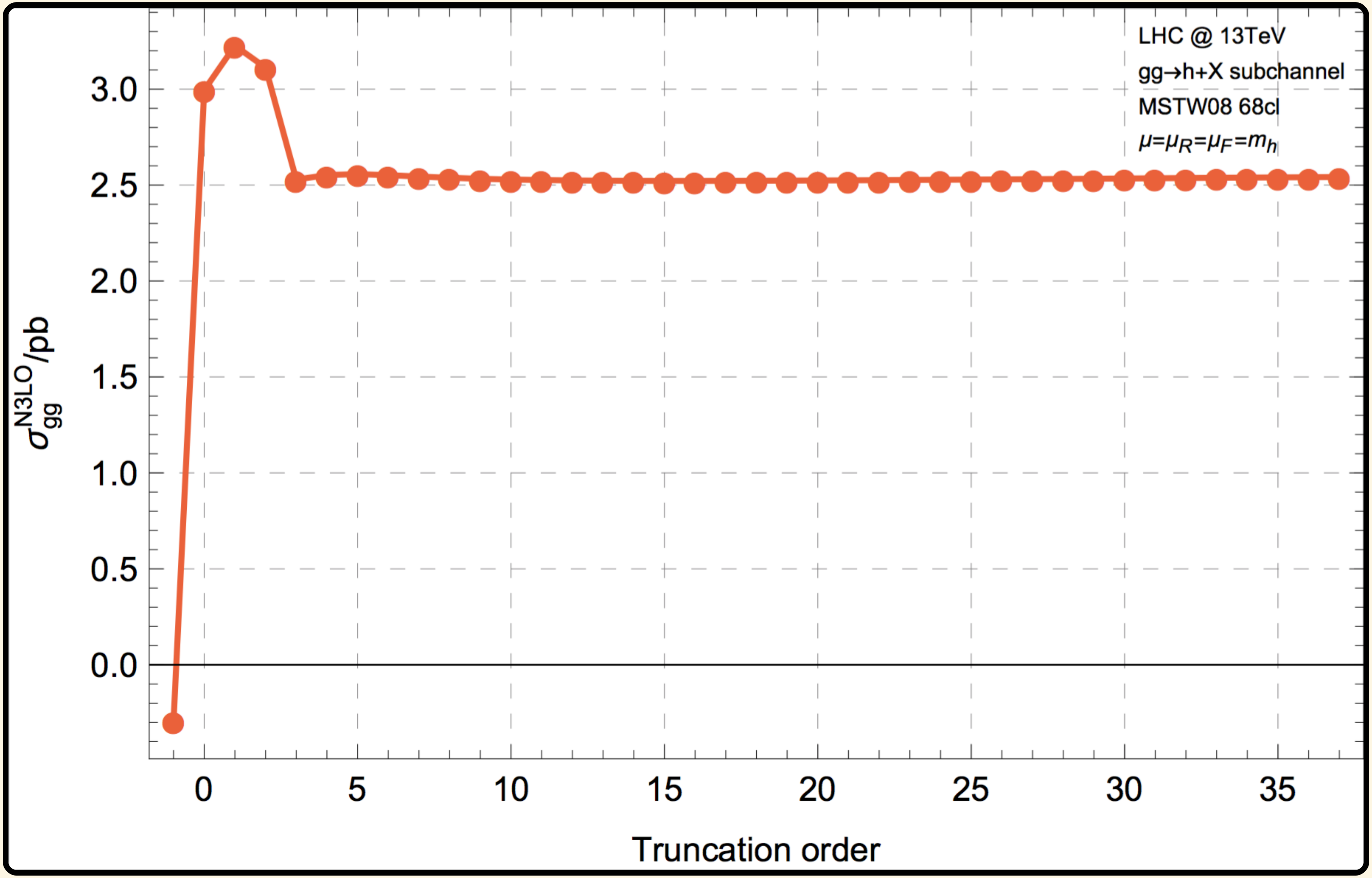
- The leading boundary conditions are linear combinations of
$$\zeta_2, \zeta_3, \zeta_4, \zeta_2\zeta_3, \zeta_5, \zeta_3^2, \zeta_6$$
with only integer coefficients.

The leading term of the cross section

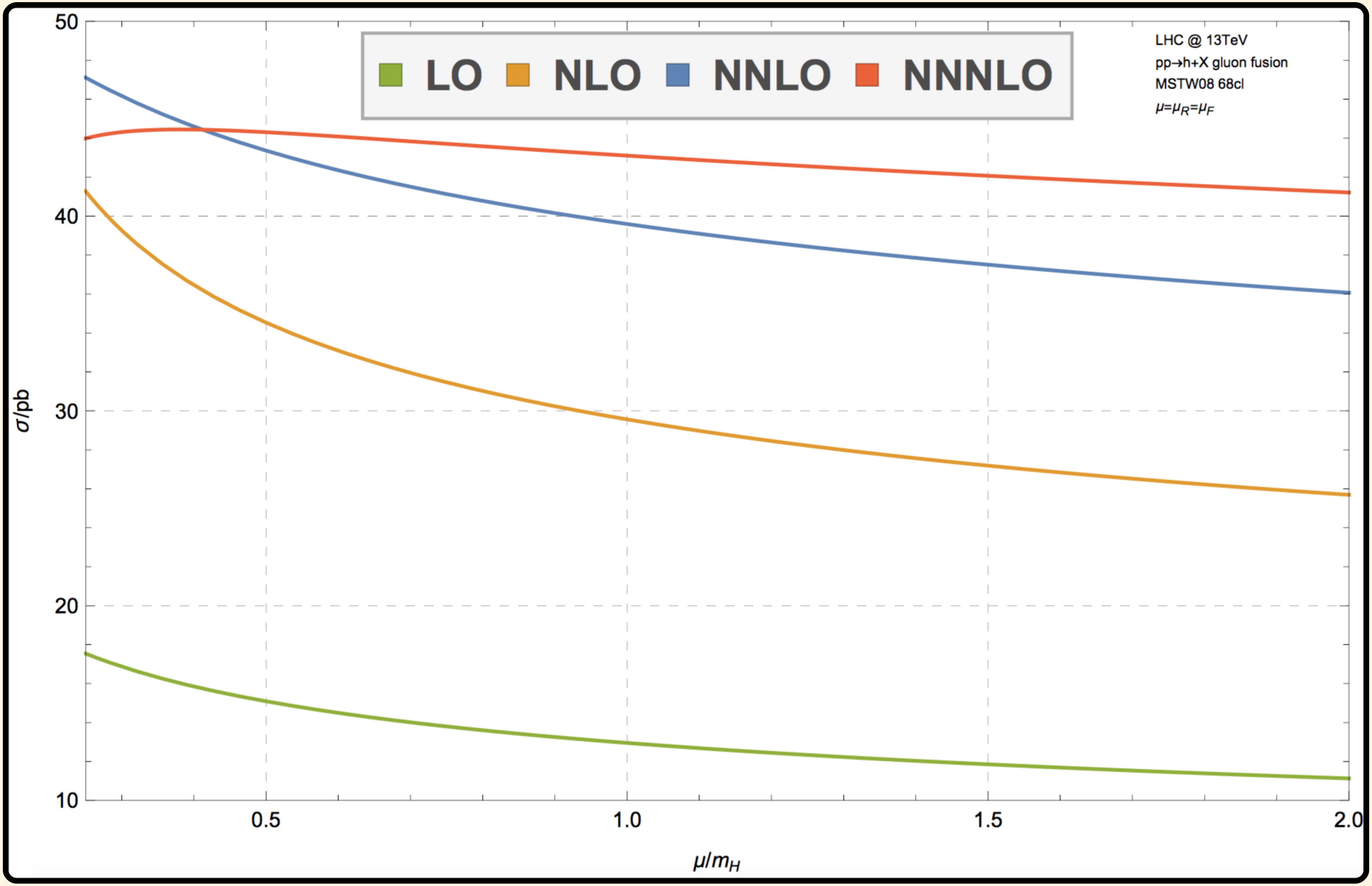
$$
\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z)\Bigg\{ & C_A^3\left(-\frac{2003}{48}\zeta_6 + \frac{413}{6}\zeta_3^2 - \frac{7579}{144}\zeta_5 + \frac{979}{24}\zeta_2\zeta_3 - \frac{15257}{864}\zeta_4 - \frac{819}{16}\zeta_3 + \frac{16151}{1296}\zeta_2 + \frac{215131}{5184}\right) \\
& + N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5 - \frac{125}{12}\zeta_3\zeta_2 + \frac{2629}{432}\zeta_4 + \frac{1231}{216}\zeta_3 - \frac{70}{81}\zeta_2 - \frac{98059}{5184}\right) \\
& \quad + C_A C_F\left(\frac{5}{2}\zeta_5 + 3\zeta_3\zeta_2 + \frac{11}{72}\zeta_4 + \frac{13}{2}\zeta_3 - \frac{71}{36}\zeta_2 - \frac{63991}{5184}\right) + C_F^2\left(-5\zeta_5 + \frac{37}{12}\zeta_3 + \frac{19}{18}\right)\Bigg] \\
& + N_F^2\left[C_A\left(-\frac{19}{36}\zeta_4 + \frac{43}{108}\zeta_3 - \frac{133}{324}\zeta_2 + \frac{2515}{1728}\right) + C_F\left(-\frac{1}{36}\zeta_4 - \frac{7}{6}\zeta_3 - \frac{23}{72}\zeta_2 + \frac{4481}{2592}\right)\right]\Bigg\} \\
& + \left[\frac{1}{1-z}\right]_+ \Bigg\{C_A^3\left(186\zeta_5 - \frac{725}{6}\zeta_3\zeta_2 + \frac{253}{24}\zeta_4 + \frac{8941}{108}\zeta_3 + \frac{8563}{324}\zeta_2 - \frac{297029}{23328}\right) + N_F^2 C_A\left(\frac{5}{27}\zeta_3 + \frac{10}{27}\zeta_2 - \frac{58}{729}\right) \\
& \quad + N_F\left[C_A^2\left(-\frac{17}{12}\zeta_4 - \frac{475}{36}\zeta_3 - \frac{2173}{324}\zeta_2 + \frac{31313}{11664}\right) + C_A C_F\left(-\frac{1}{2}\zeta_4 - \frac{19}{18}\zeta_3 - \frac{1}{2}\zeta_2 + \frac{1711}{864}\right)\right]\Bigg\} \\
& + \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(-77\zeta_4 - \frac{352}{3}\zeta_3 - \frac{152}{3}\zeta_2 + \frac{30569}{648}\right) + N_F^2 C_A\left(-\frac{4}{9}\zeta_2 + \frac{25}{81}\right) \\
& \quad + N_F\left[C_A^2\left(\frac{46}{3}\zeta_3 + \frac{94}{9}\zeta_2 - \frac{4211}{324}\right) + C_A C_F\left(6\zeta_3 - \frac{63}{8}\right)\right]\Bigg\} \\
& + \left[\frac{\log^2(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(181\zeta_3 + \frac{187}{3}\zeta_2 - \frac{1051}{27}\right) + N_F\left[C_A^2\left(-\frac{34}{3}\zeta_2 + \frac{457}{54}\right) + \frac{1}{2}C_A C_F\right] - \frac{10}{27}N_F^2 C_A\Bigg\} \\
& + \left[\frac{\log^3(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(-56\zeta_2 + \frac{925}{27}\right) - \frac{164}{27}N_F C_A^2 + \frac{4}{27}N_F^2 C_A\Bigg\} \\
& + \left[\frac{\log^4(1-z)}{1-z}\right]_+ \left(\frac{20}{9}N_F C_A^2 - \frac{110}{9}C_A^3\right) + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 8\,C_A^3 .
\end{aligned}
$$

[Anastasiou, Duhr, FD, Furlan, Gehrmann, Herzog, Mistlberger]

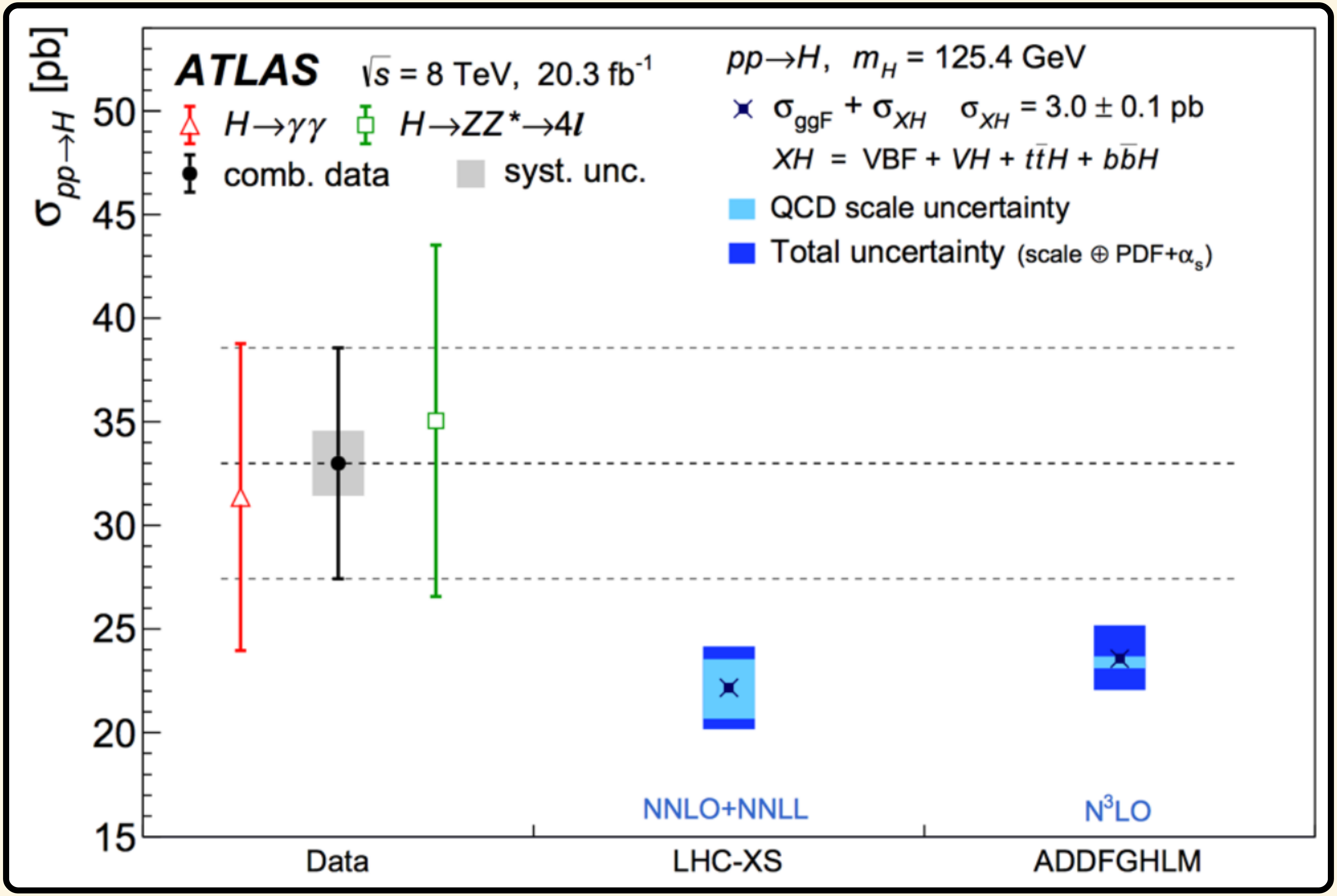
The cross section



The cross section



The cross section



Conclusions

- We finished the first ever calculation at N3LO for a hadron collider.
- New state of the art of perturbative QCD.
- Important result for Higgs physics at the LHC.
- Made possible with the use of many exciting developments in the amplitudes community.
- We will compute more processes at N3LO.
- Maybe we can learn something from comparing different orders in QCD.