Precision calculations for the LHC

or

From amplitudes to cross sections

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Amplitudes 2015, Zurich
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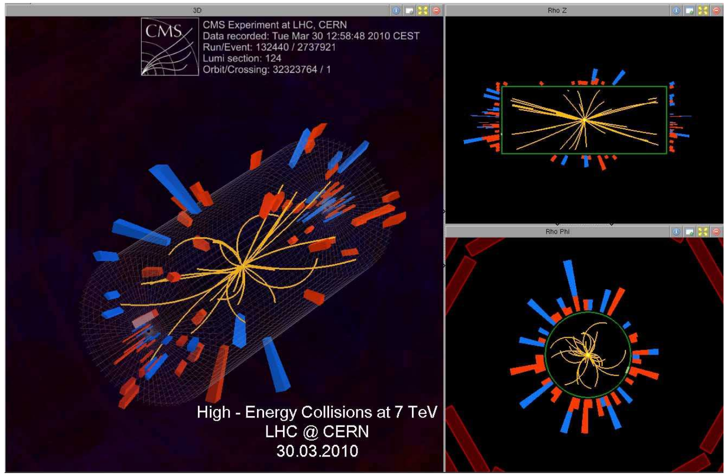
From SU(3)xSU(2)xU(1) Lagrangian to Data

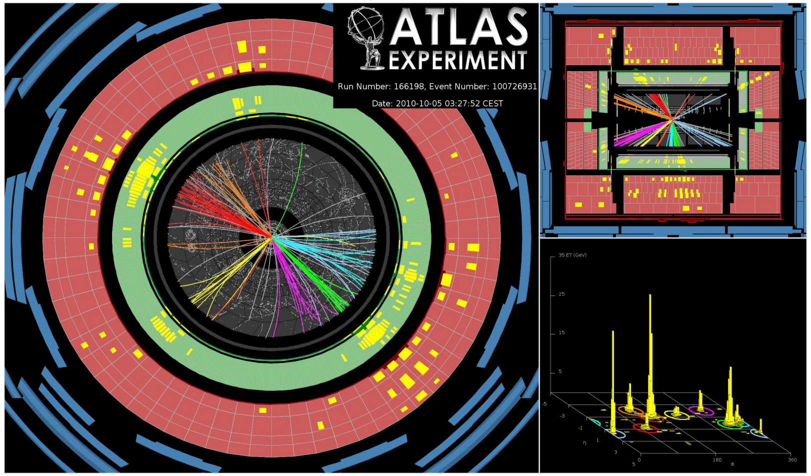
$$\mathcal{L} \sim -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi$$

$$+y_{ij}\bar{\psi}_i\phi\psi_j + h.c. + |D_\mu\phi|^2 - V(\phi)$$



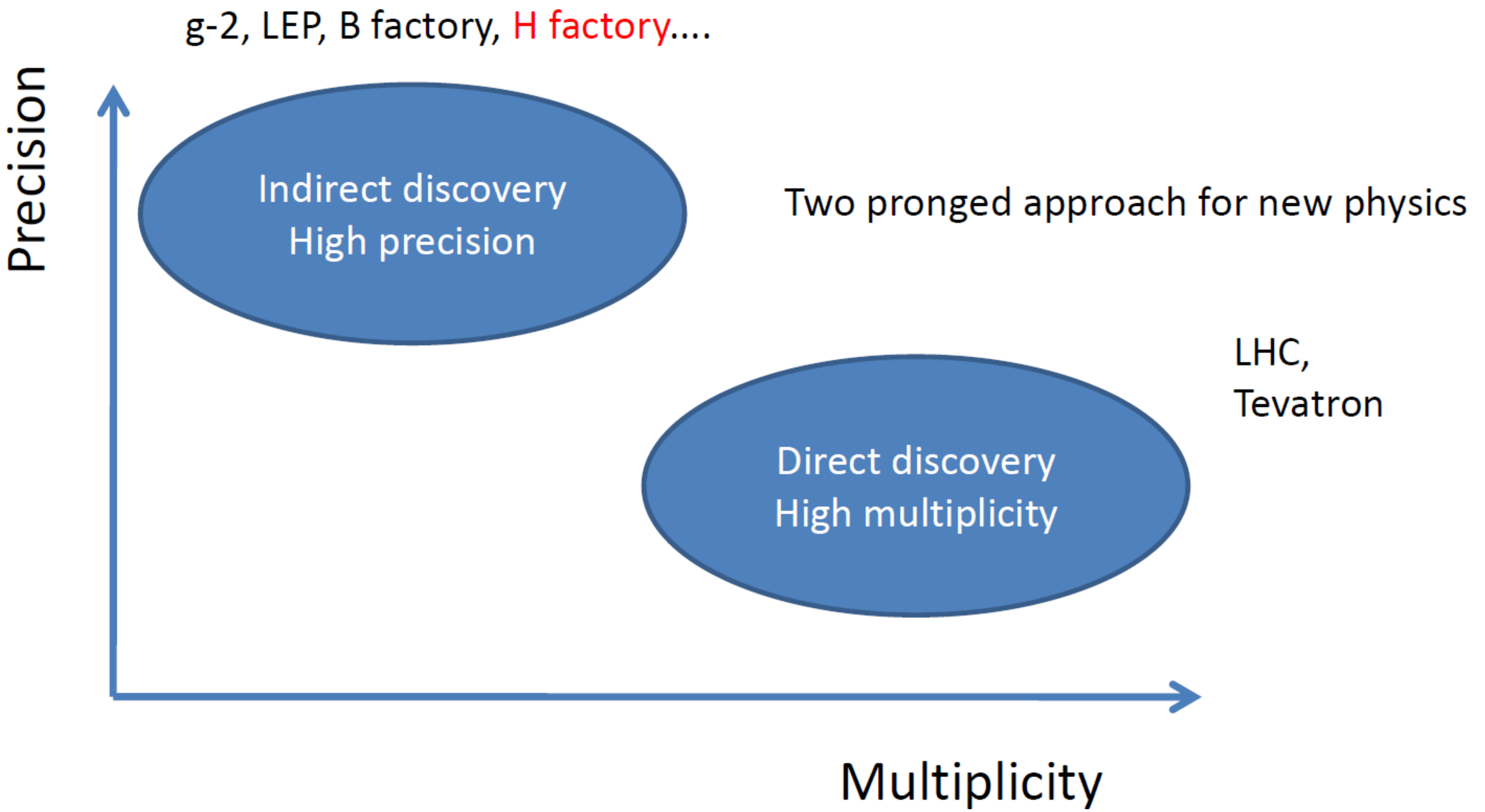
CMS Experiment at LHC, CERN
Data recorded: Tue Mar 30 12:58:48 2010 CEST
Run/Event: 132440 / 2737921
Lumi section: 124
Orbit/Crossing: 32323764 / 1

High - Energy Collisions at 7 TeV
LHC @ CERN
30.03.2010



Run Number: 166198, Event Number: 100726931
Date: 2010-10-05 03:27:52 CEST

The Task for Experimental Particle Physics



g-2, LEP, B factory, H factory....

Precision

Indirect discovery
High precision

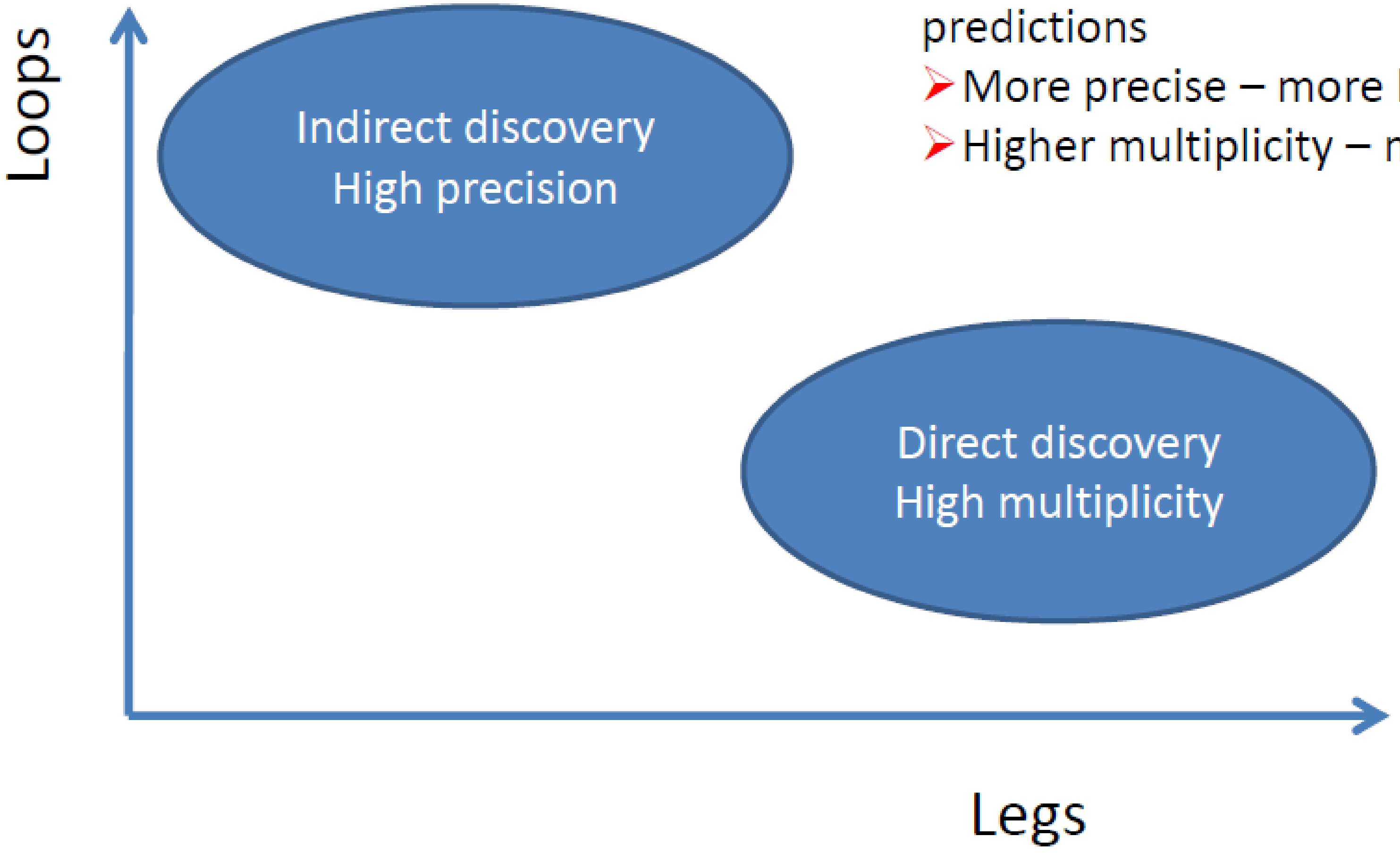
Two pronged approach for new physics

LHC,
Tevatron

Direct discovery
High multiplicity

Multiplicity

The Task for Theoretical Particle Physics

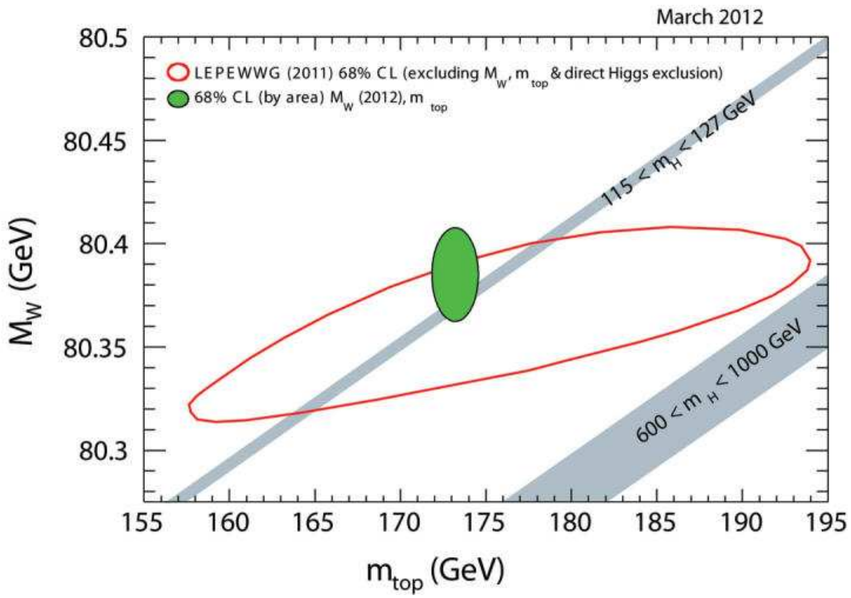


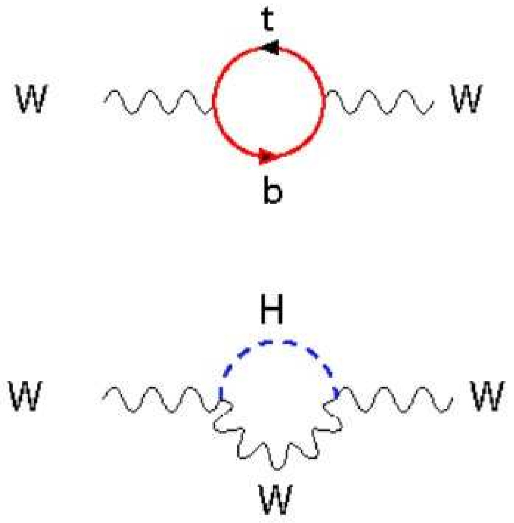
Perturbative field theory provides a systematic basis for making predictions

- More precise – more loops
- Higher multiplicity – more legs

Theoretical input

- ✓ vital in making precise perturbative predictions in quantum field theory, in general, and in the Standard Model of particle physics, in particular.
- ✓ precise data enables information on new physics to be extracted indirectly (pre-discovery)

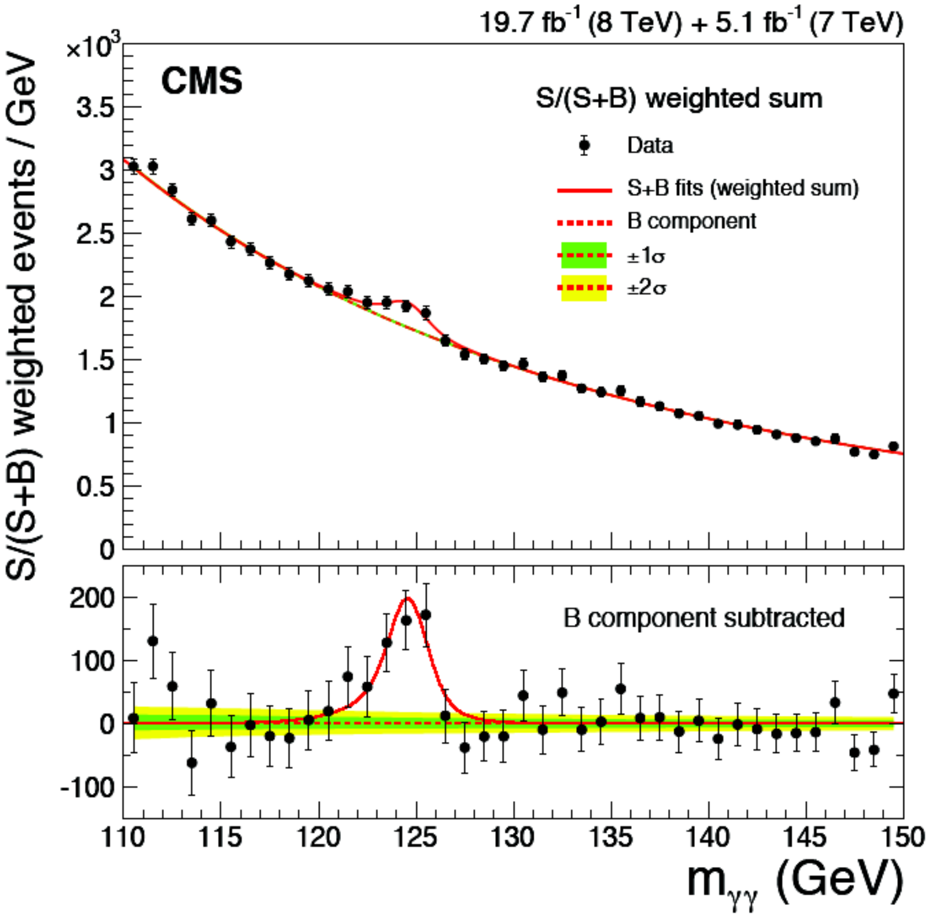




$\sim M_t^2$

$\sim \ln(m_H)$

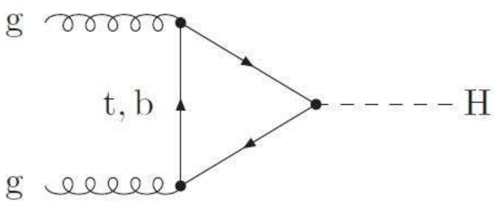
Theoretical input

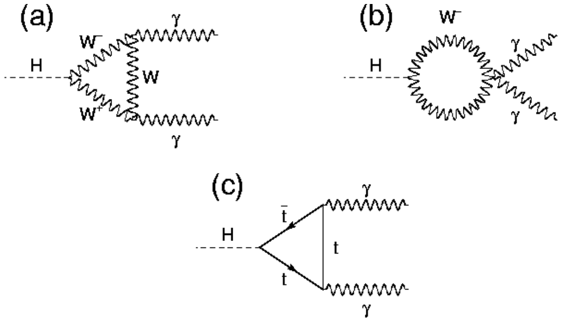


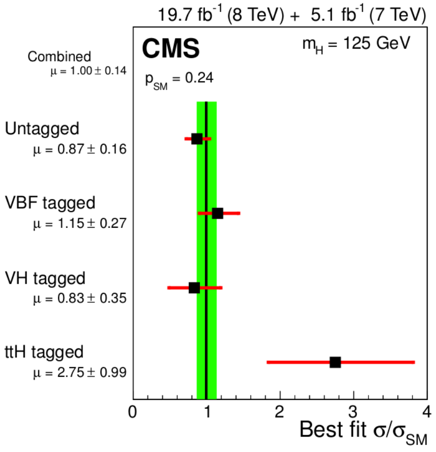
- ✓ not crucial for direct discovery!!

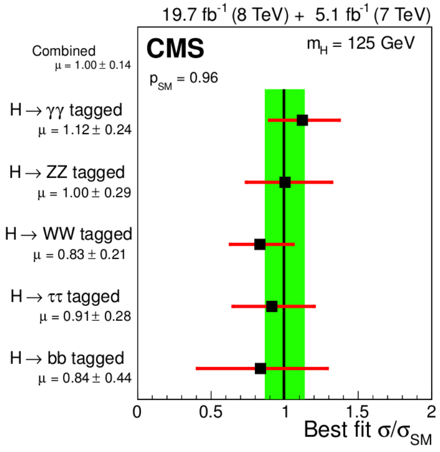
Theoretical input

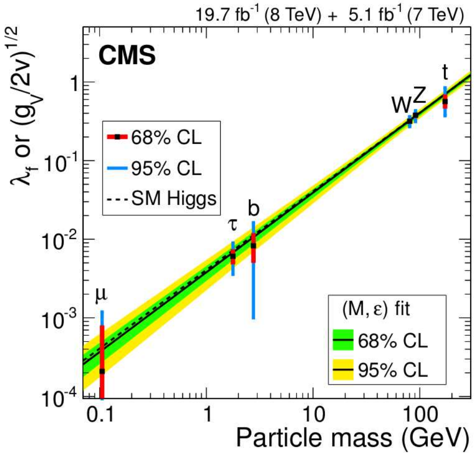
- ✓ but needed to interprete discovery as due to the production and decay of a Standard Model Scalar-like particle
- ✓ H production cross section (σ)
- ✓ H branching ratio (BR)











Why loops?

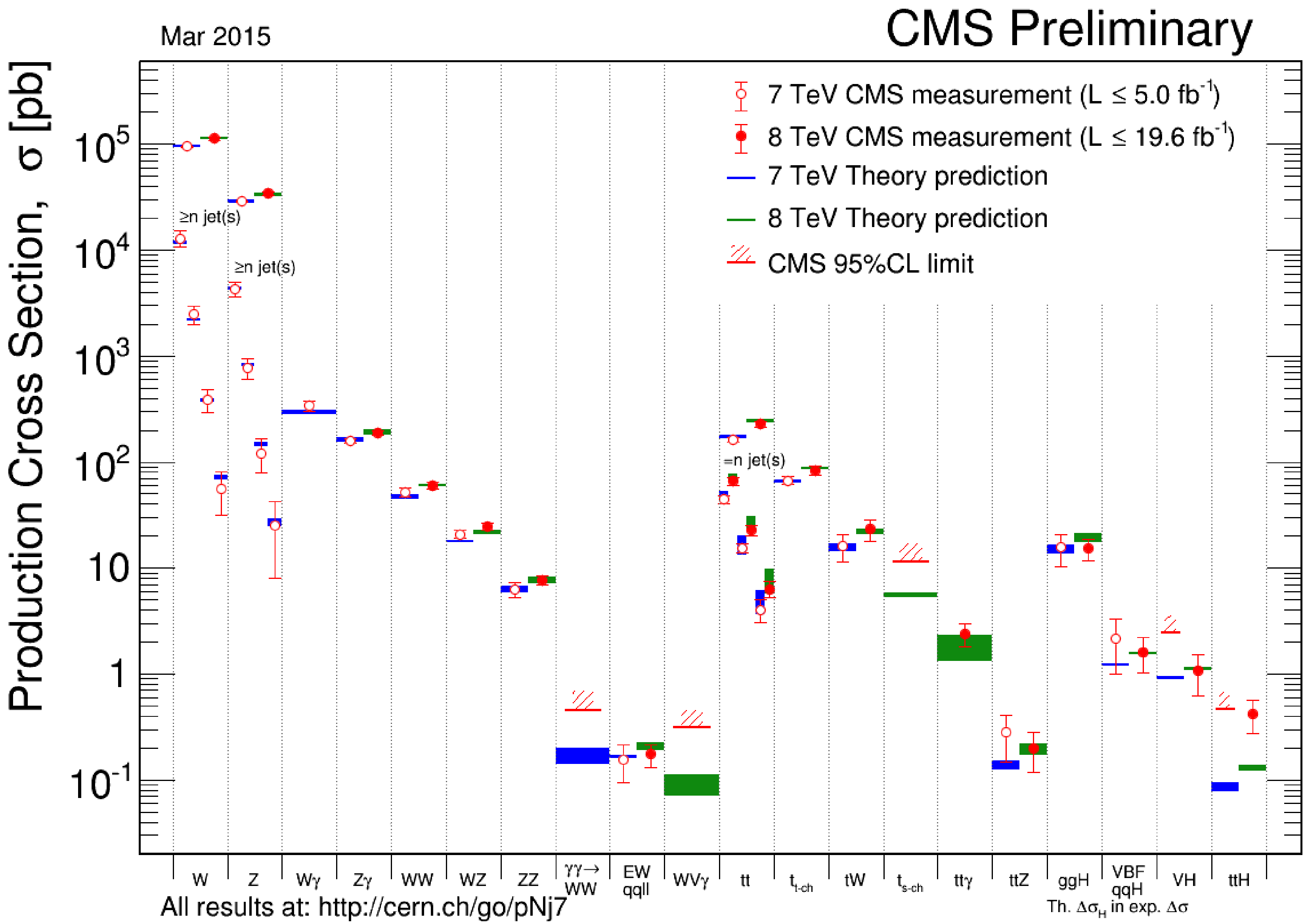
- ✓ Loop integrals play an intrinsic part in
  - (a) the interpretation of experimental discoveries at the high energy frontier
  - (b) extracting precise information from precision experiments
  - (c) in making the case for the physics potential of future high energy facilities

⇒ Importance of developments in Amplitudes that will be showcased at this conference

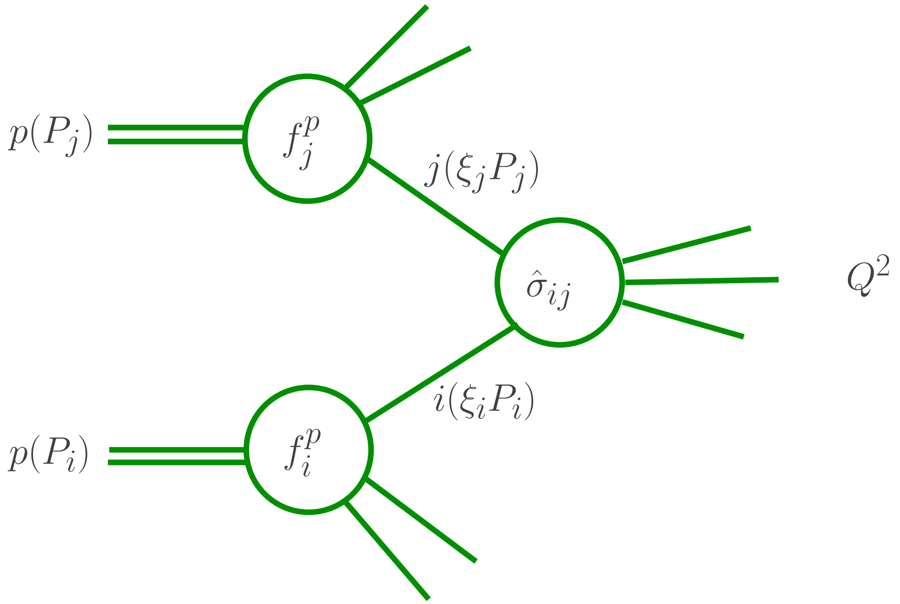
The challenge from the LHC

- ✓ Everything (signals, backgrounds, luminosity measurement) involves QCD
- ✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  - ⇒ events have high multiplicity of hard partons
  - ⇒ each hard parton fragments into a cluster of collimated particles jet
  - ⇒ higher order perturbative corrections can be large
  - ⇒ theoretical uncertainties can be large
- ✓ Processes can involve multiple energy scales: e.g. p_T^W and M_W
  - ⇒ may need resummation of large logarithms
- ✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  - ⇒ importance of infrared safe jet definition
  - ⇒ accurate modelling of underlying event, hadronisation, ...

Cross Sections at the LHC



Theoretical Framework



$$\sigma(Q^2) = \int \sum_{i,j} d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F) \otimes f_j^p(\mu_F) \qquad \left[+\mathcal{O}\left(\frac{1}{Q^2}\right)\right]$$

- ✓ partonic cross sections $d\hat{\sigma}_{ij}$
- ✓ running coupling $\alpha_s(\mu_R)$
- ✓ parton distributions $f_i(x, \mu_F)$
- ✓ renormalization/factorization scale μ_R, μ_F
- ✓ jet algorithm + parton shower + hadronisation model + underlying event + ...

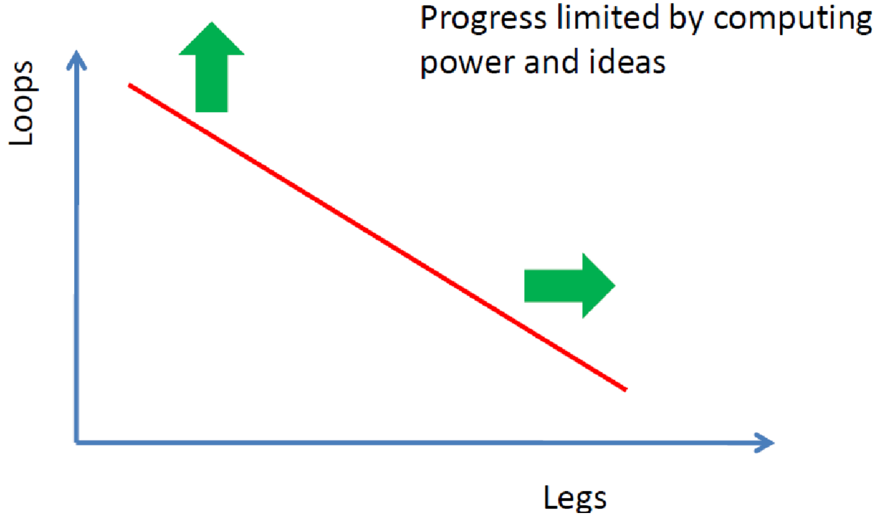
Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

Goal: Reduce theory certainties by a factor of two compared to where we are now in next decade

What is the hold up?

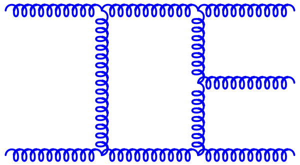
Rough idea of complexity of process $\sim$ #Loops + #Legs (+ #Scales)



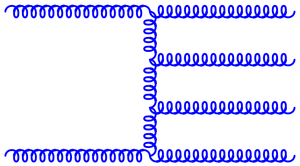
- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

Current standard: NLO

Anatomy of a NLO calculation



- ✓ one-loop $2 \rightarrow 3$ process
  - ✓ explicit infrared poles from loop integral
  - ✓ looks like 3 jets in final state



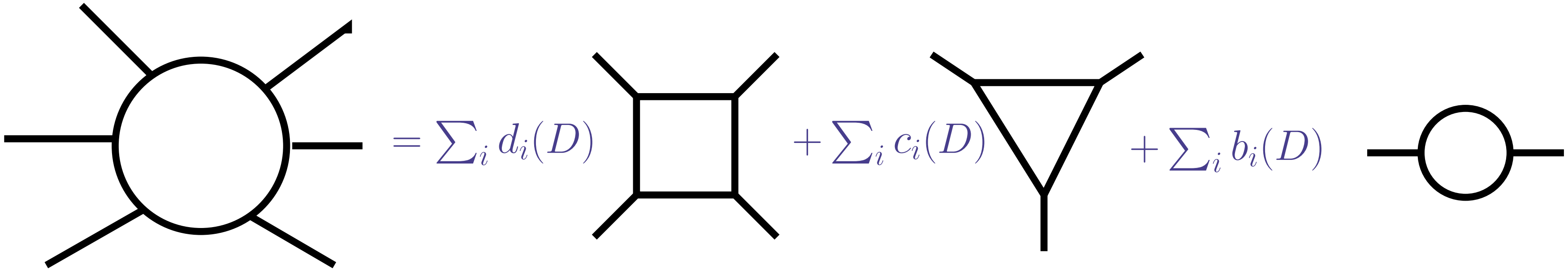
- ✓ tree-level $2 \rightarrow 4$ process
  - ✓ implicit poles from soft/collinear emission
  - ✓ looks like 3 or 4 jets in final state

- ✓ plus method for combining the infrared divergent parts
  - ✚ dipole subtraction — Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf
  - ✚ residue subtraction — Frixione, Kunszt, Signer
  - ✚ antenna subtraction — Kosower; Campbell, Cullen, NG; Daleo, Gehrmann, Maitre
- ✓ automated subtraction tools Gleisberg, Krauss (SHERPA); Hasegawa, Moch, Uwer (AutoDipole); Frederix, Gehrmann, Greiner (MadDipole); Seymour, Tevlin (TeVJet), Czakon, Papadopoulos, Worek (Helac/Phegas) and Frederix, Frixione, Maltoni, Stelzer (MadFKS)

For a long time bottleneck was the one-loop amplitudes

The one-loop problem

Any (massless) one-loop integral can be written as



$$\mathcal{M} = \sum d(D)\text{boxes(D)} + \sum c(D)\text{triangles(D)} + \sum b(D)\text{bubbles(D)}$$

- ✓ higher polygon contributions drop out
- ✓ scalar loop integrals are known analytically around $D = 4$ Ellis, Zanderighi (08)
- ✓ need to compute the D-dimensional coefficients $d(D)$ etc.

The problem is complexity - the number of terms generated is too large to deal with, even with computer algebra systems, and there can be very large cancellations.

Unitarity for one-loop diagrams

Several important breakthroughs

- ✓ Sewing trees together

  Bern, Dixon, Dunbar, Kosower (94)

- ✓ Freezing loop momenta with quadruple cuts

  Britto, Cachazo, Feng (04)

- ✓ OPP tensor reduction of integrand

  Ossola, Pittau, Papadopoulos (06)

- ✓ D-dimensional unitarity

  Giele, Kunzst, Melnikov (08)

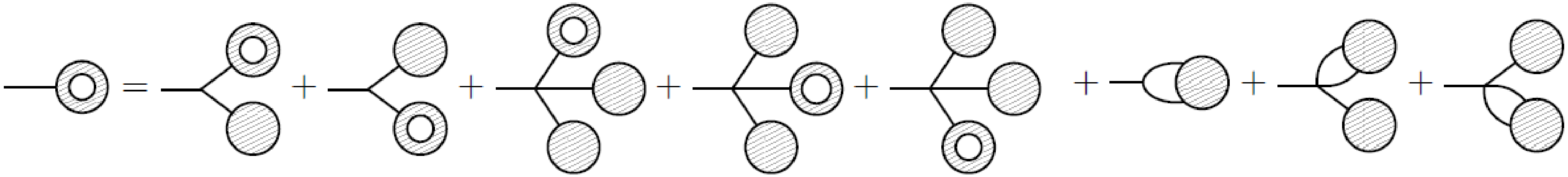
$\Longrightarrow$ automation

HELAC/CutTools, Rocket, BlackHat+SHERPA, GoSam+SHERPA/MADGRAPH,
NJet+SHERPA, MADLOOPS+MADGRAPH

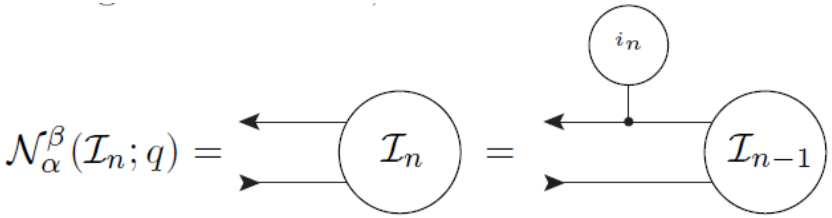
Numerical recursion for one-loop diagrams

Breakthroughs on the “traditional" side

✓ One-loop Berends-Giele recursion van Hameren (09)



✓ Recursive construction of tensor numerator Cascioli, Maierhöfer, Pozzorini (11)

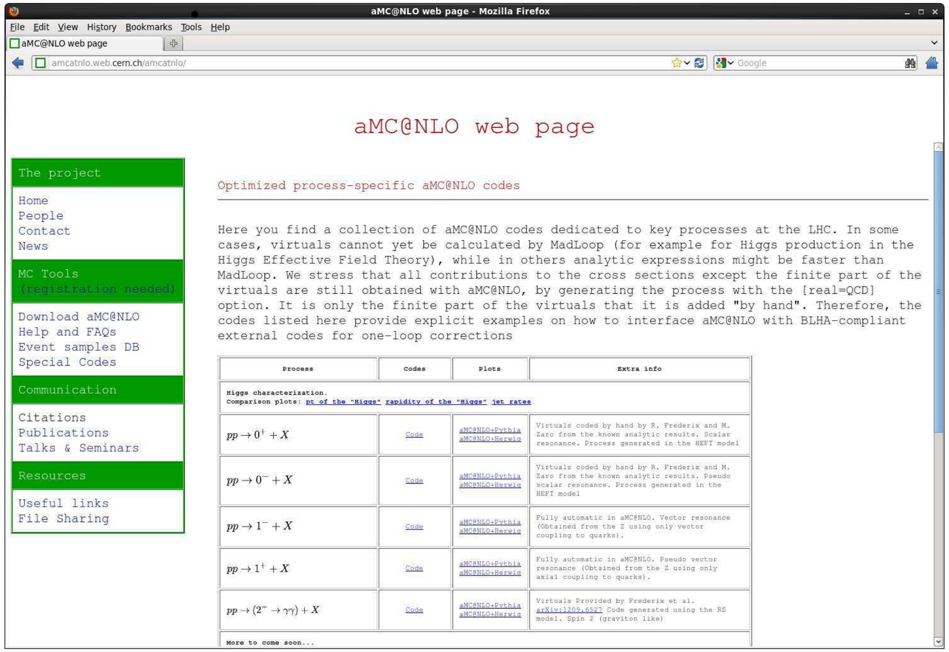


$\Longrightarrow$ automation

OpenLoops+SHERPA, RECOLA

NLO - the new standard

- ✓ A lot of progress, and the “best" solution is still to emerge. In the meantime, there are public codes with NLO capability that could only be dreamed of a few years ago.



aMC@NLO web page

Optimized process-specific aMC@NLO codes

Here you find a collection of aMC@NLO codes dedicated to key processes at the LHC. In some cases, virtuals cannot yet be calculated by MadLoop (for example for Higgs production in the Higgs Effective Field Theory), while in others analytic expressions might be faster than MadLoop. We stress that all contributions to the cross sections except the finite part of the virtuals are still obtained with aMC@NLO, by generating the process with the [real=QCD] option. It is only the finite part of the virtuals that it is added "by hand". Therefore, the codes listed here provide explicit examples on how to interface aMC@NLO with BLHA-compliant external codes for one-loop corrections

Process	Codes	Plots	Extra info
Higgs characterization. Comparison plots: pt of the "Higgs" rapidity of the "Higgs" jet rates			
$pp \to 0^+ + X$	Code	aMC@NLO+Pythia aMC@NLO+Herwig	Virtuals coded by hand by R. Frederix and M. Zaro from the known analytic results. Scalar resonance. Process generated in the HEFT model
$pp \to 0^- + X$	Code	aMC@NLO+Pythia aMC@NLO+Herwig	Virtuals coded by hand by R. Frederix and M. Zaro from the known analytic results. Pseudo scalar resonance. Process generated in the HEFT model
$pp \to 1^- + X$	Code	aMC@NLO+Pythia aMC@NLO+Herwig	Fully automatic in aMC@NLO. Vector resonance (Obtained from the Z using only vector coupling to quarks).
$pp \to 1^+ + X$	Code	aMC@NLO+Pythia aMC@NLO+Herwig	Fully automatic in aMC@NLO. Pseudo vector resonance (Obtained from the Z using only axial coupling to quarks).
$pp \to (2^+ \to \gamma\gamma) + X$	Code	aMC@NLO+Pythia aMC@NLO+Herwig	Virtuals Provided by Frederix et al. arXiv:1209.6527 Code generated using the RS model. Spin 2 (graviton like)
More to come soon...			

SHERPA

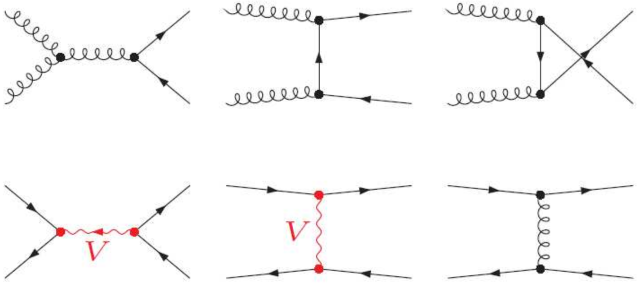
Process	BlackHat	GoSam	OpenLoops
jets	≤ 3	—	≤ 4
γ+jets	≤ 3	≤ 2	≤ 3
$\gamma\gamma$+jets	≤ 2	—	≤ 2
V+jets	≤ 4	≤ 3	≤ 3
$V + b\bar{b}$+jets	—	≤ 1	≤ 1
VV'+jets	≤ 2	≤ 2	≤ 2
$V\gamma$+jets	—	≤ 2	≤ 2
$W^\pm W^\pm qq$	—	0	0
$VV'V''$	—	—	≤ 1
$t\bar{t}$+jets	—	≤ 1	≤ 1
$t\bar{t} + V$+jets	—	—	≤ 1
$tb^\dagger$	—	—	≤ 1
$tj^\dagger$	—	—	≤ 1
$tW^\dagger$	—	—	≤ 1
h+jets	≤ 2	≤ 2	—
WBF: hqq'	—	—	≤ 1
VH	—	—	≤ 1
$t\bar{t}h$	—	—	0
$gg \to 4\ell$	—	0	0

NLO EW corrections

- ✓ Relevance and size of EW corrections
  generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW $\sim$ NNLO QCD
  but systematic enhancements possible, e.g.,
  - ✚ by photon emission, mass singular logs $\propto (\alpha)\ln(m_\ell/Q)$ for bare leptons - important for measurement of W mass
  - ✚ at high energies, EW Sudakov logs $\propto (\alpha/\sin^2\theta_W)\ln^2(M_W/Q)$
- ✓ EW corrections to PDFs at hadron colliders
  - ✚ photon PDF
- ✓ Instability of W and Z bosons
  - ✚ realistic observables have to be defined via decay products
  - ✚ off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections
- ? How to combine QCD and EW corrections in predictions?

Mixed QCD - EW corrections

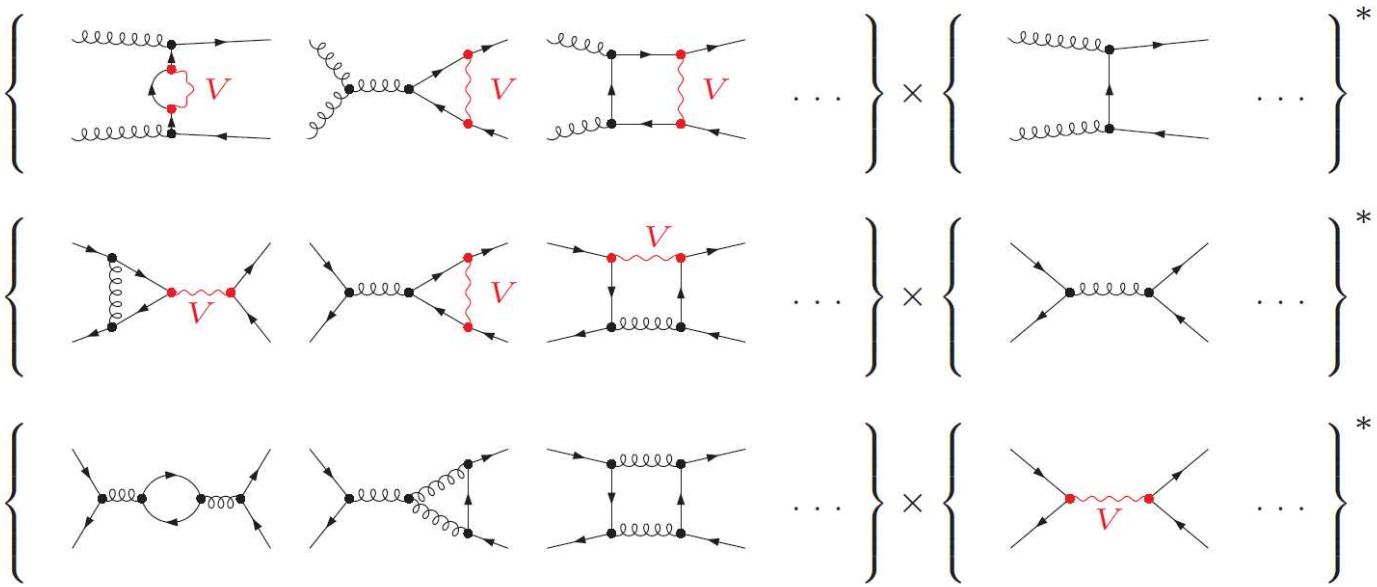
- ✓ Tree contributions: $\mathcal{O}(\alpha_s\alpha)$, $\mathcal{O}(\alpha^2)$



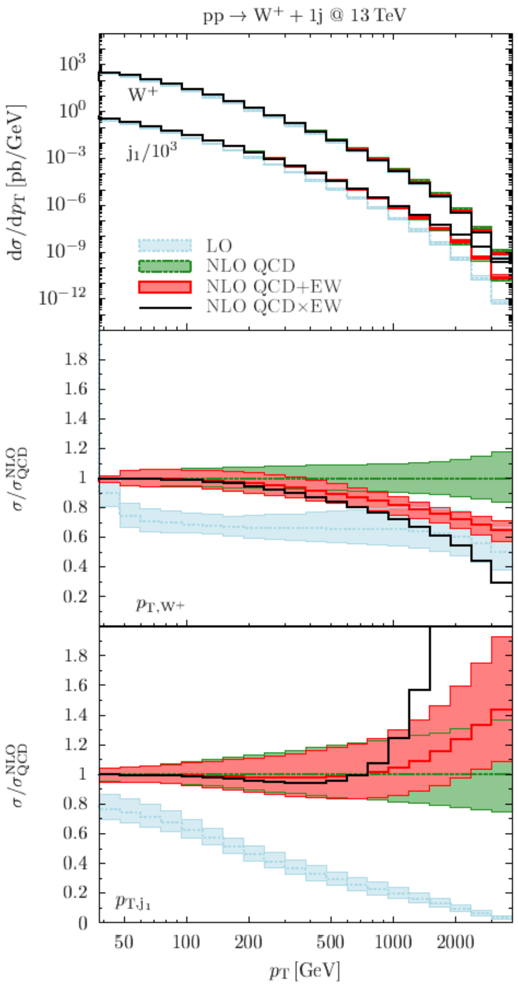
(W/Z emission suppressed in graphs)

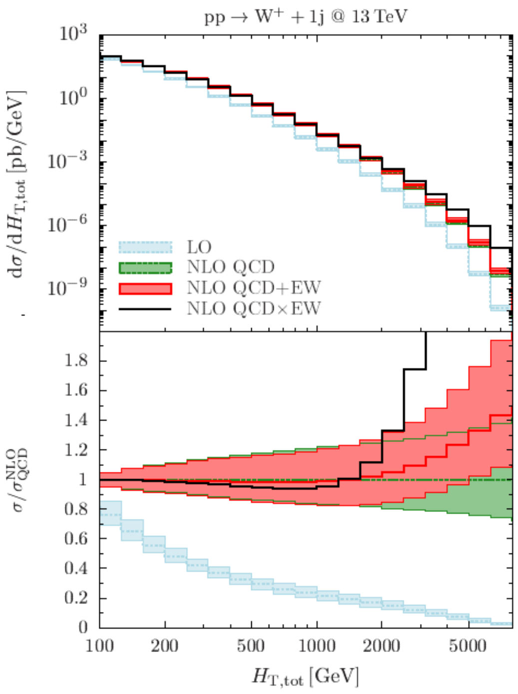
$V = \gamma, \mathrm{Z}, \mathrm{W}$

- ✓ Loop contributions: $\mathcal{O}(\alpha_s^2\alpha)$



Example: W/Z+higher jet multiplicities at NLO





- normalization to $\sigma_{\mathrm{QCD}}^{\mathrm{NLO}}$
- $\mu_{\mathrm{ren}} = \mu_{\mathrm{fact}} = \hat{H}_{\mathrm{T}} = \sum E_{\mathrm{T}}$
- $H_{\mathrm{T}}^{\mathrm{tot}} = p_{\mathrm{T,W}} + \sum p_{\mathrm{T},j_k}$

Kallweit, Lindert, Maierhoefer, Pozzorini, Schoenherr (15)

NLO precision for event simulation

Fixed order calculations

- ✓ Expansion in powers of the coupling constant
- ✓ Correctly describes hard radiation pattern
- ✓ Final states are described by single hard particles
- ✓ NLO: up to two particles in a jet, NNLO: up to three..
- ✓ Soft radiation poorly described

Parton shower

- ✓ Exponentiates multiple soft radiation (leading logarithms)
- ✓ Describes multi-particle dynamics and jet substructure
- ✓ Allows generation of full events (interface to hadronization)
- ✓ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- ✓ Fails to account for hard emissions

Ideally: combine virtues of both approaches

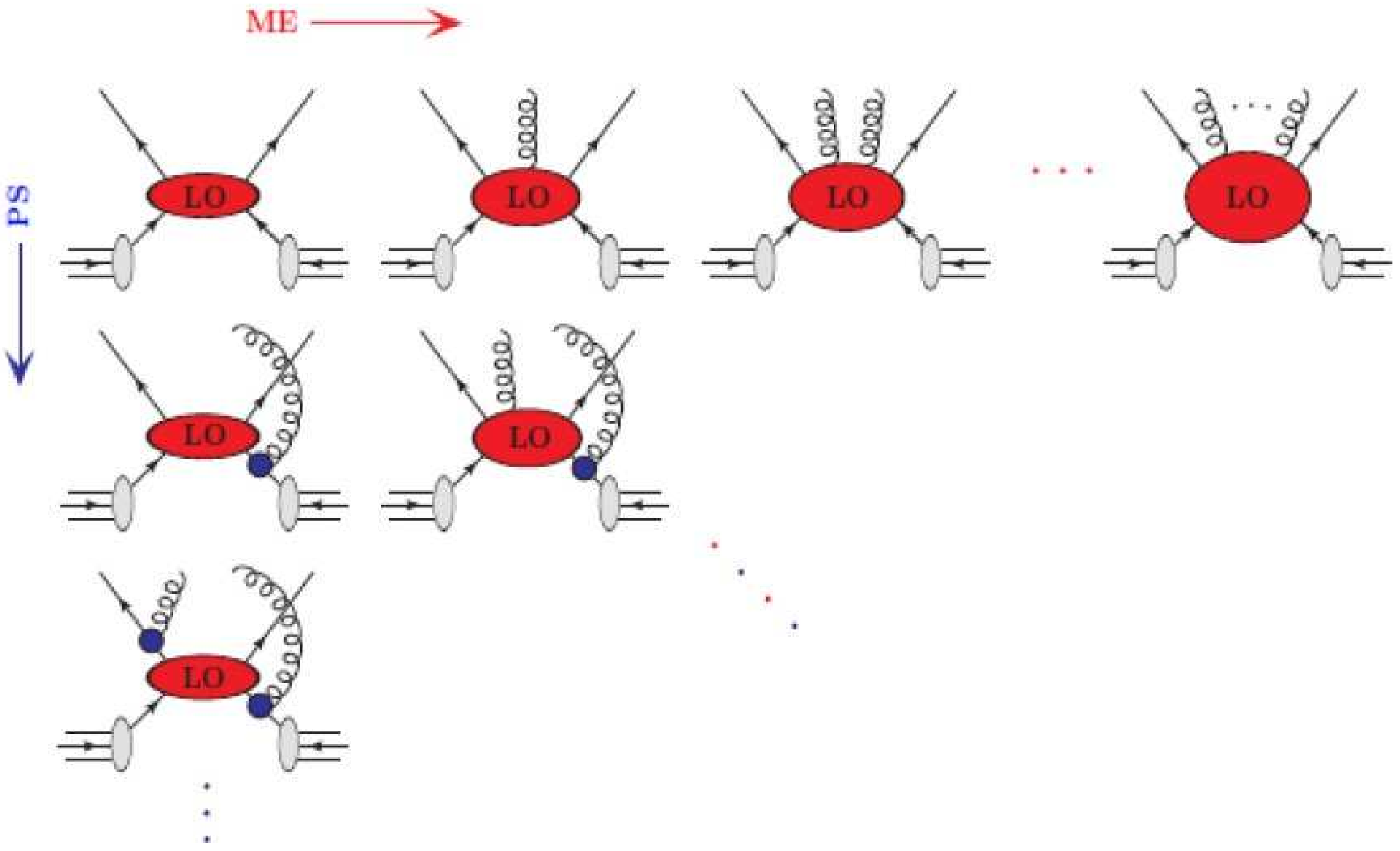
Shape: Real Radiation and Normalisation: Loops

Matrix Element improved Parton Shower

MEPS - merging

Several fixed order calculations of increasing multiplicity supplemented by PS

CKKW: Catani, Krauss, Kuhn, Webber (01); MLM: Mangano

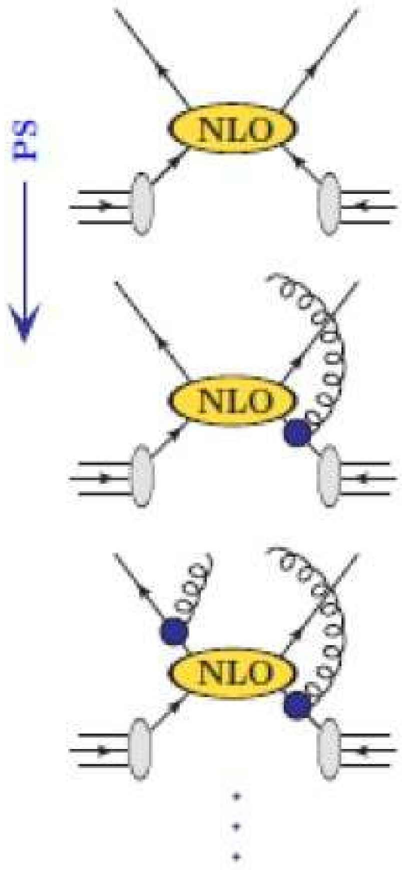


Matrix Element improved Parton Shower

NLOPS - matching

One fixed order calculation supplemented by PS

MC@NLO: Frixione, Webber (02); POWHEG: Nason, Oleari (07)

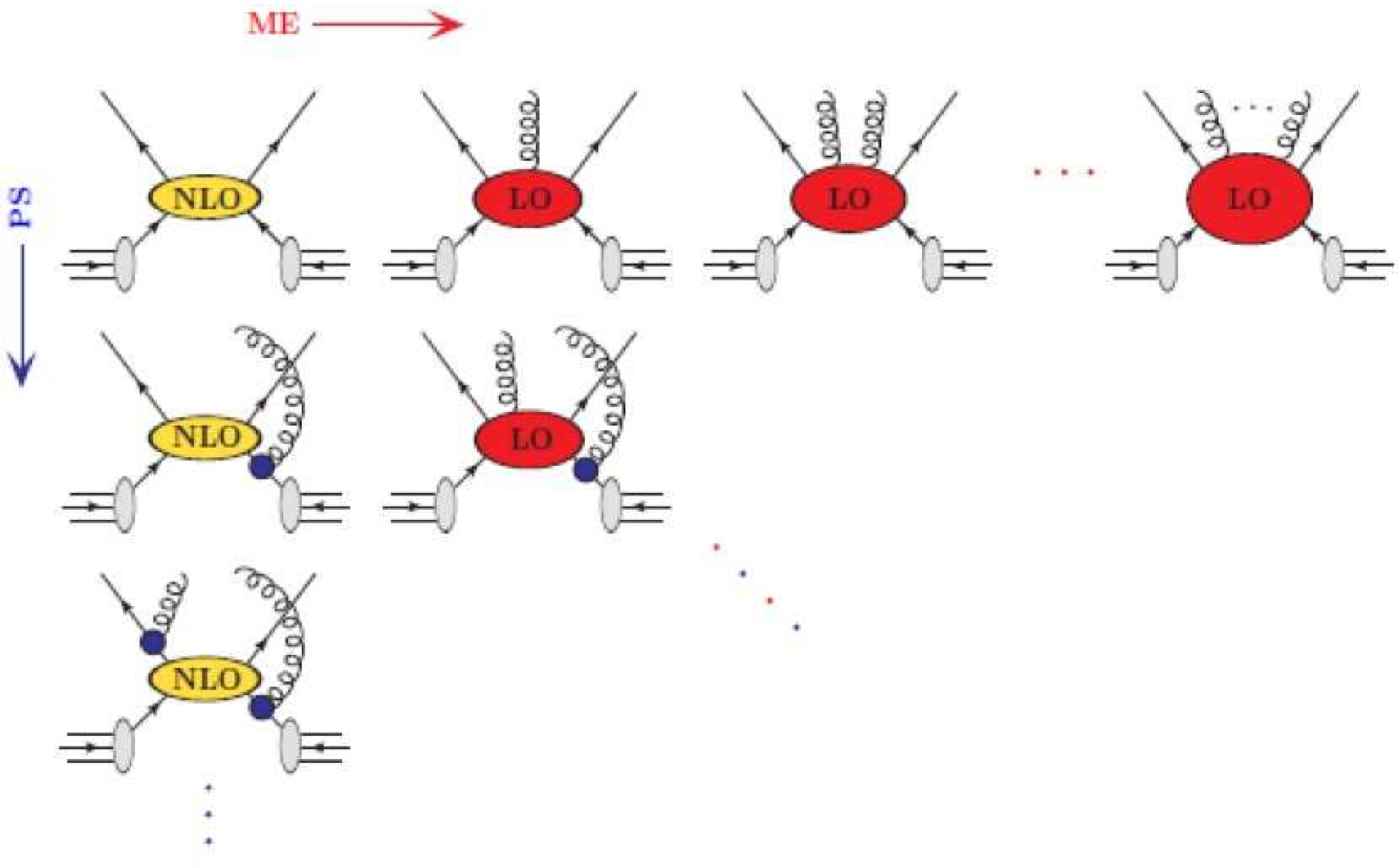


Matrix Element improved Parton Shower

MENLOPS

Supplements core NLOPS with higher multiplicity MEPS

Hamilton, Nason; Hoeche, Krauss, Schoenherr, Siegert; Lonnblad, Prestel

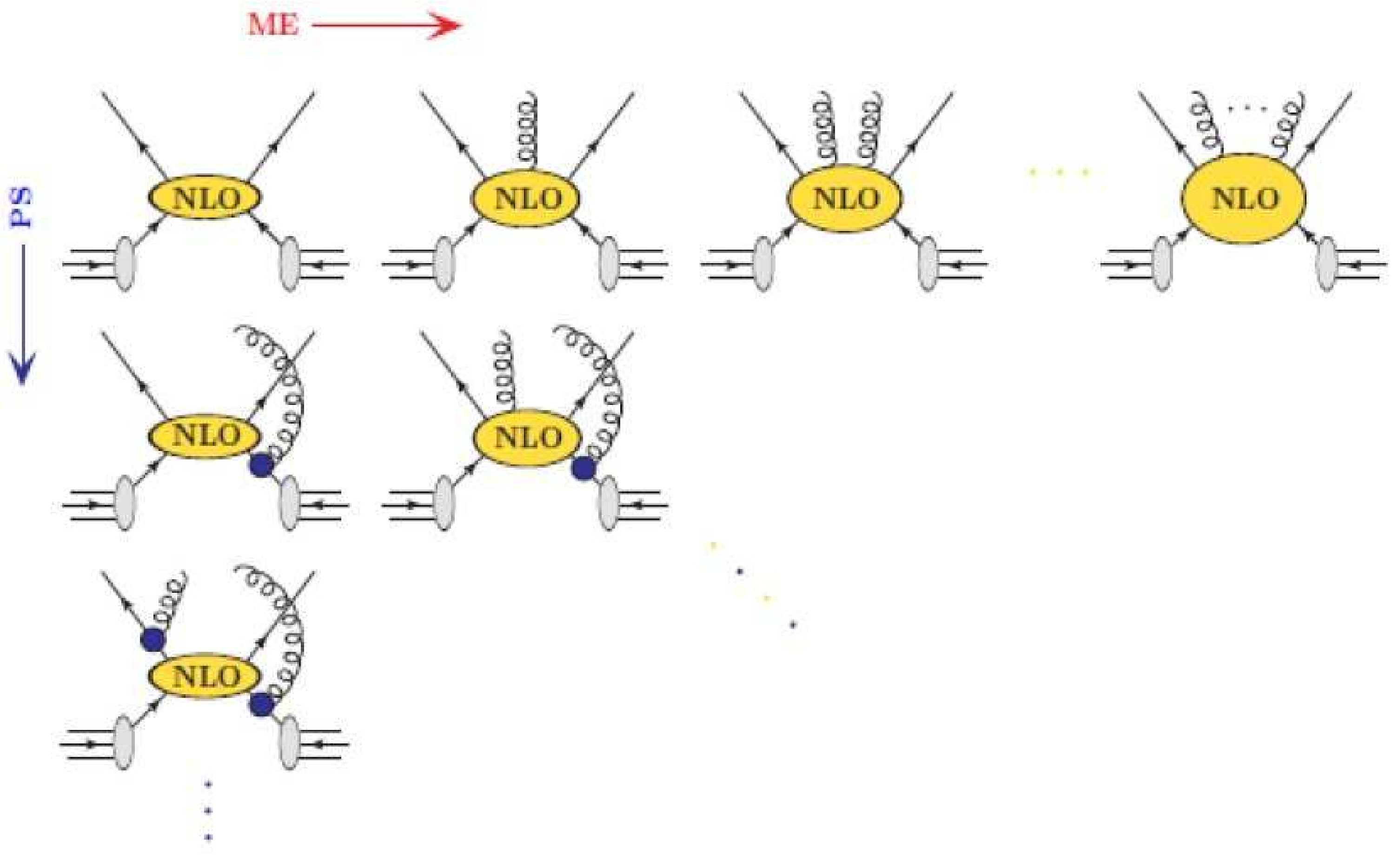


Matrix Element improved Parton Shower

MEPS@NLO (UNLOPS)

Combines multiple NLOPS

Lavesson, Lonnblad; Hoeche, Krauss, Schoenherr, Siegert; Frederix, Frixione



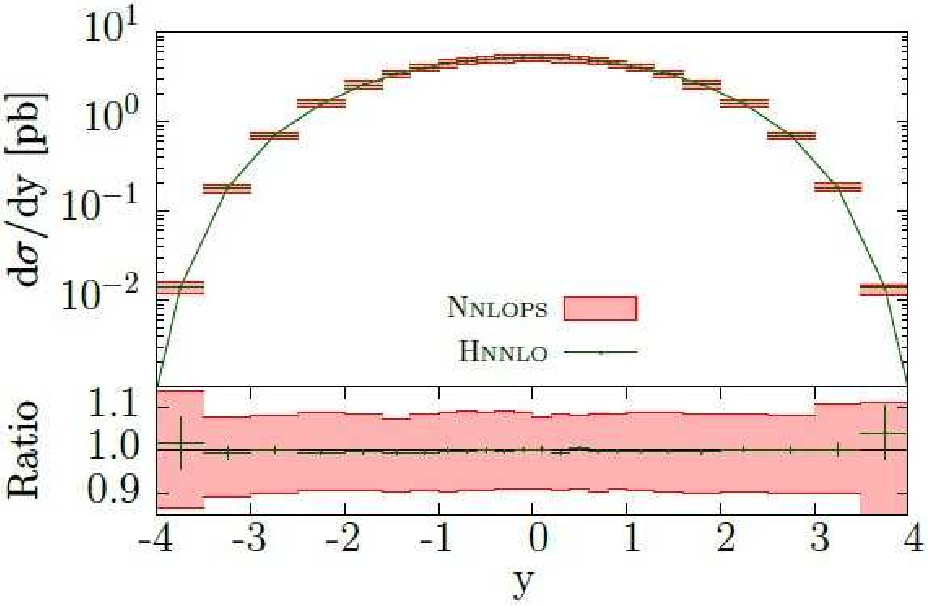
Reaching NNLOPS accuracy

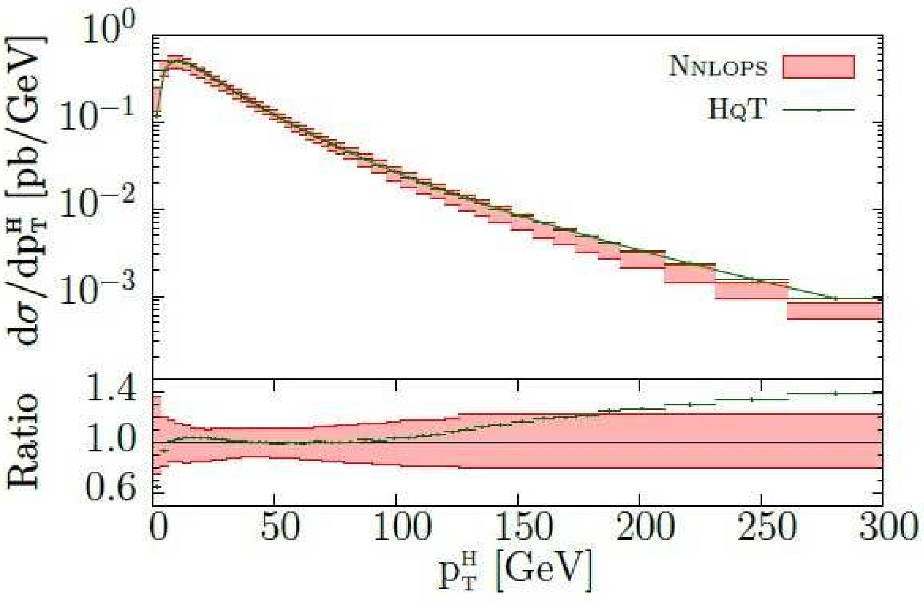
MINLO

Multiscale improved NLO CKKW scale for Born pieces
Sudakov form factors for Born functions in POWHEG

Hamilton, Nason, Zanderighi

Exciting idea! starting from HJ@NLO+PS generate H rapidity distribution at NNLO





Hamilton, Nason, Oleari, Re, Zanderighi

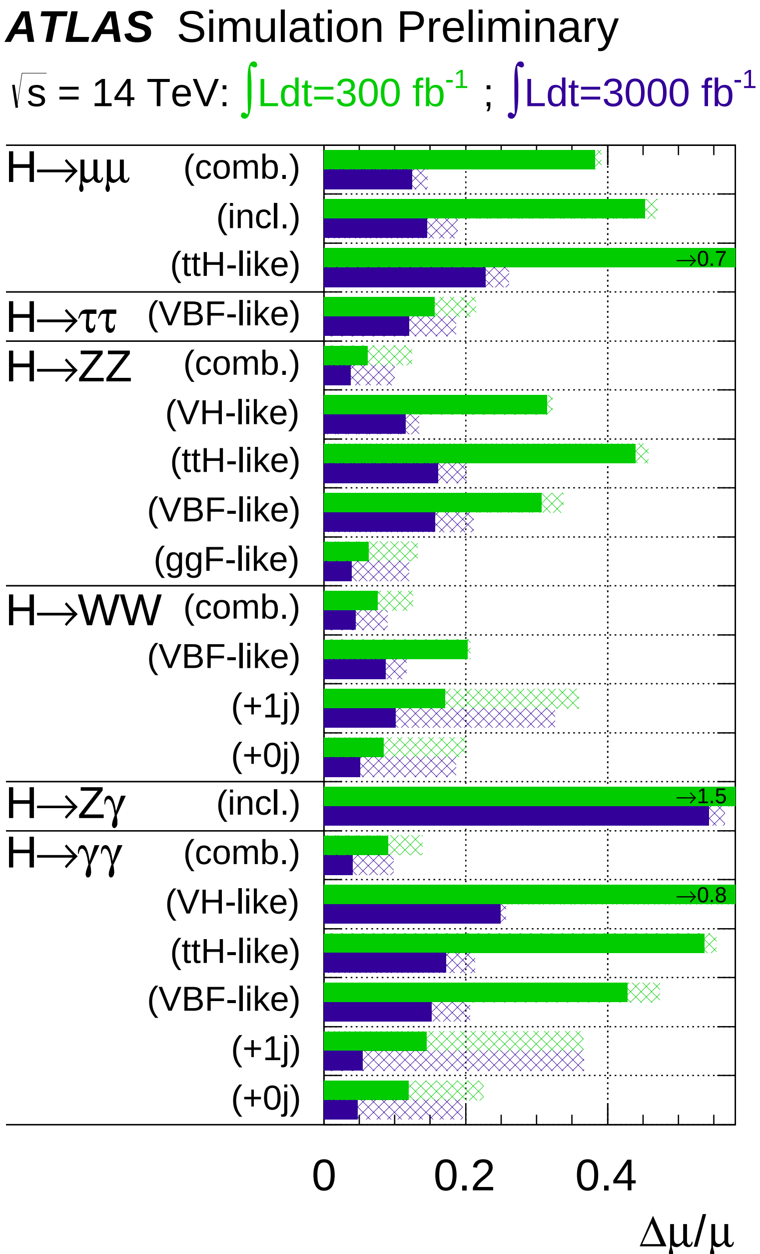
Motivation for more precise theoretical calculations

- ✓ Estimated signal strengths with larger LHC data set

ATL-PHYS-PUB-2013-014

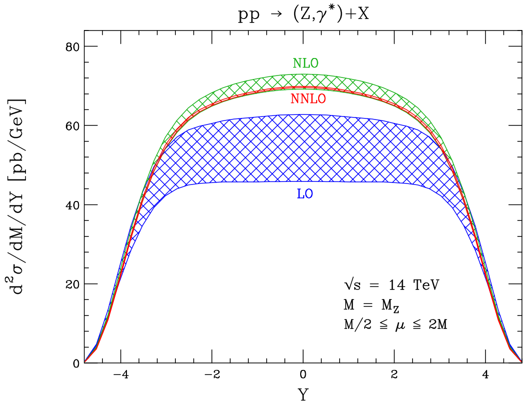
- ✓ Theory uncertainty has big impact on measurement
- ✓ Revised wishlist of theoretical predictions for
  - ✚ Higgs processes
  - ✚ Processes with vector bosons
  - ✚ Processes with heavy quarks or jets

1405.1067

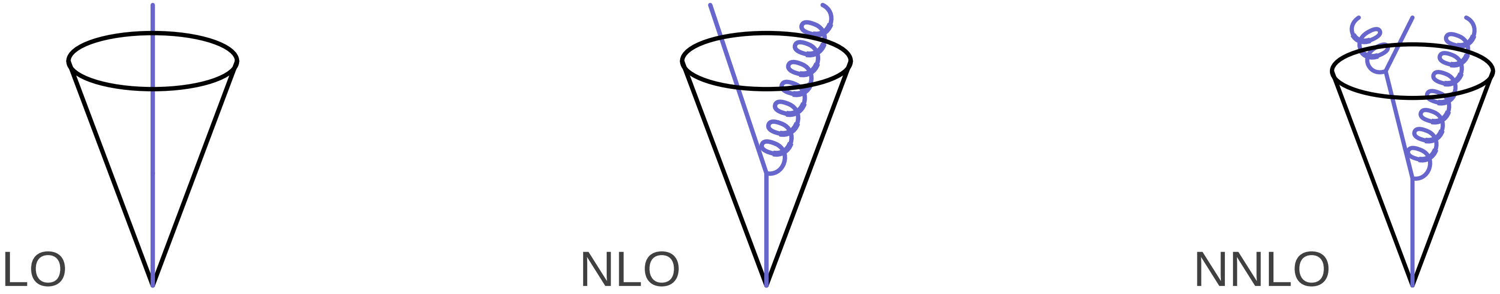


What NNLO might give you

✓ Reduced renormalisation scale dependence



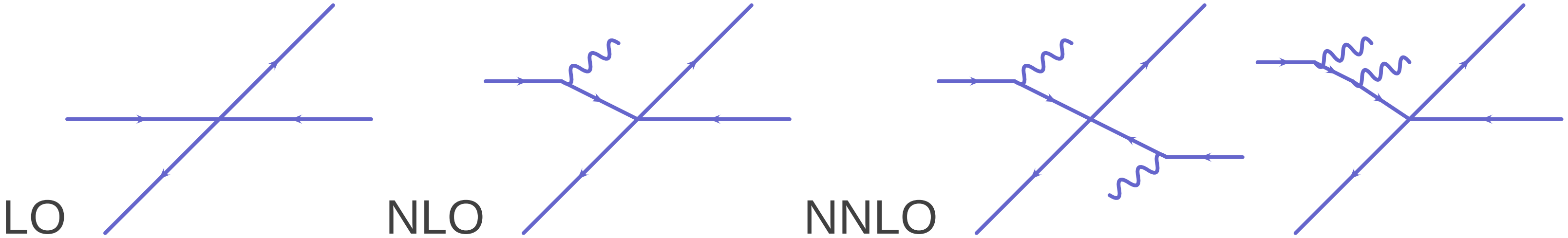
✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower
$\Rightarrow$ better matching of jet algorithm between theory and experiment



✓ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$

Motivation for NNLO

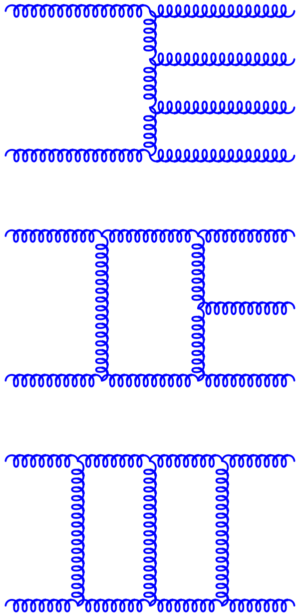
- ✓ Better description of transverse momentum of final state due to double radiation off initial state



  - ✓ At LO, final state has no transverse momentum
  - ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
  - ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state
- ✓ NNLO provides the first serious estimate of the error

✓✓✓ and most importantly, the volume and quality of the LHC data!!

Anatomy of a NNLO calculation e.g. $pp \to 2j$



- ✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$
  - ✓ implicit poles from double unresolved emission
- ✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ implicit poles from soft/collinear emission
- ✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$
  - ✓ explicit infrared poles from loop integral
  - ✓ including square of one-loop amplitude

$$\mathrm{d}\hat{\sigma}_{NNLO} \quad \sim \quad \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

NNLO - amplitudes

- ✓ small number of two loop matrix elements known
  - ✓ $2 \to 1$: $q\bar{q} \to V$, $gg \to H$, ($q\bar{q} \to VH$)
  - ✓ $2 \to 2$: massless parton scattering, e.g. $gg \to gg$, $q\bar{q} \to gg$, etc
  - ✓ $2 \to 2$: processes with one offshell leg, e.g. $q\bar{q} \to V$+jet, $gg \to H$+jet
  - ✓ $2 \to 2$: $q\bar{q} \to t\bar{t}$, $gg \to t\bar{t}$ known numerically Bärnreuther, Czakon, Mitov
  - ✓ $2 \to 2$: $q\bar{q} \to VV$, $gg \to VV$ new results in 2014 Cascioli et al
  - ✓ $2 \to 3$: $gg \to ggg$ first results in 2014 Badger, Frellesvig, Zhang
- ?? Basis set of master integrals
- ?? Efficient evaluation of master integrals
- ?? Far from Automation

✓ ✓ Eager to have input from Amplitudes community

IR subtraction at NNLO

- ✓ The aim is to recast the NNLO cross section in the form

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right]
\end{aligned}
$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

NNLO - IR subtraction schemes

We do not have a fully general subtraction scheme as we have at NLO

Five main methods:

- Antenna subtraction — Gehrmann, Gehrmann-De Ridder, NG (05)
- q_T subtraction — Catani,Grazzini (07)
- Colourful subtraction — Del Duca, Somogyi, Tronsanyi
- Stripper — Czakon (10); Boughezal et al (11)
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello (15); Gaunt, Stahlhofen, Tackmann, Walsh (15)

Each method has its advantages and disadvantages

	Analytic	FS Colour	IS Colour	Local
Antenna	✓	✓	✓	✗
qT	✓	✗	✓	✓
Colourful	✓	✓	✗	✓
Stripper	✗	✓	✓	✓
N-jettiness	✓	✓	✓	✓

Higgs production at N3LO $m_t \to \infty$

- ✓ Aim to reduce the theoretical error for the inclusive Higgs cross section via gluon fusion to $\mathcal{O}(5\%)$
  - ✘ In principle, need double box with top-quark loop! - currently not known
  - ✓ Higgs boson is lighter than the top-pair threshold
  - ✓ $1/m_t$ corrections known to be small at NNLO
- ⇒ Work in effective theory where top quark is integrated out

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 H G^a_{\mu\nu} G^{a\mu\nu}$$

- ✓ Ingredients: Three-loop H+0 parton, Two-loop H+1 parton, One-loop H+2 parton, Tree-level H+3 parton - all known as matrix elements for $m_t \to \infty$
  - key part is to extract the infrared singularities

Higgs production at N3LO $m_t \to \infty$

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\delta_{ig}\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}(1-z)^N$$

At N3LO,

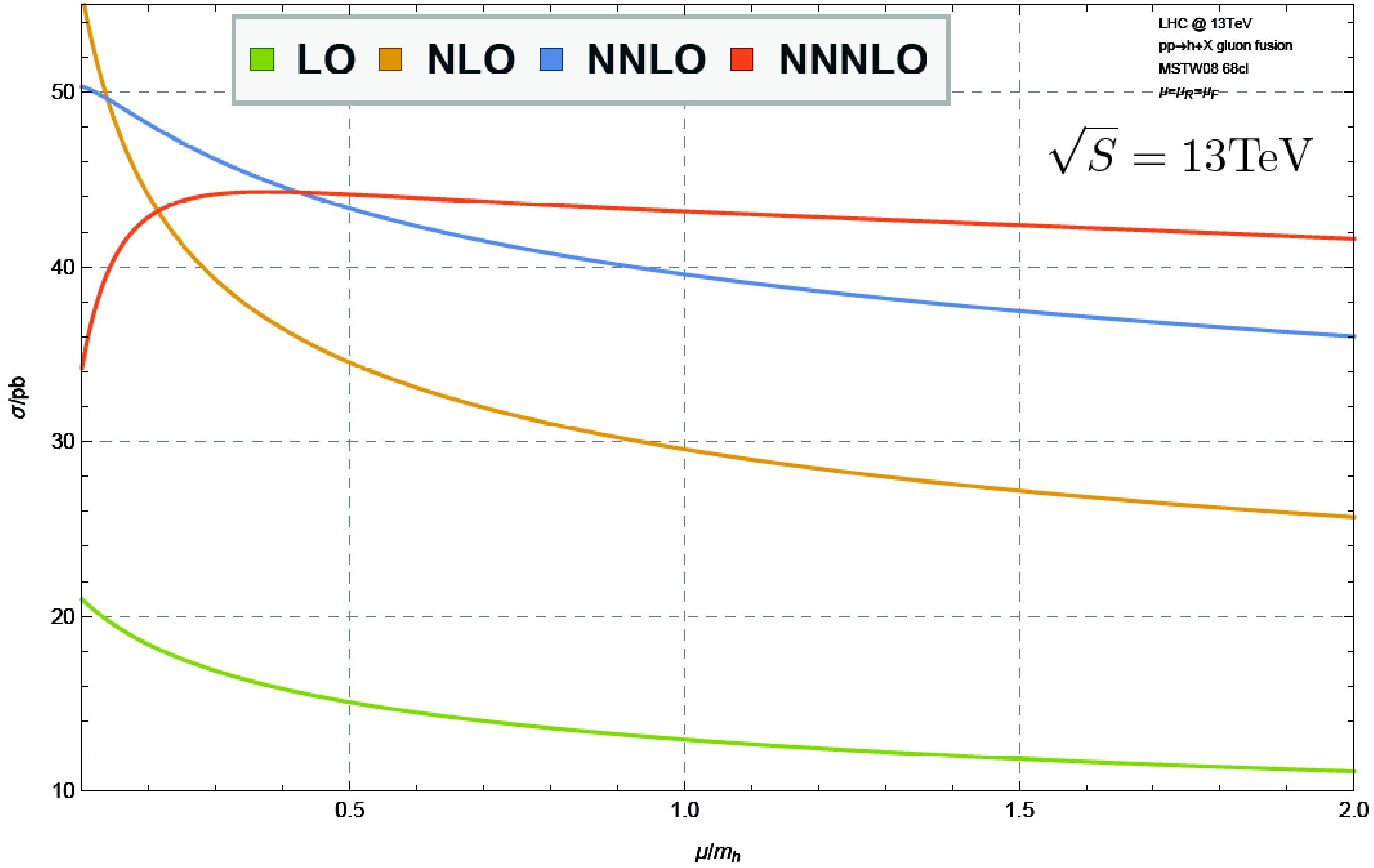
$$\hat{\sigma}^{SV} = a\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+$$

- ✓ Plus-distributions produced by soft gluon emissions and already known a decade ago
- ✓ a computed by Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger (14)

$$\sigma_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)} \log^k(1-z)$$

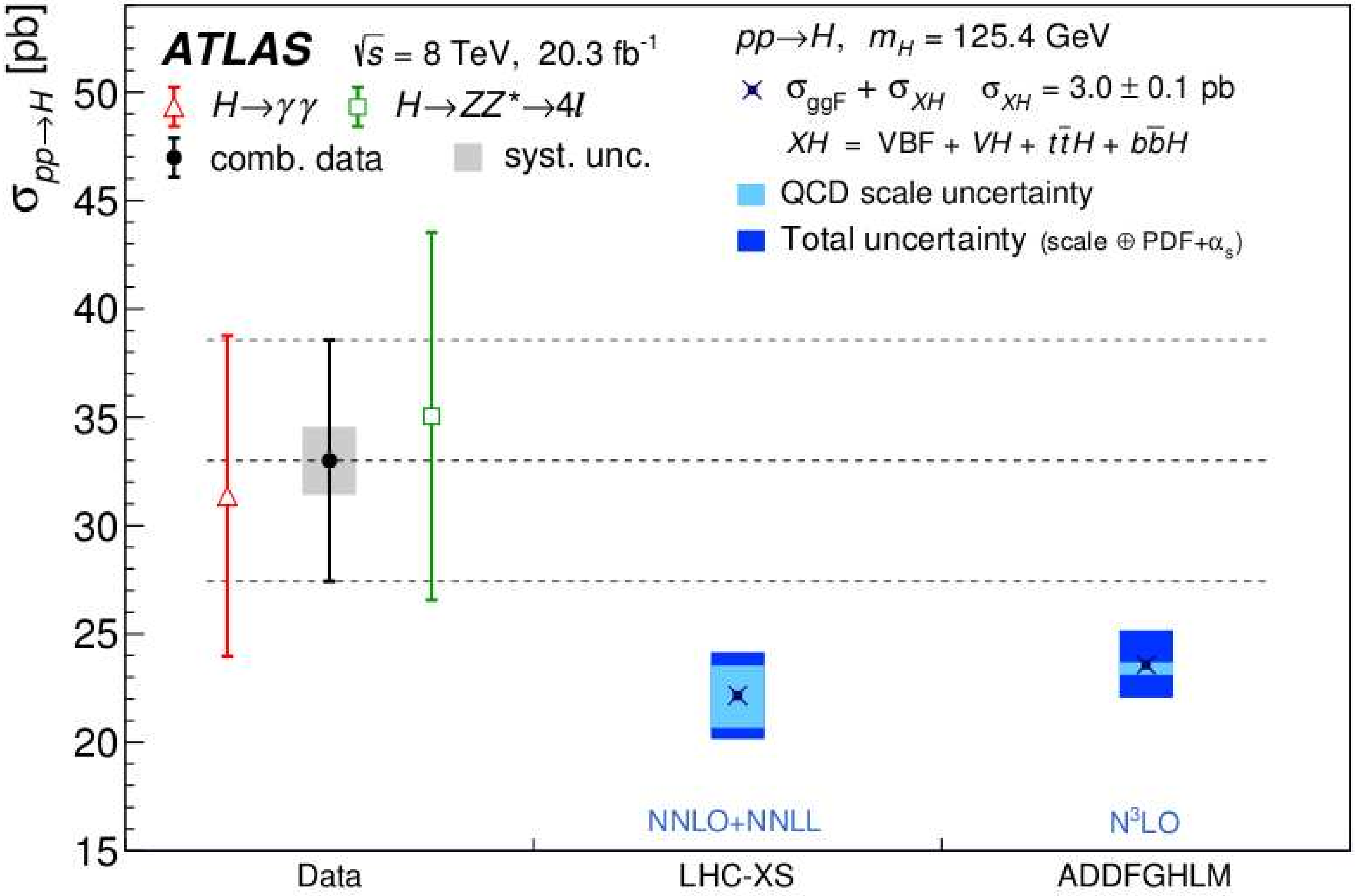
- ✓ Describes subeading soft emissions
- ✓ Single emissions known exactly, but double and triple emissions known only as an expansion Anastasiou, Duhr, Dulat, Herzog, Mistlberger (14)

Higgs cross section at N3LO



Anastasiou, Duhr, Dulat, Herzog, Mistlberger

Higgs cross section at N3LO



pp $\rightarrow$ H + jet production at NNLO $m_t \rightarrow \infty$

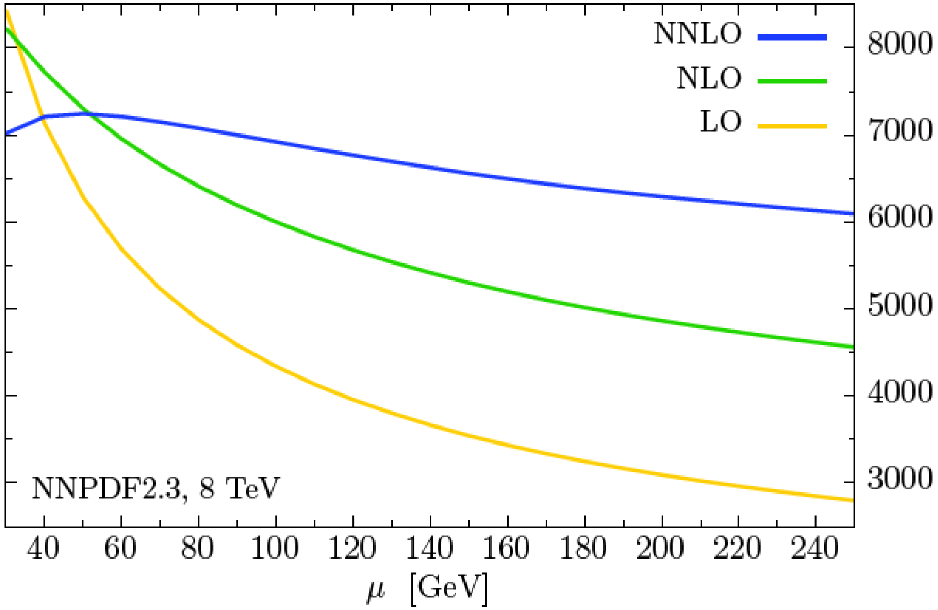
- ✓ Key goal: Establish properties of the Higgs boson!
- ✓ experimental event selection according to number of jets
  - ✓ different backgrounds for different jet multiplicities
  - ✓ H+0 jet known at NNLO

    Anastasiou, Melnikov, Petriello; Catani, Grazzini
  - ✓ H+n jets (n=1,2,3) known at NLO
  - ✓ H+0 jet and H+1 jet samples of similar size
- ✓ NNLO H+1 jet crucial, particularly for WW channel
  - ✓ Three independent computations:
    - ✚ Stripper — Boughezal, Caola, Melnikon, Petriello, Schulze
    - ✚ N-jettiness — Boughezal, Focke, Giele, Liu, Petriello
    - ✚ Antenna (gluons only) — Chen, Gehrmann, Jaquier, NG
  - ✓ Fully differential and allows for arbitrary cuts on the final state

pp $\rightarrow$ H + jet at NNLO $m_t \rightarrow \infty$

$p_T^{jet} > 30$ GeV, $\lvert Y^{jet}\rvert < 2.5$	
Leading order:	$3.1^{+1.3}_{-0.9}$ pb
Next-to-leading order:	$4.8^{+1.1}_{-0.9}$ pb
Next-to-next-to-leading order:	$5.5^{+0.3}_{-0.4}$ pb

NNPDF2.3, m_H=125GeV, anti-K_T with R = 0.5



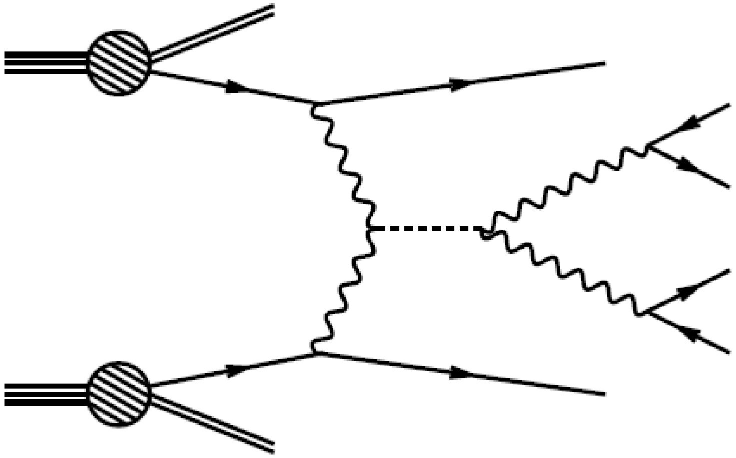
- ✓ large effects near partonic threshold
- ✓ large K-factor

$$\begin{aligned} \sigma_{NLO}/\sigma_{LO} &\sim 1.6 \\ \sigma_{NNLO}/\sigma_{NLO} &\sim 1.3 \end{aligned}$$

- ✓ significantly reduced scale dependence $\mathcal{O}(4\%)$

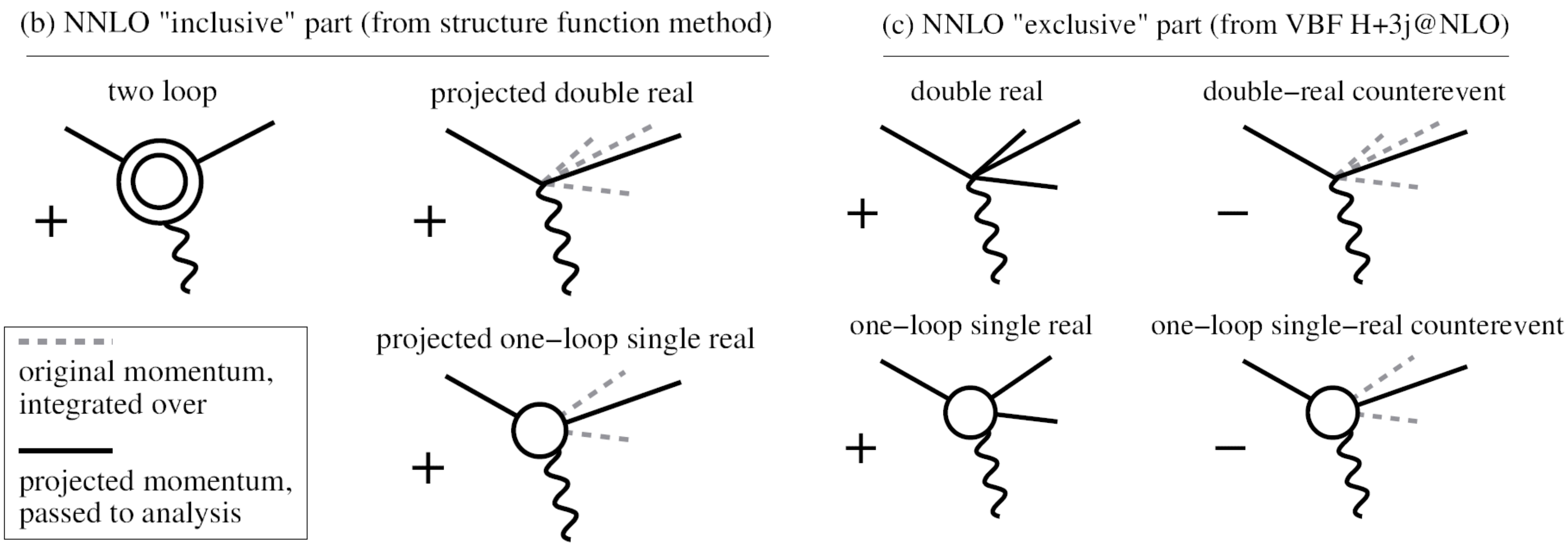
NNLO Higgs production via VBF

- ✓ Second largest source of Higgs bosons
- ✓ distinctive signature
  $\Rightarrow$ very useful for signal extraction and background suppression



- ✓ suppressed color exchange between quark lines gives rise to
  - ✚ little jet activity in central rapidity region
  - ✚ scattered quarks $\rightarrow$ two forward tagging jets
  - ✚ Higgs decay products typically between tagging jets
  - ✚ many Feynman diagrams suppressed by colour or kinematic considerations
- ✓ NLO QCD corrections moderate and well under control (order 10% or less)

NNLO Higgs production via VBF



	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032\,^{+0.057}_{-0.069}$	$0.957\,^{+0.066}_{-0.059}$
NLO	$3.929\,^{+0.024}_{-0.023}$	$0.876\,^{+0.008}_{-0.018}$
NNLO	$3.888\,^{+0.016}_{-0.012}$	$0.826\,^{+0.013}_{-0.014}$

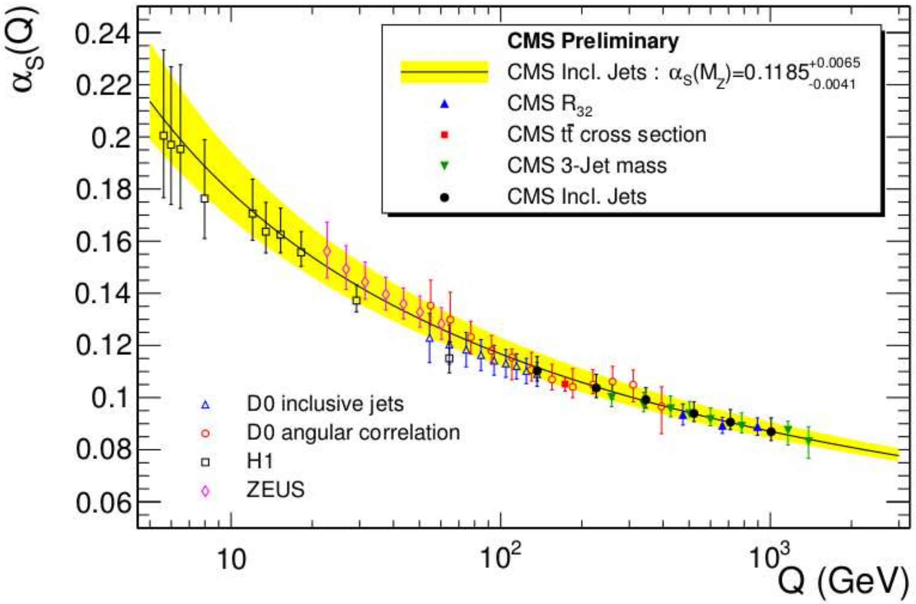
relative NNLO corrections $\sim 1\%$

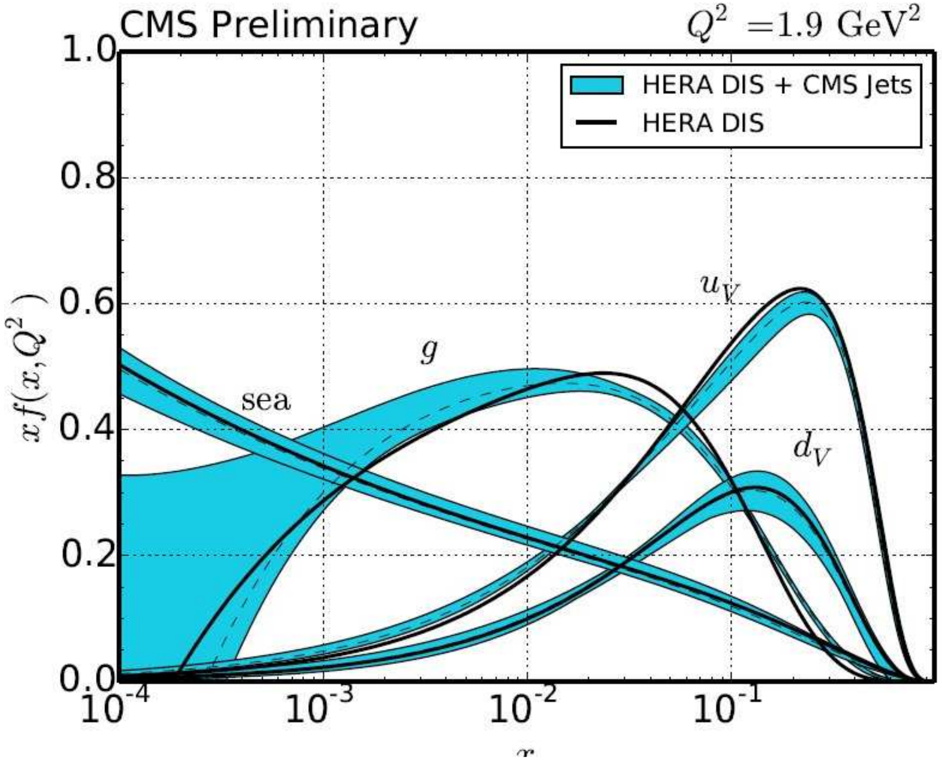
relative NNLO corrections $\sim 6\%$

Cacciari, Dreyer, Karlberg, Salam, Zanderighi

- ✓ NNLO QCD corrections are much larger in VBF setup than for inclusive cuts
- ✓ NNLO corrections appear to make jets softer, hence fewer events pass the VBF cut

Improved precision for input parameters



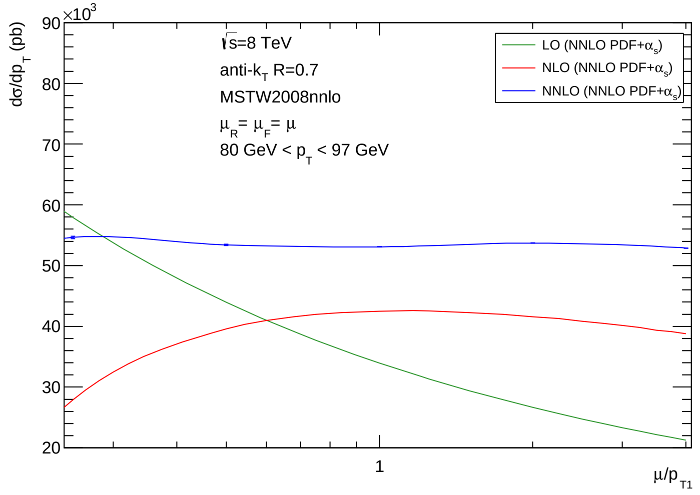


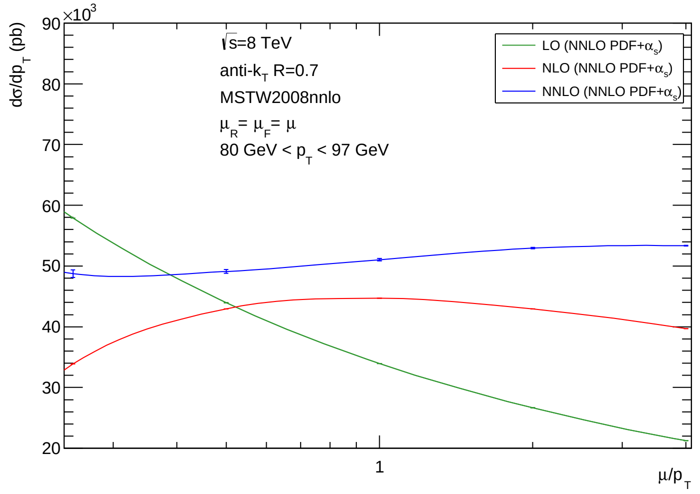
- ✓ More precise measurements of strong coupling
- ✓ Improved parton distributions

pp $\rightarrow$ 2 jets at NNLO

- ✓ One of key processes for perturbative QCD
- ✓ Current experimental precision $\mathcal{O}$(5-10%) for jets from 200 GeV/c-1 TeV/c
  - Need NNLO QCD and NLO EW
- ✘ Only process currently included in global PDF fits that is not known at NNLO
  - ✓ gg channel - leading colour

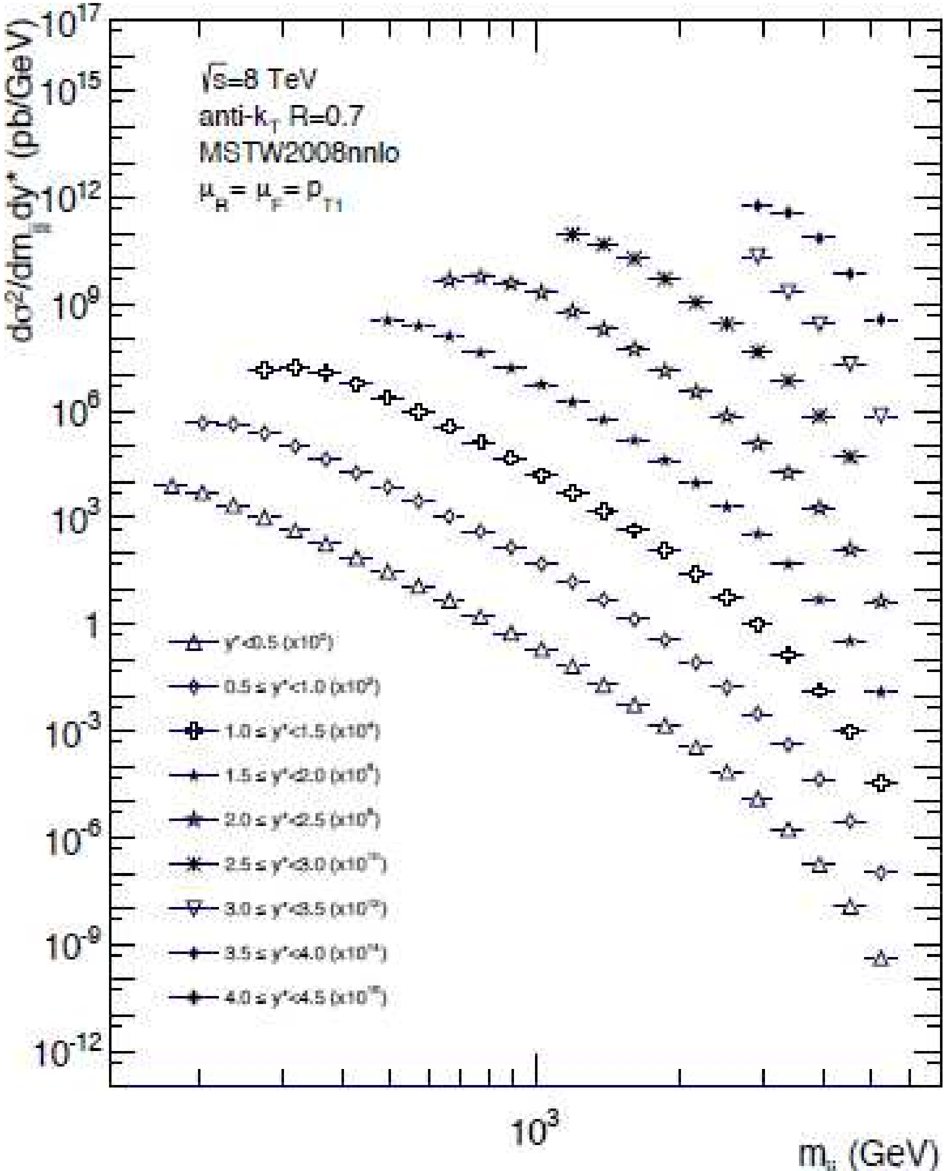
    Currie, Gehrmann-De Ridder, Gehrmann, Pires, NG
- ✓ Scale variation much reduced for $0.5 < \mu/p_T < 2$.
- ✓ Size of corrections, and uncertainty, still depends on scale choice p_{T1} v p_T.

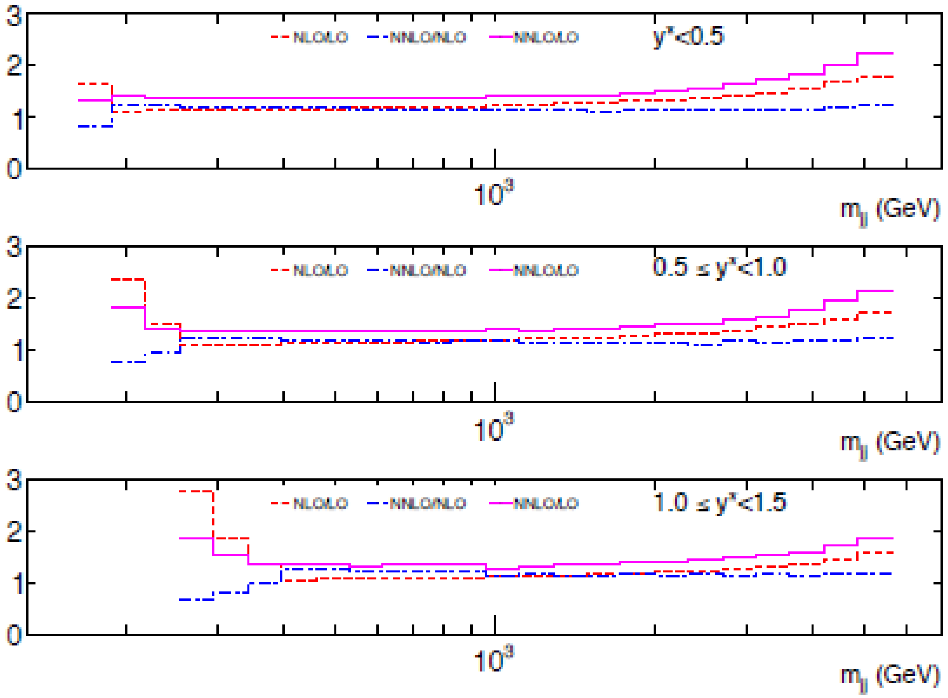




Di-jet mass distribution (gluons only) at NNLO

Gehrmann-De Ridder, Gehrmann, Pires, NG; Currie, Gehrmann-De Ridder, Pires, NG





- ✓ NNLO corrections ∼25% wrt NLO
- ✓ similar behavior for different rapidity slices

NNLO Z+1 jet production

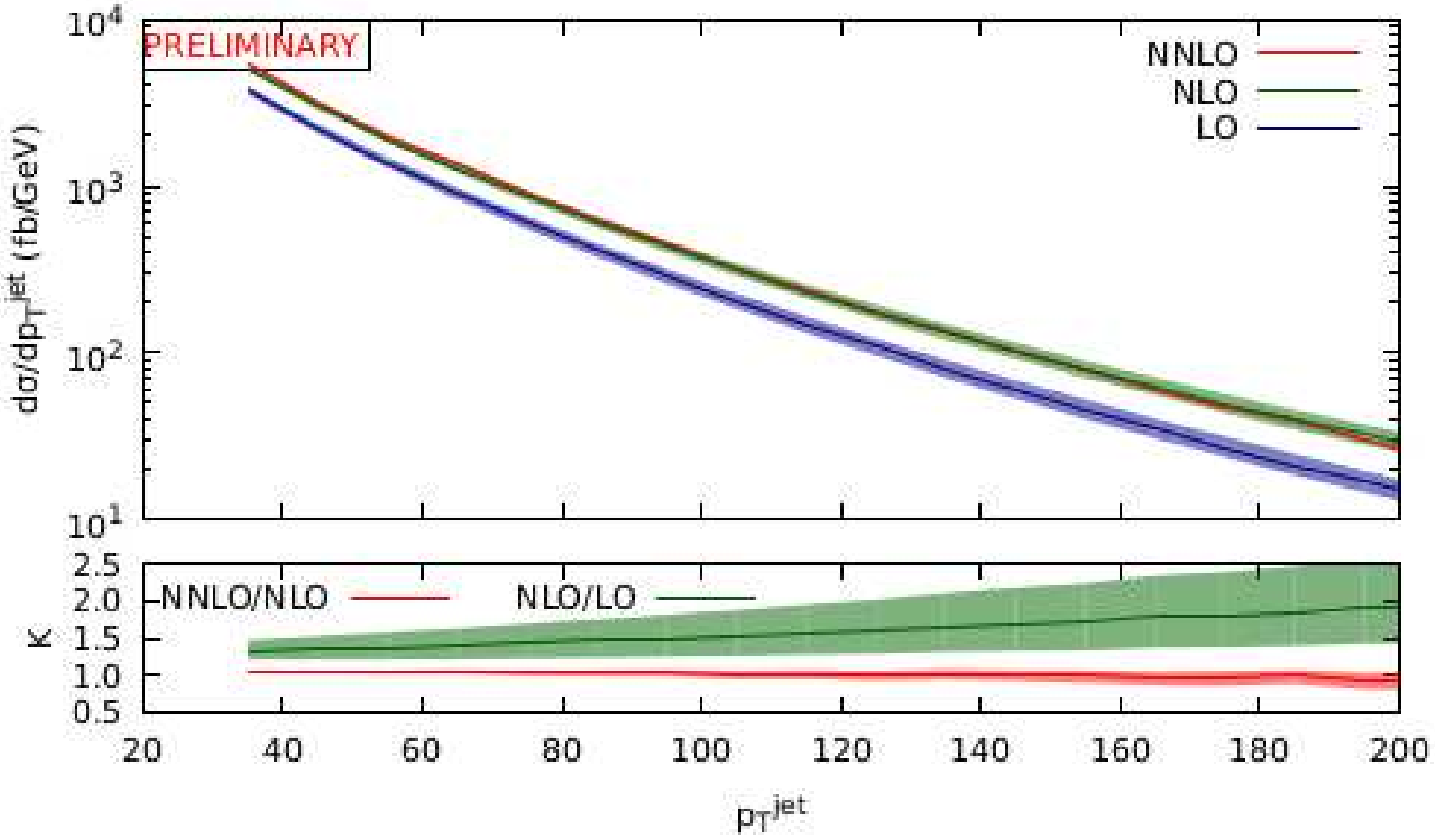
Gehrmann, Gehrmann-De Ridder, Huss, Morgan, NG

- ✓ An important background for beyond the standard model searches
- ✓ Very precise measurements can be obtained.
- ✓ Provides a fantastic testing ground for precision QCD and electroweak corrections
- ✓ Useful for detector calibration, jet energy scale can be determined from the recoil of the jet against the Z boson.
- ✓ Useful process for PDF determination

Initial State	σ (pb)	% contribution
qg	80.2	55.6
$q\bar{q}$	33.1	22.9
$\bar{q}g$	33.1	22.9
gg	-4.0	-2.7
qq	1.8	1.2
$\bar{q}\bar{q}$	0.1	0.1
Total	144.3	100.0

NNLO Z+1 jet production

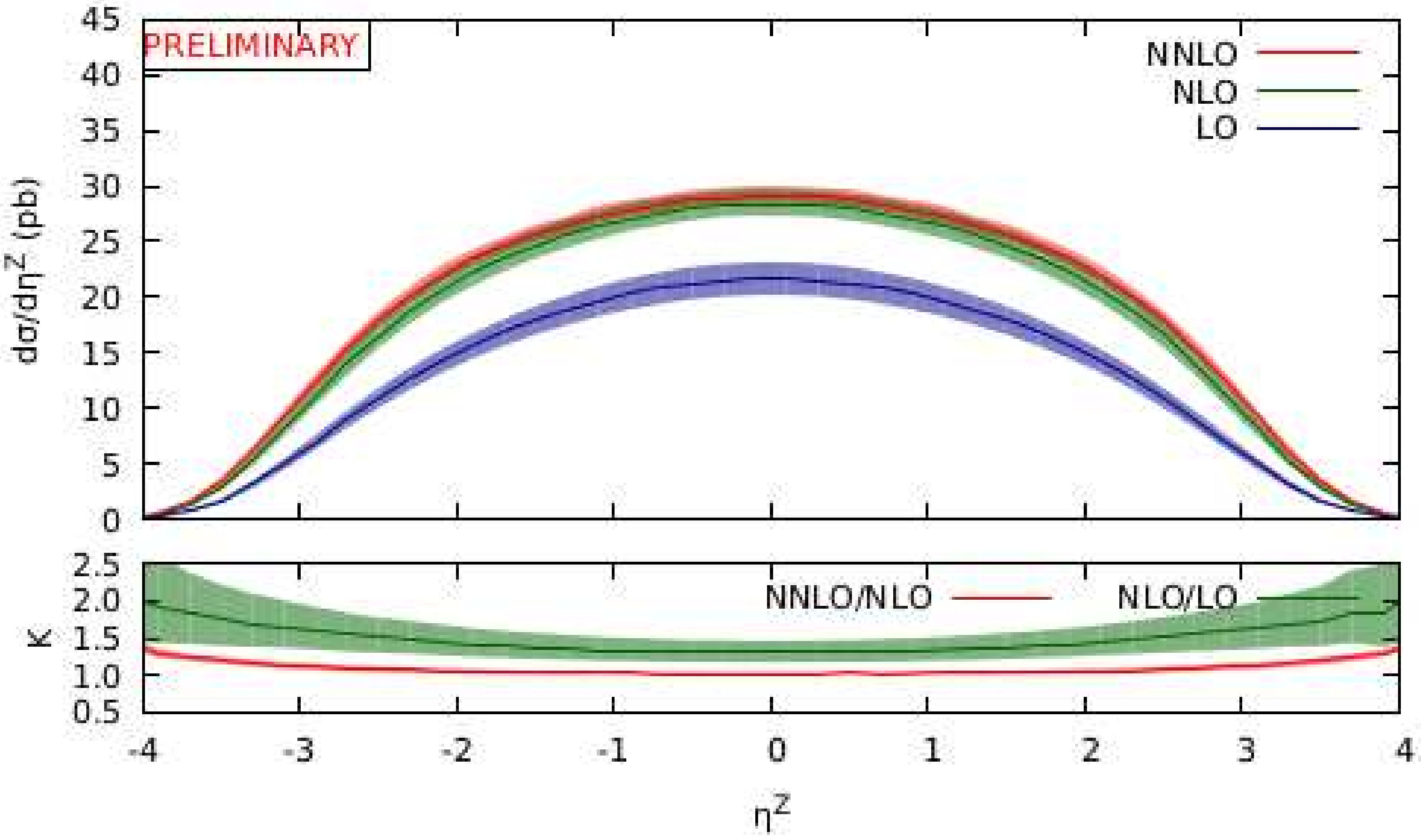
Gehrmann, Gehrmann-De Ridder, Huss, Morgan, NG



- ✓ Excellent convergence of NNLO in the jet p_T distribution
- ✓ Significant reduction in the scale uncertainty

NNLO Z+1 jet production

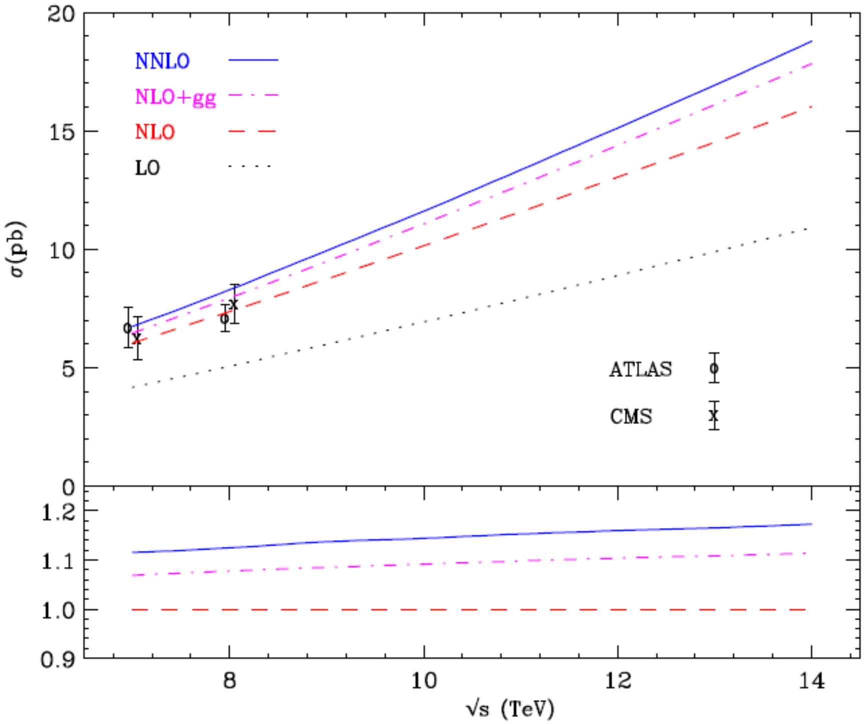
Gehrmann, Gehrmann-De Ridder, Huss, Morgan, NG



- ✓ NNLO corrections uniform in rapidity, approximately 7%
- ✓ Significant reduction in the scale uncertainty

pp $\rightarrow$ ZZ at NNLO

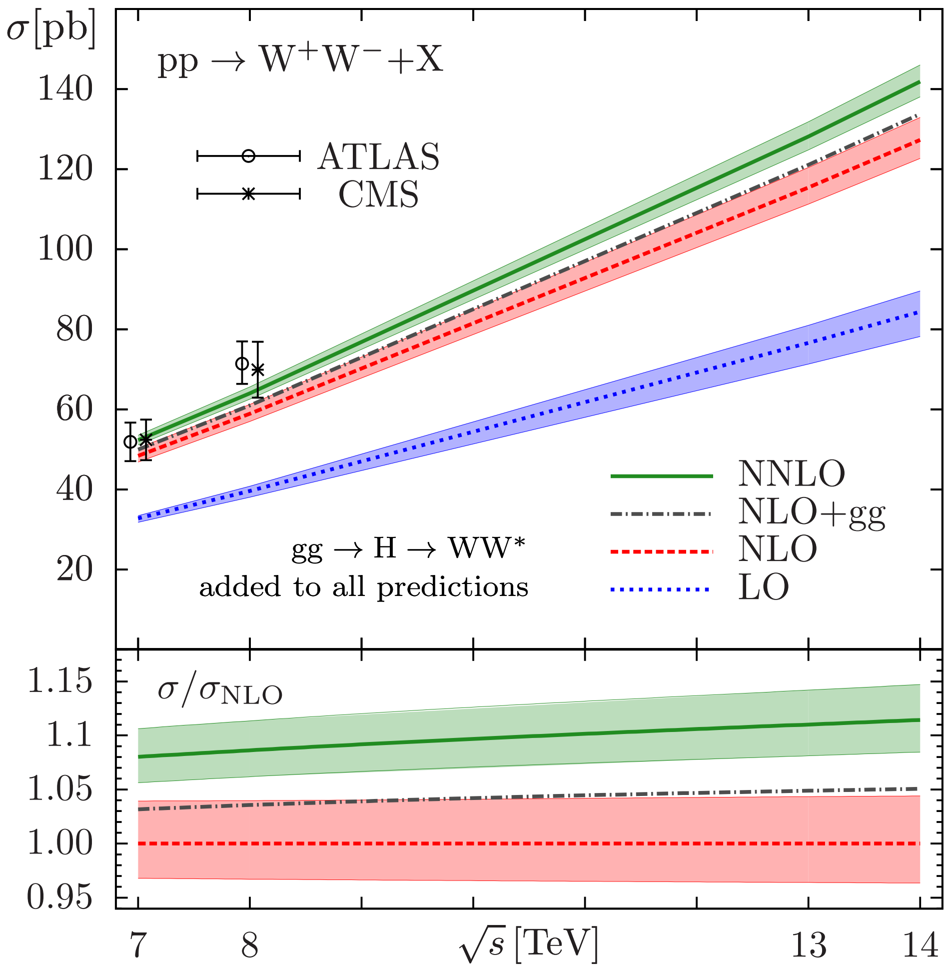
Cascioli et al



Observe

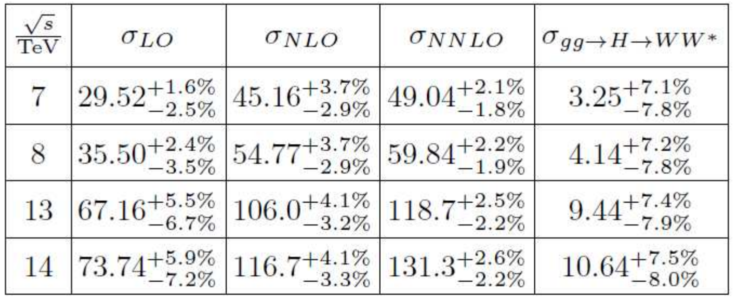
- ✓ The NNLO corrections increase the NLO result by an amount varying from 11% to 17% as $\sqrt{s}$ goes from 7 to 14 TeV.
- ✓ The loop-induced gluon fusion contribution provides about 60% of the total NNLO effect.
- ✓ When going from NLO to NNLO the scale uncertainties do not decrease and remain at the $\pm 3\%$ level.

pp $\rightarrow W^+W^-$ at NNLO



Gehrmann et al

- ✓ Provides a handle on the determination of triple gauge couplings, and possible new physics
- ✓ Severe contamination of the W^+W cross section due to top-quark resonances



$\frac{\sqrt{s}}{\rm TeV}$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{gg\to H\to WW^*}$
7	$29.52^{+1.6\%}_{-2.5\%}$	$45.16^{+3.7\%}_{-2.9\%}$	$49.04^{+2.1\%}_{-1.8\%}$	$3.25^{+7.1\%}_{-7.8\%}$
8	$35.50^{+2.4\%}_{-3.5\%}$	$54.77^{+3.7\%}_{-2.9\%}$	$59.84^{+2.2\%}_{-1.9\%}$	$4.14^{+7.2\%}_{-7.8\%}$
13	$67.16^{+5.5\%}_{-6.7\%}$	$106.0^{+4.1\%}_{-3.2\%}$	$118.7^{+2.5\%}_{-2.2\%}$	$9.44^{+7.4\%}_{-7.9\%}$
14	$73.74^{+5.9\%}_{-7.2\%}$	$116.7^{+4.1\%}_{-3.3\%}$	$131.3^{+2.6\%}_{-2.2\%}$	$10.64^{+7.5\%}_{-8.0\%}$

- ✓ The NNLO QCD corrections increase the NLO result by an amount varying from 9% to 12% as $\sqrt{s}$ goes from 7 to 14 TeV.
- ✓ Scale uncertainties at the $\pm 3\%$ level.

LHC cross sections at NNLO

- ✓ Past few years, there has been an accelerating progress in computing NNLO corrections to important LHC processes
  - ✚ H+Jet, H+2 Jet (VBF), H+W, H+Z, H+H, $\ldots$
  - ✚ W+Jet, Z+Jet, Di-Jet, Di-Gamma, WW, ZZ, Z+Gamma, single t, $t\bar{t}$, $\ldots$
- ✓ Mostly of $2 \to 2$ variety based on amplitudes computed a while ago
- ✓ Several IR subtraction technologies available
- ✓ Whenever gluons are involved, the NNLO contributions can be large
- ✓ All show a reduced scale variation

Accuracy and Precision (A. David)

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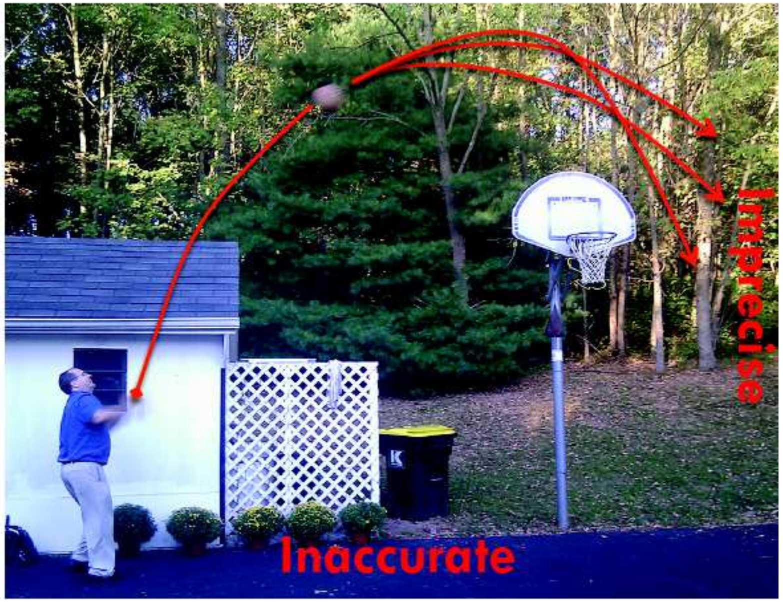

Accuracy and Precision (A. David)

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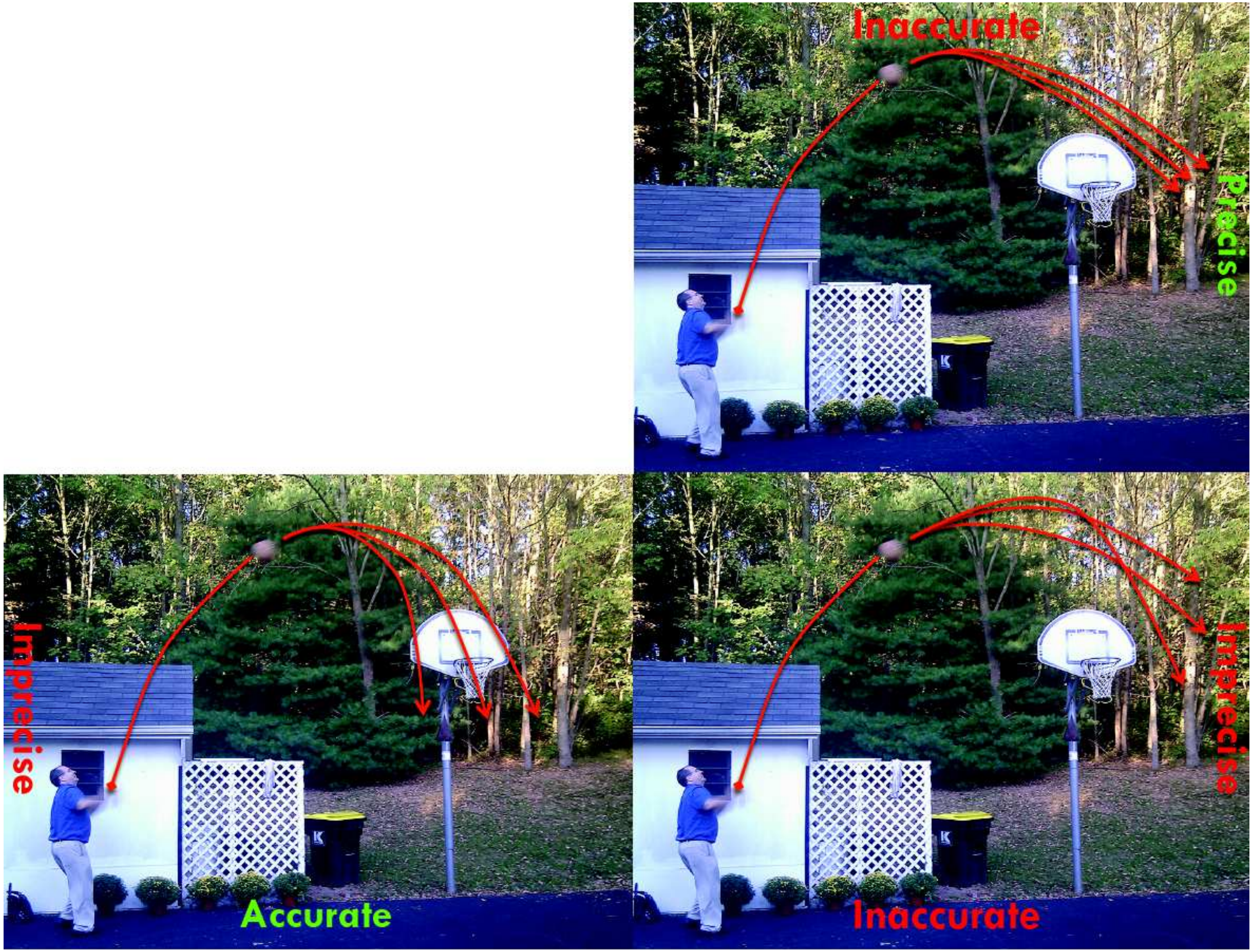


Accuracy and Precision (A. David)

Estimating uncertainties of MHO

- ✓ Consider a generic observable $\mathcal{O}$ (e.g. σ_H)

$$\mathcal{O}(Q) \quad \sim \quad \mathcal{O}_k(Q,\mu) + \Delta_k(Q,\mu)$$

where

$$\mathcal{O}_k(Q,\mu) \equiv \sum_{n=0}^{k} c_n(Q,\mu)\alpha_s(\mu)^n, \qquad \Delta_k(Q,\mu) \equiv \sum_{n=k+1}^{\cdots} c_n(Q,\mu)\alpha_s(\mu)^n$$

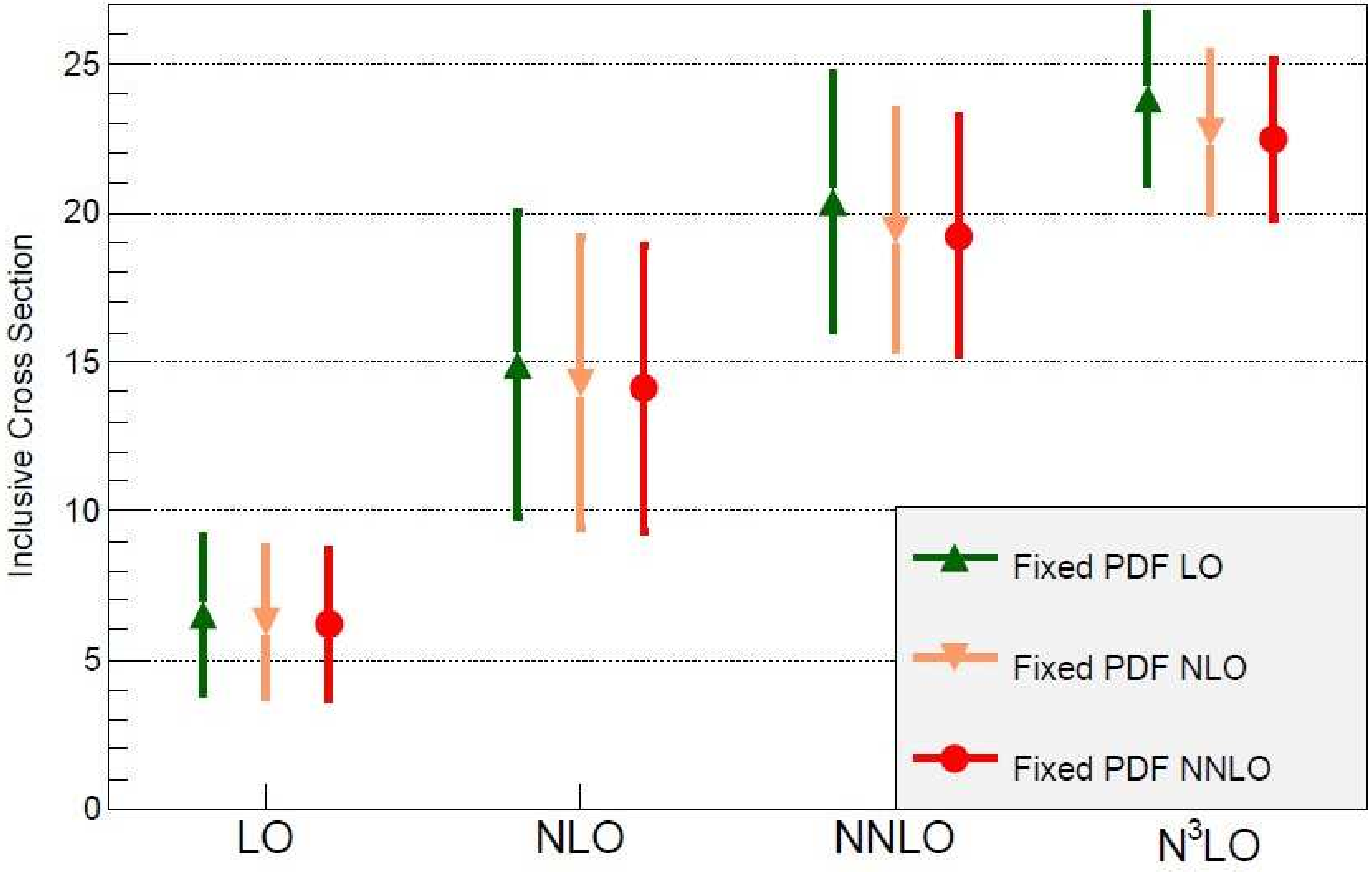
- ✓ Usual procedure is to use scale variations to estimate Δ_k,

$$\Delta_k(Q,\mu) \quad \sim \quad \max\left[\mathcal{O}_k\left(Q,\frac{\mu}{r}\right),\mathcal{O}_k(Q,r\mu)\right] \sim \alpha_s(\mu)^{k+1}$$

where μ is chosen to be a typical scale of the problem and typically $r=2$.

Choice of μ and $r=2$ is convention

Theoretical uncertainty on σ_H



Forte, Isgro, Vita

Warning: Scale variation may not give an accurate estimate of the uncertainty in the cross section!!

Going beyond scale uncertainties

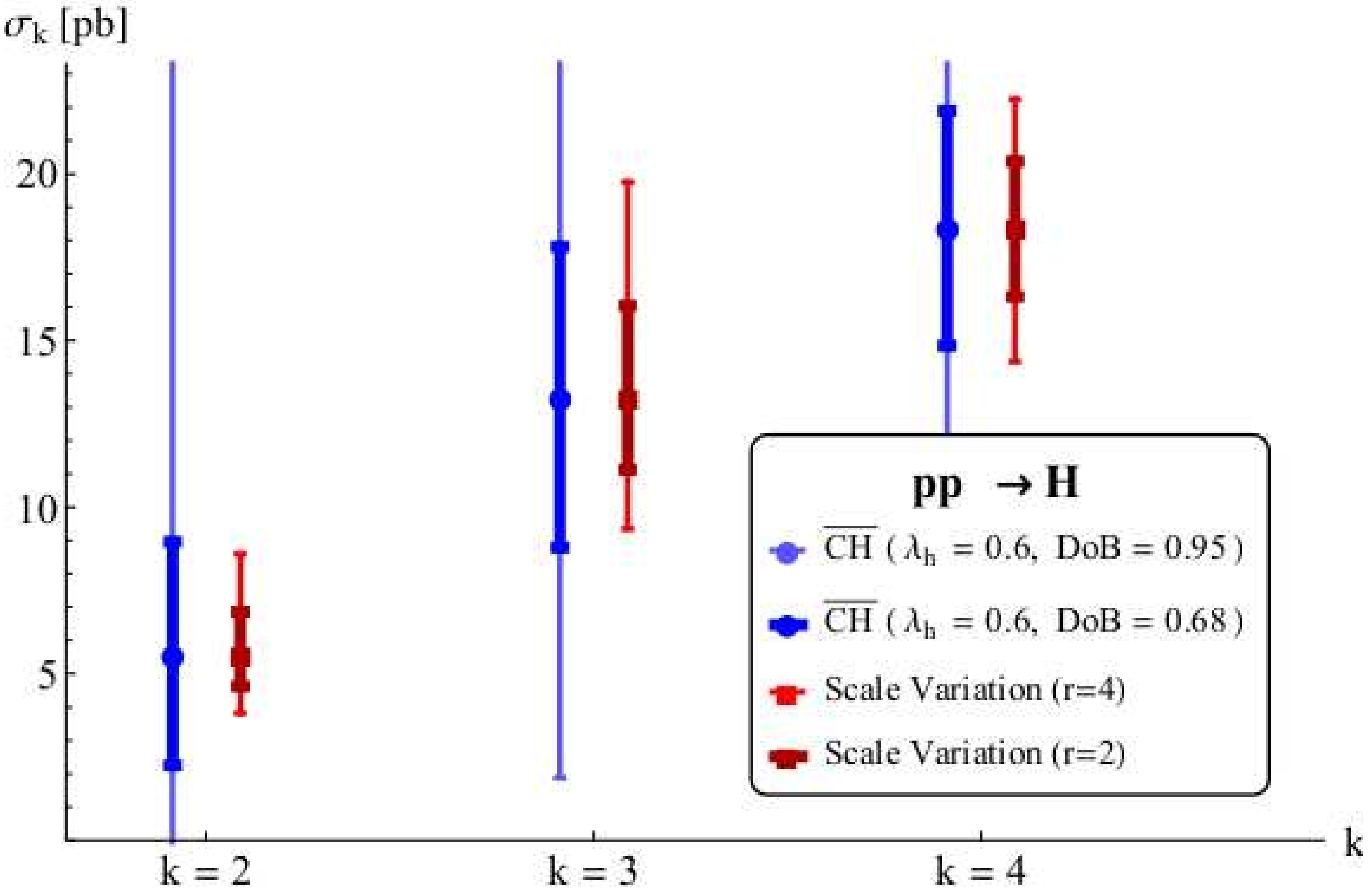
- ✓ Series acceleration David, Passarino

  sequence transformations gives estimates of some of the unknown terms in series
- ✓ Estimate coefficients using information on the singularity structure of the Mellin space cross section coming from all order resummation Ball et al
  - large N (soft gluon, Sudakov)
  - small N (high energy, BFKL)
- ✓ Bayesian estimate of unknown coefficients Cacciari, Houdeau

  make the assumption that all the coefficients c_n share a (process dependent) upper bound $\bar{c} > 0$ leading to density functions $f(c_n|\bar{c})$ and $f(\ln \bar{c})$
  recent refinement of method Bagnaschi, Cacciari, Guffanti, Jenniches
- ✓ Accepting that scale variation does not give reliable error estimate, can predict the part of the N^{3}LO cross section coming from scale variations.

Pressure is building to better estimate MHO

Theoretical uncertainty on σ_H revisited



Bagnaschi, Cacciari, Guffanti, Jenniches

Uncertainty in Modified CH approach larger but more realistic!!

Where are we now?

- ✓ Witnessed a revolution that has established NLO as the new standard
  - previously impossible calculations now achieved
  - very high level of automation for numerical code
  - standardisation of interfaces - linkage of one-loop and real radiation providers
  - take up by experimental community
- ✓ Substantial progress in NNLO in past couple of years
  - several different approaches for isolating IR singularities
  - several new calculations available
  - codes typically require significant CPU resource

Where are we going?

- ✓ NNLO automation?
  - as we gain analytical and numerical experience with NNLO calculations, can we benefit from (some of) the developments at NLO, and the improved understanding of amplitudes
  - automation of two-loop contributions?
  - automation of infrared subtraction terms?
  - standardisation of interfaces - linkage to one-loop and real radiation providers?
  - interface with experimental community

Next few years:

- ✓ Les Houches wishlist to focus theory attention
- ✓ New high precision calculations that will appear such as, e.g. N3LO σ_H, could reduce Missing Higher Order uncertainty by a factor of two
- ✓ NNLO will emerge as standard for benchmark processes such as dijet production leading to improved pdfs etc. could reduce theory uncertainty due to inputs by a factor of two