4d Ambitwistor Strings

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July 9, 2015

1404.6219: Geyer,Lipstein,Mason

1406.1462: Geyer,Lipstein,Mason

1504.01364: Lipstein

to appear: Lipstein,Schomerus

Spinor-Helicity

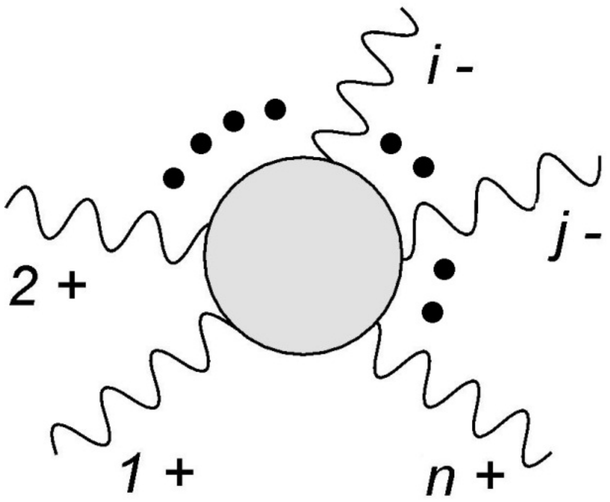
- 4d null momentum:

$$p^{\alpha\dot{\alpha}} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$$

- Expressing amplitudes in terms of these spinors leads to very simple expressions.

MHV Amplitudes

At tree-level:



$$A_n = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

(Parke,Taylor)

where $\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$

Twistor String Theory

- The simplicity of MHV amplitudes suggests a deeper mathematical structure.
- Is there a way to reformulate Yang-Mills theory to make this structure manifest?
- Nair/Berkovits/Witten: N=4 SYM is equivalent to string theory with target space $CP^{3|4}$

Twistors

- Twistors: (Penrose)

$$\begin{pmatrix} Z^A \\ \chi^a \end{pmatrix}, \quad Z^A = \begin{pmatrix} \lambda_\alpha \\ \mu^{\dot\alpha} \end{pmatrix}$$

- Incidence relations:

$$\mu^{\dot\alpha} = -i x^{\dot\alpha\alpha} \lambda_\alpha, \quad \chi^a = -i\theta^{a\alpha}\lambda_\alpha$$

- Combining insights from AdS/CFT and twistor string theory has lead to powerful techniques for computing amplitudes of N=4 super-Yang-Mills, which have revealed new symmetries, dualities, and mathematical structures.

- Question: Can these ideas be extended to other theories such as gravity or N<4 SYM?

  (see also Casali,Geyer,Mason,Monteiro,Roehrig)

Extension to Gravity

- Hodges formula for tree-level MHV:

$$\mathcal{M}_{n,0} = \langle i, j \rangle^8 \det{}'(\mathrm{H})$$

$$\mathrm{H}_{ij} = \frac{[i,j]}{\langle i,j \rangle} \qquad \text{for } i \neq j, \qquad\qquad \mathrm{H}_{ii} = -\sum_{j \neq i} \frac{[i,j]}{\langle i,j \rangle} \frac{\langle a,j \rangle \langle b,j \rangle}{\langle a,i \rangle \langle b,i \rangle}$$

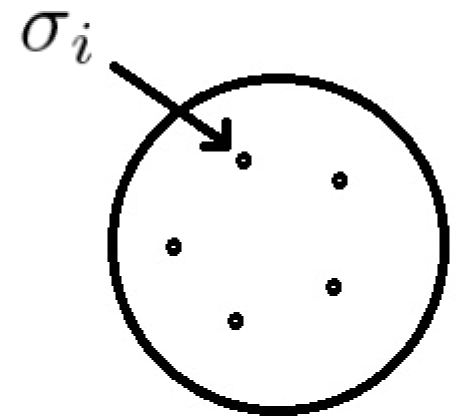
- Skinner: N=8 SUGRA is equivalent to string theory with target space $CP^{3|8}$

Scattering Equations

external momentum

$$\sum_{i \neq j} \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$$

point on 2-sphere



- Gross/Mende: These equations arise from the tensionless limit of string amplitudes

- Cachazo/He/Yuan: They also arise in the amplitudes of massless point particles!

Ambitwistor Strings

- Mason,Skinner: Amplitudes of complexified massless point particles can be computed using a chiral, infinite tension limit of the RNS string:

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu + \frac{e}{2} P_\mu P^\mu + \ldots$$

- Correlation functions of vertex operators reproduce the CHY formulae!

- Critical in d=26 (bosonic) and d=10 (superstring)

4d Ambitwistor Space

- Twistors:

$$Z^A = \begin{pmatrix} \lambda_\alpha \\ \mu^{\dot\alpha} \\ \chi^a \end{pmatrix}, \quad W_A = \begin{pmatrix} \tilde\mu^\alpha \\ \tilde\lambda_{\dot\alpha} \\ \tilde\chi_a \end{pmatrix}$$

- Incidence Relations:

$$\mu^{\dot\alpha} = i(x^{\alpha\dot\alpha} + i\theta^{a\alpha}\tilde\theta_a^{\dot\alpha})\lambda_\alpha \qquad \chi^a = \theta^{a\alpha}\lambda_\alpha$$

$$\tilde\mu^{\alpha} = -i(x^{\alpha\dot\alpha} - i\theta^{a\alpha}\tilde\theta_a^{\dot\alpha})\tilde\lambda_{\dot\alpha} \qquad \tilde\chi_a = \tilde\theta_a^{\dot\alpha}\tilde\lambda_{\dot\alpha}$$

4d Ambitwistor Strings

- Action:

$$\mathcal{L} = Z^A \bar{\partial} W_A + \rho^A \bar{\partial} \tilde{\rho}_A + u^B K_B$$

$$K_B = \left\{ Z^A W_A, \rho^A \tilde{\rho}_A, \rho^\alpha \rho_\alpha, \tilde{\rho}^{\dot{\alpha}} \tilde{\rho}_{\dot{\alpha}}, \rho^A W_A, Z^A \tilde{\rho}_A, \lambda^\alpha \rho_\alpha, \tilde{\lambda}^{\dot{\alpha}} \tilde{\rho}_{\dot{\alpha}} \right\}$$

- Fields are worldsheet spinors (Geyer,Lipstein,Mason).

4d Vertex Operators

$$\mathcal{V}_h = \int \left[W, \frac{\partial h}{\partial Z}\right] + \left[\tilde{\rho}, \frac{\partial}{\partial Z}\right] \rho \cdot \frac{\partial h}{\partial Z}$$

$$\widetilde{\mathcal{V}}_{\tilde{h}} = \int \left\langle Z, \frac{\partial \tilde{h}}{\partial W}\right\rangle + \left\langle \rho, \frac{\partial}{\partial W}\right\rangle \tilde{\rho} \cdot \frac{\partial \tilde{h}}{\partial W}$$

where

$$h_a = \int \frac{\mathrm{d}s_a}{s_a^3} \bar{\delta}^{2|\mathcal{N}}(\lambda_a - s_a\lambda | \eta_a - s_a\chi) \mathrm{e}^{is_a[\mu\,\tilde{\lambda}_a]}$$

$$\tilde{h}_a = \int \frac{\mathrm{d}s_a}{s_a^3} \bar{\delta}^{2}(\tilde{\lambda}_a - s_a\tilde{\lambda}) \mathrm{e}^{is_a\left(\langle\tilde{\mu}\,\lambda_a\rangle + \tilde{\chi}_I \eta_a^I\right)}\,.$$

$$\langle Z_1 Z_2\rangle \equiv \langle \lambda_1 \lambda_2\rangle\,, \quad [W_1 W_2] \equiv \left[\tilde{\lambda}_1 \tilde{\lambda}_2\right]$$

Correlation Functions

- Consider N^{k-2}MHV amplitude:

$$\mathcal{A} = \left\langle \widetilde{\mathcal{V}}_1 \ldots \widetilde{\mathcal{V}}_k \mathcal{V}_{k+1} \ldots \mathcal{V}_n \right\rangle$$

- Bringing exponentials into the action gives

$$\int_\Sigma \sum_{i=1}^{k} i s_i (\langle \tilde{\mu} \lambda_i \rangle + \tilde{\chi} \cdot \eta_i) \bar{\delta}(\sigma - \sigma_i) + \sum_{p=k+1}^{n} i s_p [\mu \, \tilde{\lambda}_p] \bar{\delta}(\sigma - \sigma_p)$$

- Equations of motion:

$$\bar{\partial}_\sigma Z = \bar{\partial}\left(\lambda, \mu, \chi\right) = \sum_{i=1}^{k} s_i \left(\lambda_i, 0, \eta_i\right) \bar{\delta}\left(\sigma - \sigma_i\right),$$

$$\bar{\partial}_\sigma W = \bar{\partial}\left(\tilde{\mu}, \tilde{\lambda}, \tilde{\chi}\right) = \sum_{p=k+1}^{n} s_p \left(0, \tilde{\lambda}_p, 0\right) \bar{\delta}(\sigma - \sigma_p)$$

- Solution:

$$Z(\sigma) = (\lambda, \mu, \chi) = \sum_{i=1}^{k} \frac{s_i \left(\lambda_i, 0, \eta_i\right)}{\sigma - \sigma_i}$$

$$W(\sigma) = \left(\tilde{\mu}, \tilde{\lambda}, \tilde{\chi}\right) = \sum_{p=k+1}^{n} \frac{s_p \left(0, \tilde{\lambda}_p, 0\right)}{\sigma - \sigma_p}$$

- Scattering equations (refined by MHV degree):

$$[\tilde{\lambda}_i \, \tilde{\lambda}(\sigma_i)] = 0\,,\ i = 1 \ldots k\,, \quad \langle \lambda_p \, \lambda(\sigma_p) \rangle = 0,\ p = k{+}1 \ldots n$$

Amplitudes

- Geyer/Lipstein/Mason: 4d ambitwistor string theory gives rise to tree-level formulae for gauge and gravity amplitudes with any amount of susy!

$$\mathcal{A} = \int \frac{1}{\mathrm{Vol}\,\mathrm{GL}(2,\mathbb{C})} \prod_{a=1}^{n} \frac{\mathrm{d}^2\sigma_a}{(a\,a+1)} \prod_{i=1}^{k} \bar{\delta}^2(\tilde{\lambda}_i - \tilde{\lambda}(\sigma_i))$$

$$\prod_{p=k+1}^{n} \bar{\delta}^{2|\mathcal{N}}(\lambda_p - \lambda(\sigma_p), \eta_p - \chi(\sigma_p))$$

$$\mathcal{M} = \int \frac{\prod_{a=1}^{n} \mathrm{d}^2\sigma_a}{\mathrm{Vol}\,\mathrm{GL}(2,\mathbb{C})} \det{}'(\mathcal{H}) \prod_{i=1}^{k} \bar{\delta}^2(\tilde{\lambda}_i - \tilde{\lambda}(\sigma_i))$$

$$\prod_{p=k+1}^{n} \bar{\delta}^{2|\mathcal{N}}(\lambda_p - \lambda(\sigma_p), \eta_p - \chi(\sigma_p))$$

- Much simpler than previous formulae.

Soft Theorems

- Soft Graviton Theorem:

$$\lim_{k_n \to 0} \mathcal{A}_n = \left(S^{(-1)} + S^{(0)} + S^{(1)} \right) \mathcal{A}_{n-1}$$

$$S^{(-1)} = \sum_{i=1}^{n-1} \frac{(\epsilon \cdot k_i)^2}{k_n \cdot k_i}, \quad S^{(0)} = \sum_{i=1}^{n-1} \frac{\epsilon \cdot k_i k_{n,\mu} \epsilon_\nu J_i^{\mu\nu}}{k_n \cdot k_i}, \quad S^{(1)} = \sum_{i=1}^{n-1} \frac{\left(k_{n,\mu} \epsilon_\nu J_i^{\mu\nu}\right)^2}{k_n \cdot k_i}$$

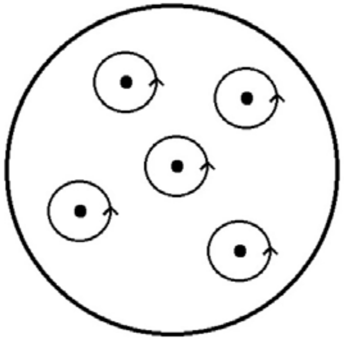
  (Weinberg/White/Cachazo,Strominger)
- Similar theorems for YM (Weinberg,Low/Burnett,Kroll/Casali)

- Recently, Strominger and collaborators proposed a new way of understanding soft limits of scattering amplitudes in terms of BMS symmetry.

Soft Limits from 2d CFT

- Key idea: Take a vertex operator in correlator to be soft. Each term in the Taylor expansion gives rise to a soft theorem (Geyer,Lipstein,Mason).

- Soft theorem follows from integrating the soft vertex operator around all of the hard ones and adding up the residues:



- Adamo, Casali, Skinner also studied soft theorems using a closely related model.

Worldsheet Charges

- Soft theorems correspond to Ward identities associated with charges obtained by expanding soft vertex operators:

$$\int \mathrm{d}^2\sigma \mathcal{V}_{YM}(\sigma) = \sum_{l=-1}^{\infty} \frac{1}{(l+1)!} q_{YM}^{(l)} \qquad \int \mathrm{d}^2\sigma \mathcal{V}_{GR}(\sigma) = \sum_{l=-1}^{\infty} \frac{1}{(l+1)!} q_{GR}^{(l)} + \sum_{l=0}^{\infty} \frac{1}{l!} q_{\rho\tilde{\rho}}^{(l)}$$

where

$$q_{YM}^{(l)} = \frac{1}{2\pi i} \oint \frac{1}{\langle s\lambda\rangle} \left(\frac{\langle \xi s\rangle}{\langle \xi\lambda\rangle}\right)^{l} [\mu s]^{l+1}\, j$$

$$q_{GR}^{(l)} = \frac{1}{2\pi i} \oint \frac{1}{\langle s\lambda\rangle} \left(\frac{\langle \xi s\rangle}{\langle \xi\lambda\rangle}\right)^{l-1} \left[\tilde{\lambda} s\right] [\mu s]^{l+1}$$

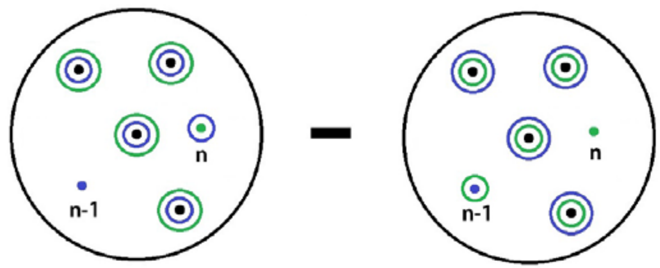
$$q_{\rho\tilde{\rho}}^{(l)} = \frac{1}{2\pi i} \oint \frac{1}{\langle s\lambda\rangle} \left(\frac{\langle \xi s\rangle}{\langle \xi\lambda\rangle}\right)^{l-1} [\mu s]^{l} \left[\tilde{\rho}\tilde{\lambda}_i\right] \left[\rho\tilde{\lambda}_i\right]$$

- YM charges can be mapped into GR charges by replacing

$$j \to \left[\tilde{\lambda} s\right] [\mu s]$$

Algebra of Soft Limits

- Consider the commutator of two consecutive soft limits:



- Discard contributions which arise from integrating one soft vertex operator around the other one before it becomes soft.

- This can be achieved by choosing $(\xi_{n-1}, \xi_n) = (\lambda_n, \lambda_{n-1})$

Symmetries of the S-matrix

- Using this prescription, the algebra of soft limits can be encoded in the OPE soft vertex operators (Lipstein).

- These results have been confirmed by explicit field theory calculations (Bern,Davies,Di Vecchia,Nohle/Broedel,de Leeuw,Plefka,Rosso/Klose,McLoughlin,Nandan,Plefka,Travaglini/Volovich,Wen,Zlotnikov)

- Whereas the algebra of leading soft graviton limits is abelian, the algebra of subleading soft graviton limits is nonabelian and appears to be different than Virasoro!

N=8 SUGRA

- If one does not gauge-fix Virasoro symmetry, the 4d ambitwistor string describing N=8 supergravity is critical and non-anomalous.
- Recall that for a standard string theory, imposing Virasoro symmetry removes unphysical states from the spectrum.
- Remarkably, for the critical 4d ambitwistor string theory, imposing global conformal symmetry along with the other gauge symmetries is powerful enough to remove all unphysical states (Lipstein,Schomerus).

1-Loop Amplitudes

- Furthermore, the 1-loop amplitudes appear to be sensible:

$$\mathcal{A}^{(1)}_{even} = \int d\tau \frac{d^2\lambda_0 d^2\tilde{\lambda}_0 d^8\eta_0}{GL(1)} \Pi_{i=1}^n \frac{dz_i dt_i}{t_i^3}$$
$$\delta^2\left(R_\lambda\right)\delta^2\left(R_{\tilde{\lambda}}\right)\delta^8\left(R_\chi\right)\left(\Pi_{l=1}^k \delta_l\right)\left(\Pi_{r=k+1}^n \delta_r\right) M$$

where

$$\begin{aligned}
\delta_l &= \delta^2\left(\tilde{\lambda}_l - t_l \tilde{\lambda}\left(z_l\right)\right) \\
\delta_r &= \delta^{2|8}\left(\lambda_r - t_r \lambda\left(z_r\right) | \eta_r - t_r \chi\left(z_r\right)\right) \\
M &= \sum_{\alpha=2,3,4} (-1)^\alpha \det H_\alpha \left(\theta_\alpha(0,\tau)/\eta(\tau)^3\right)^{-4}
\end{aligned}$$

$$R_\lambda = \sum_{l=1}^k t_l \lambda_l, \;\; R_{\tilde{\lambda}} = \sum_{r=k+1}^n t_r \tilde{\lambda}_r, \;\; R_\chi = \sum_{l=1}^k t_l \eta_l$$

Summary

- 4d ambitwistor string theory gives rise to new tree-level formulae for gauge and gravity amplitudes with any amount of susy.

- Provides new insight into soft theorems and symmetries of the gravitational S-matrix.

- May also provide new insight into finiteness of N=8 SUGRA.
  (Bern,Carrasco,Dixon,Johansson,Kosower,Roiban)

Thank You