Sagittarius A - Chandra X-ray observatory*

Classical Double Copies

Donal O'Connell
University of Edinburgh

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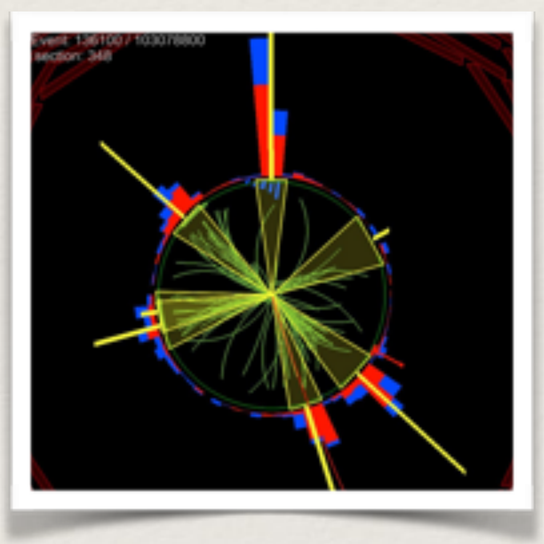
Work with Andrés Luna Godoy (Glasgow), Ricardo Monteiro (Oxford) and Chris White (Glasgow)

Overview

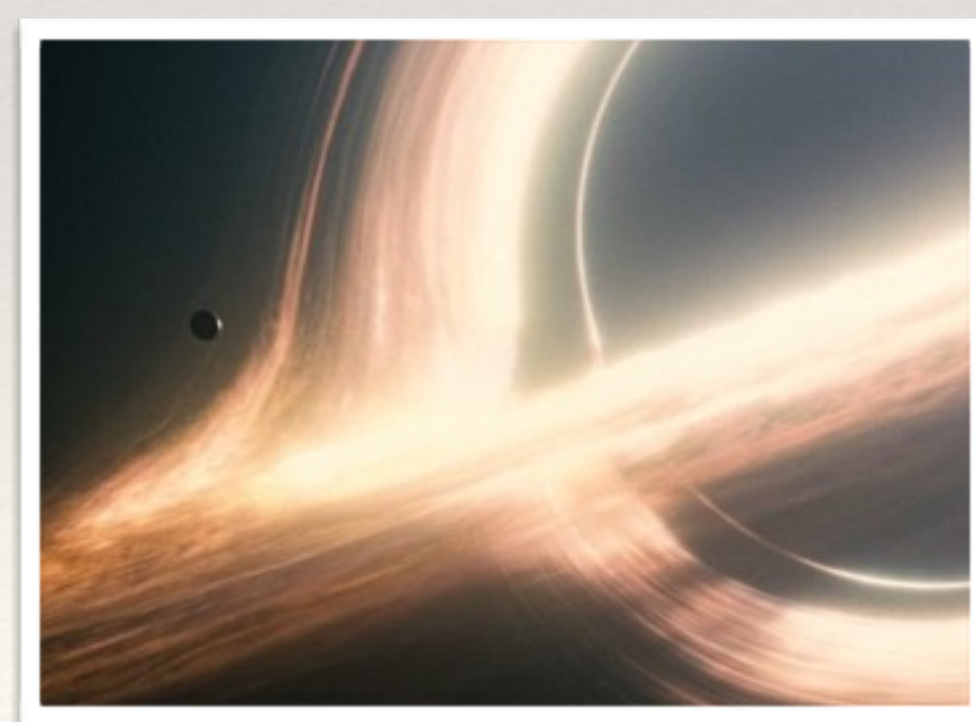
- We know a lot about tree amplitudes
- KLT/BCJ: close connection between tree gauge theory and gravity

$$-\frac{1}{4}\,\mathrm{tr}\;F^2 \Rightarrow$$



$$\xrightarrow{\text{KLT}} \mathcal{M}_n \qquad \Rightarrow S = \int d^4x\sqrt{-g}R \Rightarrow$$

(Gravity amplitudes)



BCJ basics

- Organise Yang-Mills amplitudes into a sum over three-point diagrams

Gauge group colour factors: $f^{abe}f^{cde}$

Kinematic numerator: $\epsilon_1 \cdot (p_2 - p_3) \cdots$

$$\mathcal{A} = \sum_{i \in \text{cubic}} \frac{c_i n_i}{D_i}$$

Scalar propagator denominators

- Choose numerators which have a "colour-dual" property:

$$\begin{aligned} c_i + c_j + c_k &= 0 \\ \Rightarrow n_i + n_j + n_k &= 0 \end{aligned} \Bigg\}$$ some kind of gauge choice

- Then get gravity

$$\mathcal{M} = \sum_{i \in \text{cubic}} \frac{n_i n_i}{D_i}$$

BCJ basics

- At tree level, double copy implies KLT (and is proven)

Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Bern, Dennen, Huang, Kiermaier;

Mafra, Schlotterer, Stieberger; Bjerrum-Bohr, Damgaard, Monteiro, DOC; Cachazo, He, Yuan

- Advantage: loop level generalisation

$$\mathcal{A} = \sum_{i \in \text{cubic}} \int \frac{d^{DL}\ell}{(2\pi)^{DL}} \frac{1}{S_i} \frac{c_i n_i(\ell)}{D_i(\ell)} \qquad \mathcal{M} = \sum_{i \in \text{cubic}} \int \frac{d^{DL}\ell}{(2\pi)^{DL}} \frac{1}{S_i} \frac{n_i(\ell)\tilde{n}_i(\ell)}{D_i(\ell)}$$

Bern, Carrasco & Johansson, 2010

- Quite a lot of evidence for this → Carrasco's talk

Kerr-Schild metrics

- BCJ identified a gauge: we propose a gauge for a class of spacetimes
  - Will see similar structure to BCJ kinematic Jacobi gauge
- Kerr-Schild (KS) class of spacetimes admit metric of form

$$g_{\mu\nu} = \eta_{\mu\nu} + k_\mu k_\nu \phi$$

  - Not an approximation!
- Vector *k* is special: $k_\mu \eta^{\mu\nu} k_\nu = 0 \quad k^\mu g_{\mu\nu} k^\nu = 0$
- Therefore, inverse metric also simple $g^{\mu\nu} = \eta^{\mu\nu} - k^\mu k^\nu \phi$

Kerr-Schild metrics

- Define "graviton"

$$h_{\mu\nu} = k_\mu k_\nu \phi = g_{\mu\nu} - \eta_{\mu\nu}$$

  - No linear approximation, not necessarily any kind of wave

- KS metrics enjoy an astonishing property: linear Einstein equations!

$$2R^{\mu}{}_{\nu} = \partial^\mu \partial_\alpha h^\alpha_\nu + \partial_\nu \partial^\alpha h^\mu_\alpha - \partial^2 h^\mu_\nu = 0.$$

Kerr-Schild metrics

$$h_{\mu\nu} = k_\mu k_\nu \phi$$

- KS graviton $h_{\mu\nu} = k_\mu k_\nu \phi$ looks a little like a double copy?
    - Tensor product of two vectors, some scalar playing a role
- Maybe $A_\mu = k_\mu \phi$?
- Take stationary case. Easy to see that $R^\mu{}_\nu = 0$

$$\Rightarrow \partial^\mu F_{\mu\nu} = 0$$

$$\partial^2 \phi = 0$$

→ Scalar field $\phi \sim \frac{1}{\partial^2}$ in presence of point source!

- Other cases work too - no completely systematic story yet

Kerr-Schild & BCJ

- Structural similarities between Kerr-Schild & BCJ double copy
  - Distinguished role of a scalar $\partial^2\phi = 0$
  - One k_μ for gauge theory, $k_\mu k_\nu$ in gravity
  - BCJ manifest in self-dual sector, fields Kerr-Schild-like

$$A^a_\mu = \hat{k}_\mu \phi, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \hat{k}_\mu \hat{k}_\nu \phi$$

  - Shockwave solutions are Kerr-Schild, known relationship to amplitudes

Saotome & Akhoury

Examples

1. Black holes

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$$

- Schwarzschild metric is very simple in KS coordinates

$$\phi = \frac{2GM}{r} = \kappa \frac{M}{4\pi r}, \quad k = (1, \hat{r})$$

- Take single copy

$$k_\mu k_\nu \to k_\mu, \quad \kappa \to g, \quad M \to q \quad \frac{1}{4\pi r} \to \frac{1}{4\pi r}$$

- So gauge solution

$$A_\mu = g \frac{q}{4\pi r} k_\mu$$

- Coulomb charge in a (slightly) funny gauge
- Kerr / higher dimensional cases: similar

2. (Anti) de Sitter

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda r^2 k_\mu k_\nu, \quad k_\mu = (1, \hat{r})$$

- Single copy: $A_\mu = \lambda r^2 k_\mu$
- Can transform to $A_\mu = (\lambda r^2, 0, 0, 0)$
  - Uniform charge density filling space
- Same *k* as Schwarzschild, so (A)dS-Schwarzshild is just

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(\lambda r^2 + \frac{2M}{r}\right) k_\mu k_\nu$$

- Single copy: point charge located in a uniform charge density

3. Taub-NUT

- Celebrated solution: mass m, also NUT charge n
  - Gravitational instanton (for appropriate parameters)
- Not Kerr-Schild! - But is "Double Kerr-Schild"

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \phi k_\mu k_\nu + \psi l_\mu l_\nu$$

Fiducial metric, eg Minkowski

$$k^2 = 0$$
$$l^2 = 0$$
$$k \cdot l = 0$$
$$k \cdot Dk = 0$$
$$l \cdot Dl = 0$$

- More generally, can study dS-Kerr-Taub-NUT
- Single copy? Charge, and …?

3. Taub-NUT

- Metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa m\ \phi k_\mu k_\nu + \kappa n\ \psi l_\mu l_\nu$
- Use, $m\kappa \to e, n\kappa \to g$, get gauge field $A_\mu = e\phi k_\mu + g\psi l_\mu$
  - Find $$F = \frac{e}{4\pi r^2} dt \wedge dr + * \frac{g}{4\pi r^2} dt \wedge dr$$
- NUT charge maps to magnetic charge!
  - Mass maps to electric charge, exactly as in Schwarzschild/Kerr
- Dirac quantisation condition maps into known condition on mass and NUT charge

Conclusion

- Rich relationship between gauge theory and gravity
- Relationship depends on identifying appropriate gauges

- Point particles map as expected
- Would like to understand more non-trivial “interacting” solutions
    - We’re pretty good at interactions in YM theory!
    - Potential impact on understanding of gravitational physics