Cosmological Singularities in String Theory

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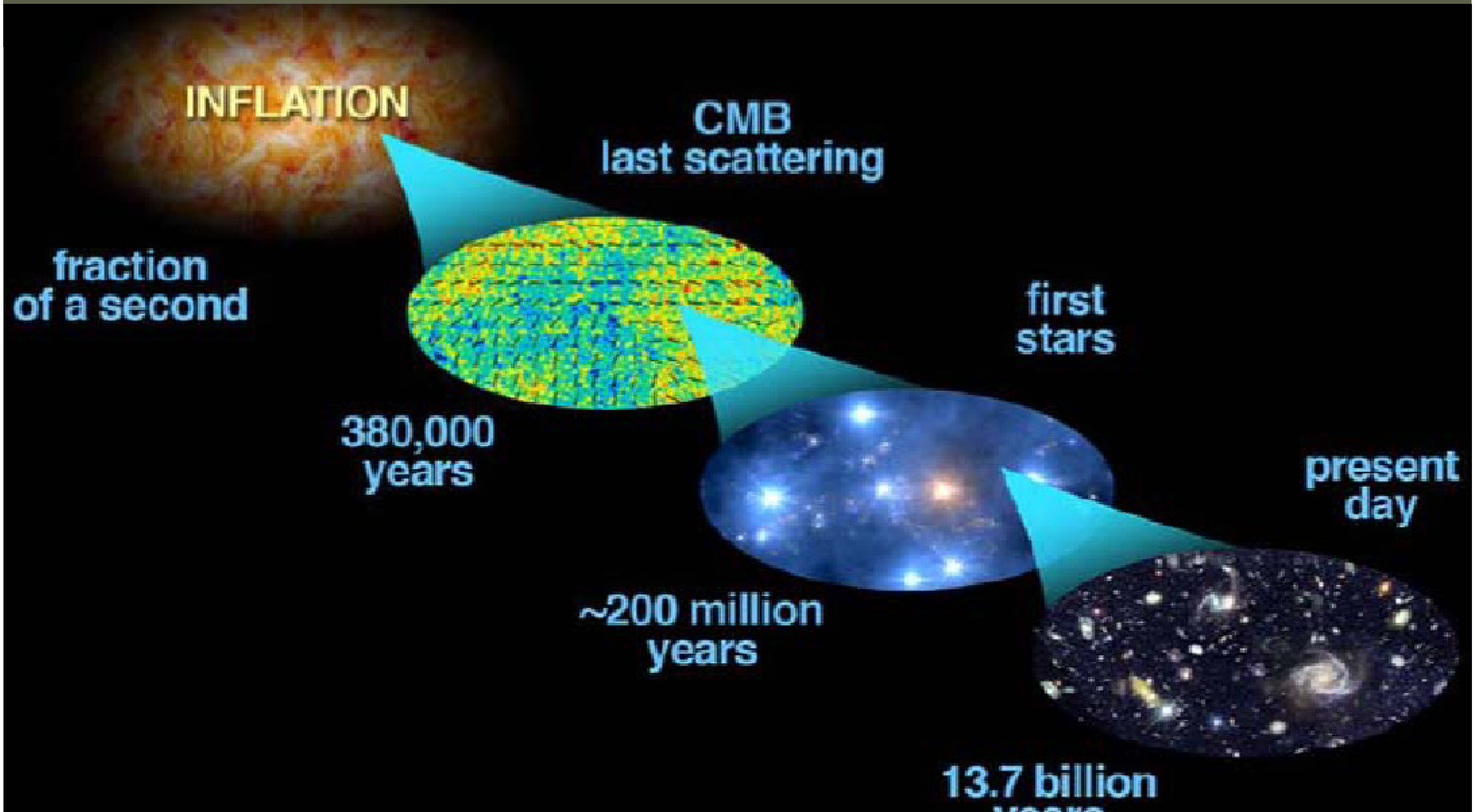
Outline

Inflation?



Task Force On Cosmic Microwave Background Research, National Academies 2005.

Successes of inflation

If one assumes that inflation started and lasted long enough, inflation explains the flatness and homogeneity of the universe. It also solves the monopole problem.

“Simple” (single-field, slow-roll) inflation predicts nearly scale-invariant, nearly Gaussian adiabatic density perturbations. These are the seeds of large scale structure and are visible as temperature anisotropies in the CMB. This is a great success of inflation.

New predictions of inflation?

Two holy grails of observational cosmology in the coming years:
- non-gaussianities in the CMB
- CMB polarization due to primordial gravitational waves (tensor modes)

The "simplest" inflationary models (single-field, slow-roll, two-derivative) predict that non-gaussianities are too small to be observed. However, other models allow for observable levels of non-gaussianity.

Simple field theory models of inflation predict a level of tensor modes that might be observable (though it is not guaranteed). Many other models predict a non-detectable level of tensor modes.

→ Strong model-dependence: the details matter

Inflationary models need UV completions

- "Slow-roll" conditions can be destroyed by $(1/M_P)^2$ quantum gravity corrections to the potential. Sufficient control over UV of the theory needed to compute such terms.

- Models with observable primordial gravity waves: inflaton moves over distances in field space much larger than M_P. Terms involving arbitrarily high powers of φ/M_P are thus important. Their coefficients depend on the UV of the theory.

- Single field, slow-roll inflation: only higher derivative models can give observable non-gaussianity. Need to know the UV theory to argue why other higher derivative terms are not present.

<u>Conclusion</u>: satisfactory theory of inflation requires theory beyond GR coupled to matter.

Inflation in string theory

- Important progress on constructing inflaton actions in string theory
  - Need to stabilize moduli
  - Need to forbid or compute all relevant Planckian corrections

- But how did inflation start?

  → how was fine-tuned initial state selected?

- In GR, inflationary solutions are (generically) past geodesically incomplete

  → does singularity resolution constrain possible string theory solutions?

An alternative? The cyclic universe

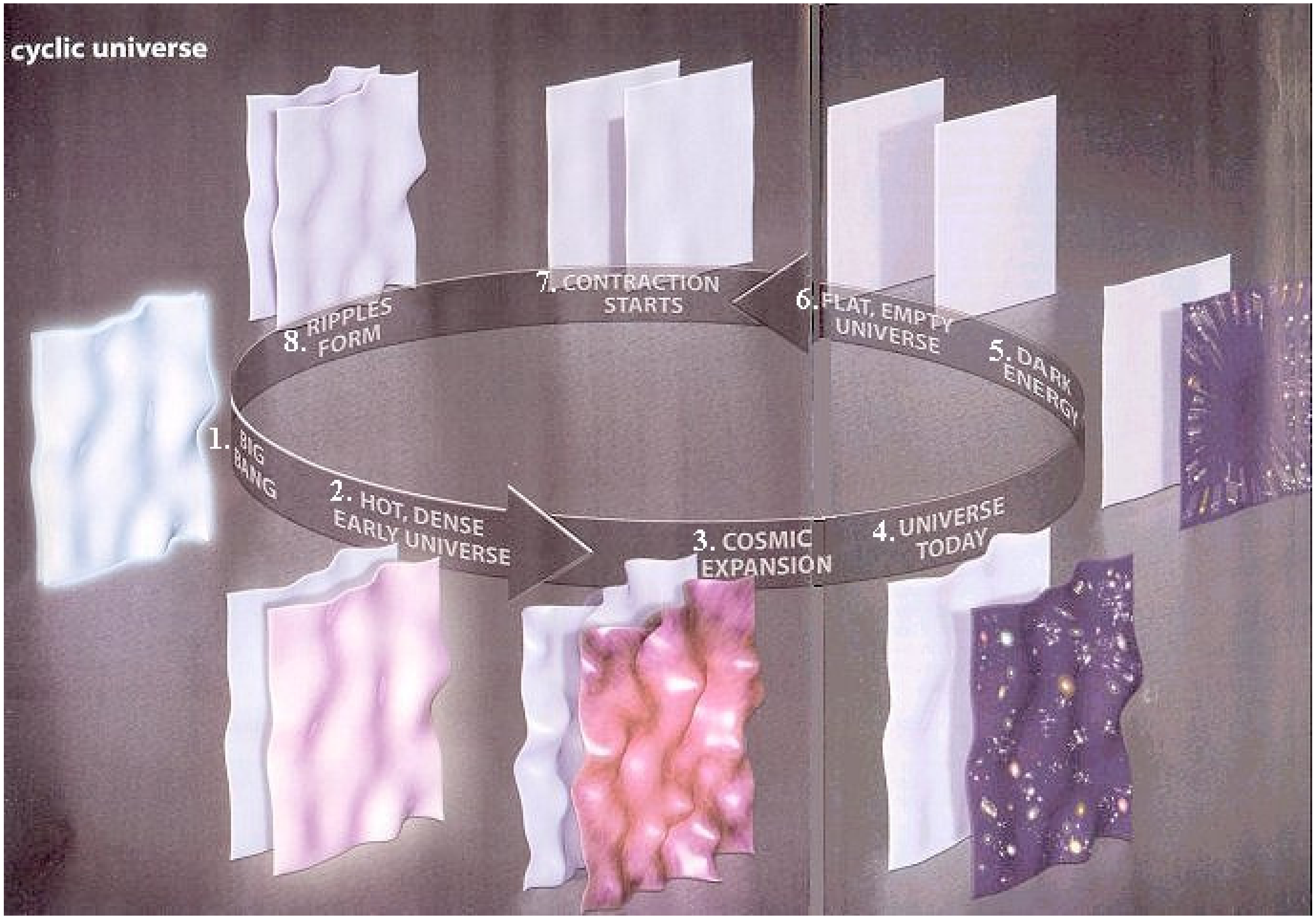


Figure: Astronomy Magazine, April 2009

A big crunch/big bang transition?

Inflation (ultra rapid expansion) is the only known mechanism to dynamically generate the required density perturbation in an expanding universe.

In a contracting universe, the ekpyrotic mechanism (ultra slow contraction) generates a spectrum of perturbations very similar to that of inflation.

Transition from contracting to expanding (spatially flat) universe → singularity in GR. Unclear whether possible and whether perturbations would go through essentially unchanged.

Answer will have to come from a theory beyond GR.

Questions motivating this work

- Can we describe the big bang itself?
- How do space and time emerge from the big bang?
- Is it consistent to have a contracting universe before the big bang?
- Does the universe have a natural initial state, and does it lead to inflation?

Outline

I. Motivation: cosmology

II. Static singularities in string theory

III. Perturbative string theory

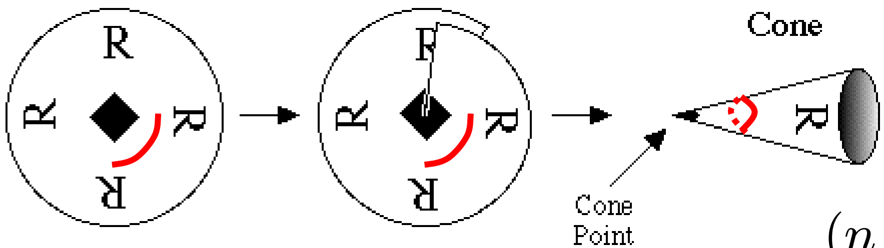
IV. Matrix theory

V. AdS/CFT

String theory resolves static orbifold singularities

$$\mathbb{C}/\mathbb{Z}_n\colon\ z \sim e^{2\pi i/n} z$$

Conical singularity at z=0



$(n = 4)$

Twisted closed strings make string perturbation theory smooth, and thus resolve the singularity perturbatively.

Dixon, Harvey, Vafa, Witten

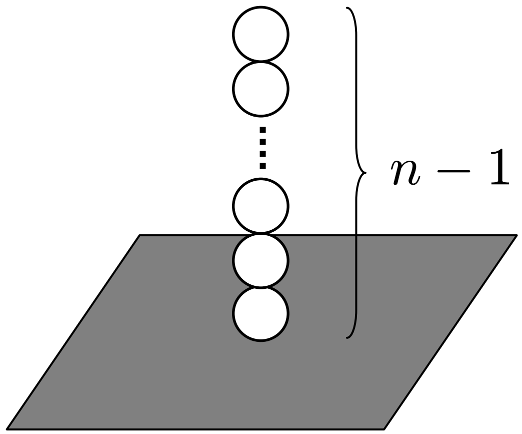
String theory resolves static orbifold singularities

$$\mathbb{C}^2/\mathbb{Z}_n\colon\ (z^1, z^2) \sim (e^{2\pi i/n} z^1, e^{-2\pi i/n} z^2)$$

A_{n-1} singularity at $(z^1, z^2) = (0,0)$

String perturbation theory is smooth due to twisted closed strings.

Geometrically, the A_{n-1} singularity can be resolved into n-1 intersecting two-spheres.



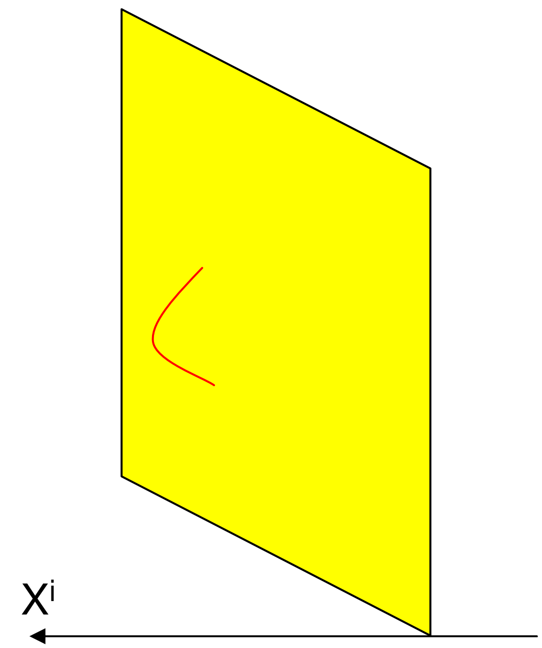
Dixon, Harvey, Vafa, Witten

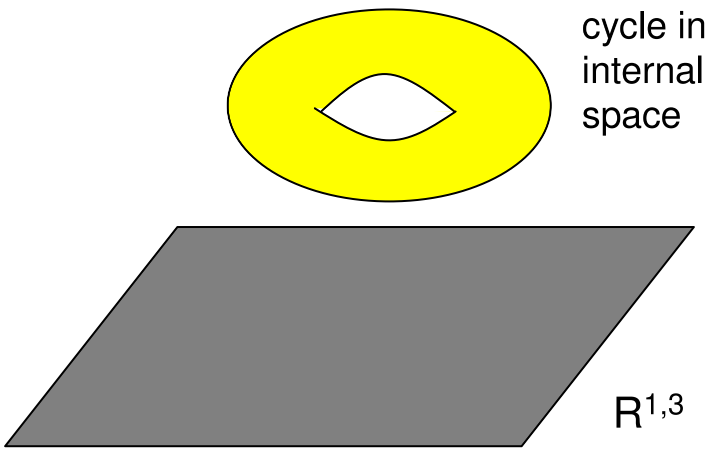
D-branes: non-perturbative objects in string theory

D-branes are extended objects on which open strings can end.

The oscillation modes of the open strings are the degrees of freedom of the brane. They include scalar fields X^i describing the location/profile of the brane in its transverse dimensions.

The tension of a D-brane is proportional to $1/g_s$, which is very large at weak coupling.



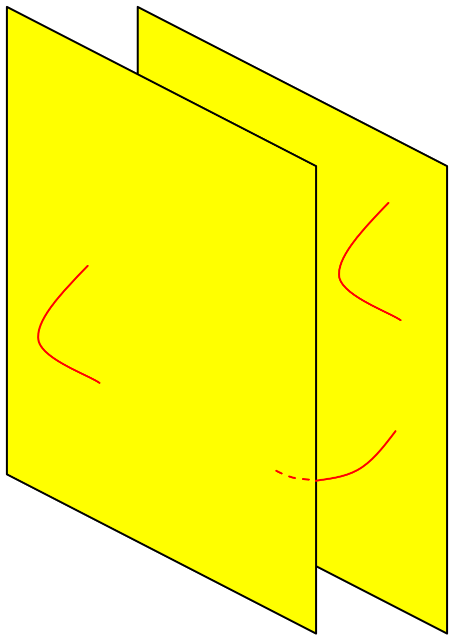


A D-brane can wrap a cycle in the internal space. At weak coupling, wrapped D-branes are very heavy, unless the cycle has very small size.

Polchinski

Multiple D-branes have matrix degrees of freedom

For two D-branes, one expects fields $(X^i)_{11}$ and $(X^i)_{22}$, describing the transverse positions/profiles of the two branes.



However, one finds more fields, corresponding to open strings stretching between the two branes: $(X^i)_{12}$ and $(X^i)_{21}$.

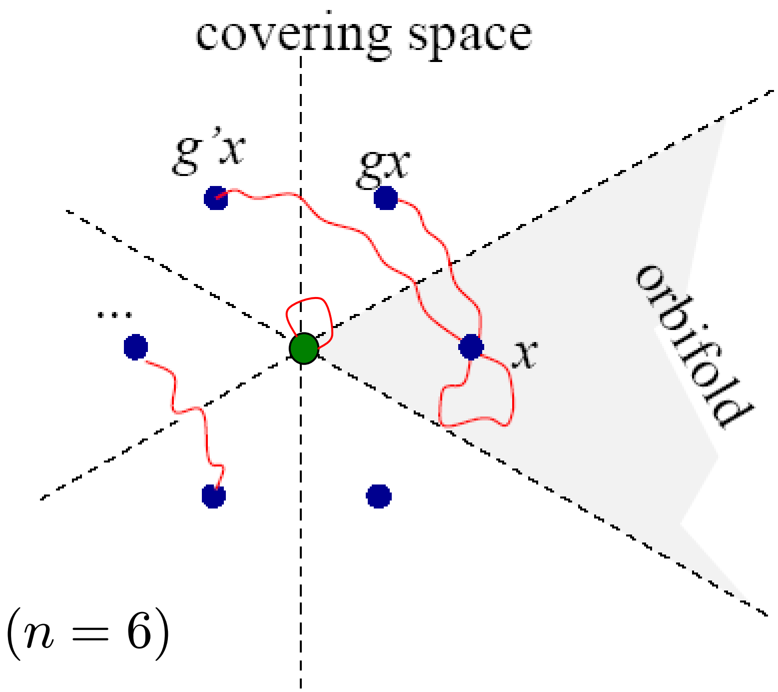
The fields combine in a 2 x 2 matrix $X^i = \begin{pmatrix} X^i_{11} & X^i_{12} \\ X^i_{21} & X^i_{22} \end{pmatrix}$

It turns out that there is a potential $V \sim \mathrm{Tr}[X^i, X^j]^2$

This implies that the off-diagonal modes (stretched strings) are very massive when the branes are well-separated. Then only the diagonal modes (brane positions/profiles) are light. When the branes are close to each other, all the matrix degrees of freedom are light!

The notion of spacetime becomes fuzzy at short distances

D-branes as probes

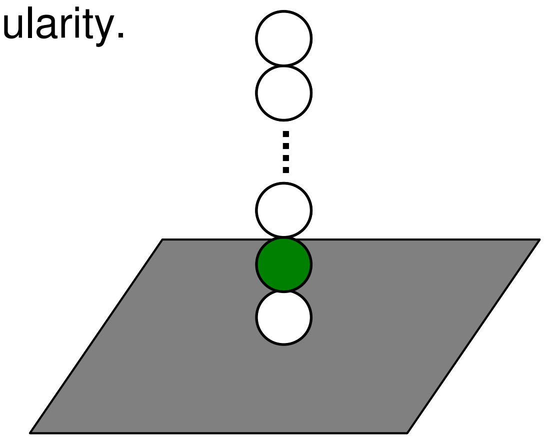


A fundamental tool to describe D-branes is boundary conformal field theory.

Recknagel, Schomerus; Fuchs, Schweigert

Bulk D-branes correspond to n image branes in the covering space. They can move anywhere in the orbifold.

Fractional D-branes correspond to one or more image D-branes at a fixed point. They are stuck at the singularity.



There exists a very precise correspondence between fractional D-branes of the orbifold and D-branes wrapping two-spheres of the resolved orbifold.

D-branes as probes of the singularity!

Douglas, Moore; Polchinski; Diaconescu, Douglas, Gomis; Diaconescu, Gomis; Billo, BC, Roose; ...

Why are the wrapped D-branes heavy?

We have seen that orbifolds singularities are resolved within perturbative string theory, which includes twisted closed strings in the description.

However, we have also seen that the theory contains fractional branes, corresponding to D-branes wrapping vanishing cycles. Geometrically, one would expect these to be massless. The presence of massless non-perturbative states would imply that string perturbation theory was singular, in contradiction with the previous observation.

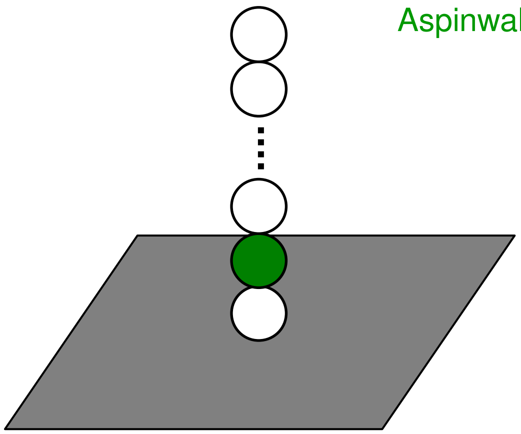
Resolution: in string theory, cycles are characterized not only by their size, but also by fluxes through them. Turns out: the orbifold CFT corresponds to a non-vanishing flux of the NS-NS two-form B through the vanishing cycles. This B-flux gives the wrapped branes a mass.

Aspinwall

One way to see that B-flux gives rise to a mass:

$$S_{WZ}^{D2} \sim \int C_3 + B \wedge C_1 + \dots$$

B-flux induces D0-brane charge on a wrapped D2-brane. Because of the BPS-bound, this implies a non-vanishing mass.



ALE singularities without B-flux

So what happens if the B-flux is turned off? This can be done by a marginal deformation in CFT and corresponds to going to a different point in moduli space.

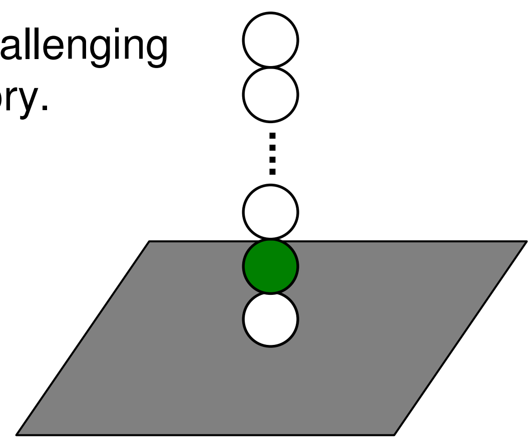
In the absence of B-flux, the D-branes wrapping vanishing cycles are now massless. Since these are non-perturbative degrees of freedom, string perturbation theory is expected to break down.

Indeed, it turns out that string perturbation theory is now singular! The conclusion is that wrapped D-branes are light and need to be included in the description.

This is very similar to what happens for the conifold. Strominger

The quantitative study of these singularities in string theory is more challenging than for orbifolds. Techniques include (double scaled) little string theory.

Including wrapped branes in the description resolves the singularity in non-perturbative string theory



Summary [Static singularities in string theory]:

- At (static) orbifold singularities, perturbative string modes (twisted closed strings) are light. They resolve the singularity in perturbative string theory.

- Without the B-flux, there are also light non-perturbative modes (wrapped D-branes). The singularity is then resolved in non-perturbative string theory.

- String theory successfully resolves (important classes of) static singularities.

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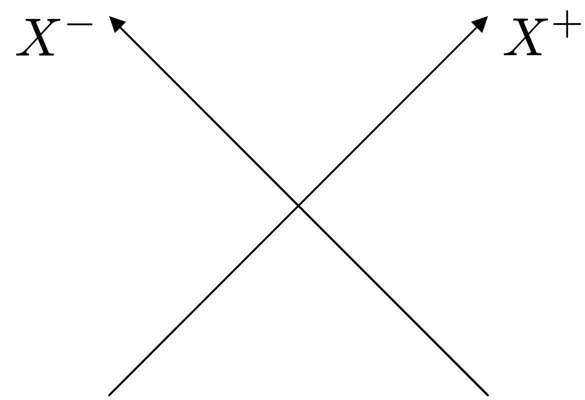
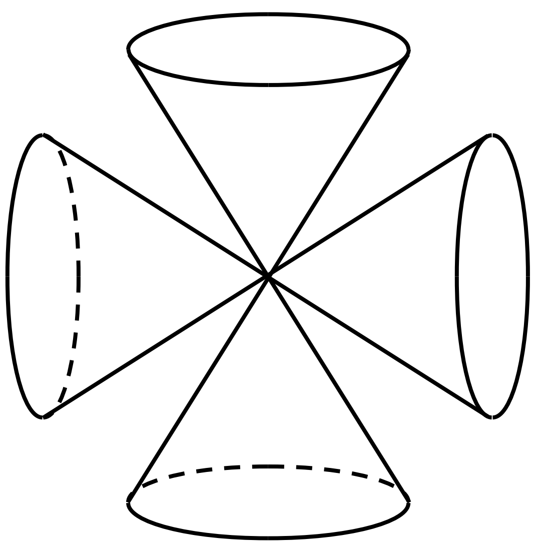
The Milne-orbifold: a big crunch/big bang singularity

$$ds^2_{10} = -2dX^+dX^- + (dX^i)^2$$



Boost identification: $X^\pm \sim e^{\pm 2\pi Q} X^\pm$

Singularity at $X^\pm = 0$



Does propagation through big crunch/ big bang singularity make sense?

Study propagation of strings from past to future cone through singularity using standard string perturbation theory.

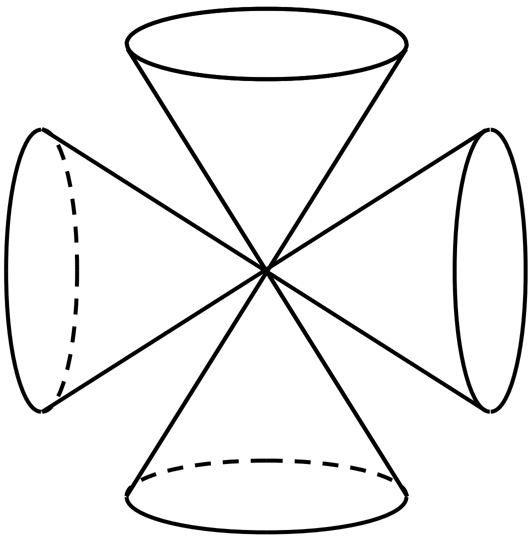
E.g. tree-level 2 → 2 scattering amplitude

Khoury, Ovrut, Seiberg, Steinhardt, Turok; Cornalba, Costa; Nekrasov; Tolley, Turok; Berkooz, BC, Kutasov, Rajesh; …

Gravitational backreaction spoils the perturbative picture

$2 \rightarrow 2$ tree level scattering amplitude diverges

⇨ the singularity is not resolved in perturbative string theory



As circle shrinks, infinite blue-shift of perturbations creates large gravitational field.

Tree-level gravitational interaction with the second perturbation causes the divergence.

A non-perturbative manifestation of gravitational backreaction: formation of a large black hole

Liu, Moore, Seiberg; Horowitz, Polchinski; Berkooz, BC, Kutasov, Rajesh

Plane wave space-times

We will consider the spacetimes

$$ds^2 = -2dx^+dx^- + \mu_{ij}(x^+)dx^i dx^j$$

These are lightlike analogues of Friedmann cosmologies if μ is diagonal and power-law.

By a coordinate transformation from Rosen to Brinkmann coordinates:

$$ds^2 = -2dx^+ dx^- + K_{ij}(x^+)x^i x^j (dx^+)^2 + (dx^i)^2$$

In pure gravity, $K_{ii} = 0$. Need to include matter to find isotropic solutions.

<u>Virtues</u>:

- No α' corrections
- Lightlike isometry allows matrix theory description

Plane waves as Penrose limits

(zoom in on null geodesic)

Penrose limit of FRW → homogeneous singular plane wave

(with $\rho > |p|$)

Blau, Borunda, O'Loughlin, Papadopoulos

$$ds^2 = -2dX^+dX^- - \frac{k}{(X^+)^2}(X^i)^2(dX^+)^2 + (dX^i)^2$$

- Solves string e.o.m. when supplemented by $\phi = \phi_0 - cX^+ + 4k \ln X^+ \qquad (D = 10)$

$g_s \sim e^{\phi}$:

- $k > 0$: weak coupling singularity at $X^+ = 0$
- $k < 0$: strong coupling singularity at $X^+ = 0$
- $k = 0$: flat space with lightlike linear dilaton
  strong coupling singularity at $X^+ \to -\infty$ (if $c > 0$)

Papadopoulos, Russo, Tseytlin

Free particles and strings in plane wave geometry

To discuss propagation of free particles or strings in a plane wave, need to solve Schrödinger eq for time-dependent harmonic oscillators.

To solve this Schrödinger eq, it is sufficient to solve classical e.o.m. for time-dependent harmonic oscillator.

Strings in singular homogeneous plane waves: modes near singularity

$$ds^2 = -2dX^+dX^- - \frac{k}{(X^+)^2}(X^i)^2(dX^+)^2 + (dX^i)^2$$

String modes satisfy $\ddot{X}_n^i + \omega_n^2(\tau)X_n^i = 0$ with $\omega_n^2(\tau) = n^2 + \frac{k}{\tau^2}$

Behavior for $\tau \downarrow 0$: $X_n^i(\tau) \approx a_n^i \tau^\nu + b_n^i \tau^{1-\nu}$ with $\nu = \frac{1}{2}\left(1 + \sqrt{1-4k}\right)$

Modes and/or first derivatives are generically singular at $\tau = 0$

Q: Is there consistent and well-motivated way to continue beyond $\tau = 0$?

- Earlier work: prescription based on analytic continuation — Papadopoulos, Russo, Tseytlin
- More recent proposal: view singular plane wave as limit of smooth ones — Evnin, Nguyen; BC, De Roo, Evnin

“Single scale” geometric resolutions

$$ds^2 = -2dX^+dX^- - \frac{1}{\epsilon^2}\Omega\left(\frac{X^+}{\epsilon}\right)(X^i)^2(dX^+)^2 + (dX^i)^2$$

ϵ: parameter of dimension length (no other scales). Geometry for $\epsilon > 0$ is smooth.
Singularity develops at $X^+ = 0$ as $\epsilon \to 0$.

Example: $$\frac{1}{\epsilon^2}\Omega\left(\frac{X^+}{\epsilon}\right) = \frac{k}{(X^+)^2 + \epsilon^2}$$

Strategy: solve e.o.m. for string modes for $\epsilon > 0$ and take $\epsilon \to 0$ limit of solutions (which may or may not exist)

Virtue: for any $\epsilon > 0$, background e.o.m. are exactly satisfied.

Result:

- $\epsilon \to 0$ limit of individual mode functions exists for discrete values of k.
- For $k > 0$ (weak coupling singularity), the total excitation energy of the string always diverges.

Evnin, Nguyen; BC, De Roo, Evnin

Holographic descriptions of string theory

- Matrix theory. Quantum mechanics of large matrices
  - Large distances: space-time, gravity
  - Small distances: non-commuting matrices

- The AdS/CFT correspondence

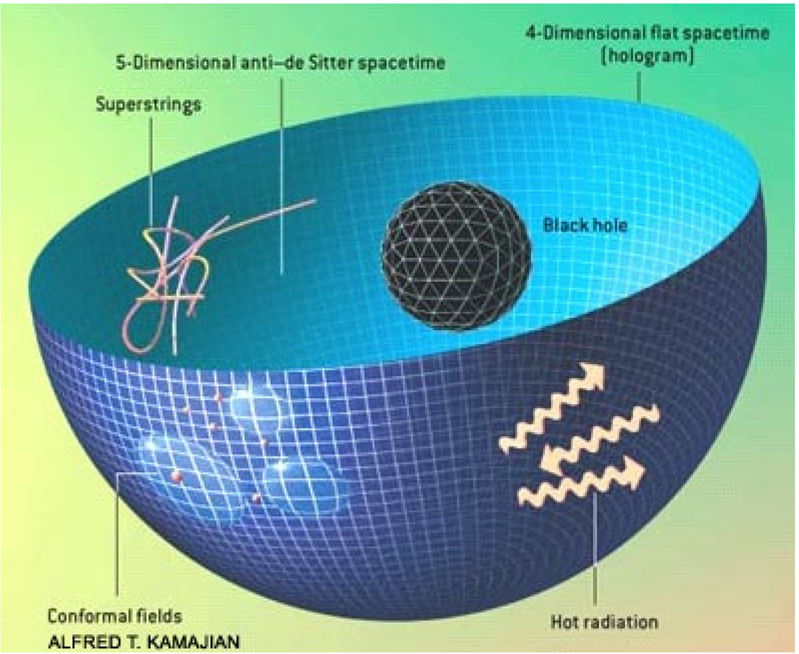


Figure: universe-review

Summary [Perturbative string theory]:

- In simple time-dependent orbifolds, string perturbation theory is invalidated by divergences due to large backreaction.

- Free strings get infinitely excited when crossing a (weakly coupled) singularity of a singular homogeneous plane wave (using a geometric resolution prescription).

- This motivates the use of non-perturbative string theory.

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Matrix (string) theory: holography in asymptotically flat space-time

Matrix string theory: non-perturbative formulation of type IIA string theory in 10d Minkowski space. Described by $\mathcal{N}=8$ U(N) super-Yang-Mills theory in 2d, in a large N limit:

$$S = \int d\tau d\sigma \,\mathrm{Tr}\left((D_\mu X^i)^2 + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \text{fermions} \right)$$

World-sheet: infinite cylinder. Coordinates (τ, σ), where $\sigma \sim \sigma + 2\pi$.

X^i: N x N hermitean matrices; index i labels 8 dimensions transverse to worldsheet.

<u>At weak string coupling</u> $g_s \to 0$: $[X^i, X^j] = 0$

- Eigenvalues of X^i correspond to coordinates of (pieces of) superstring.
- Off-diagonal matrix elements: very massive, can be integrated out.
- Space-time arises dynamically from the “moduli space” of vacua.

Banks, Fischler, Shenker, Susskind; Motl; Banks, Seiberg; Dijkgraaf, Verlinde, Verlinde

Light-like linear dilaton background

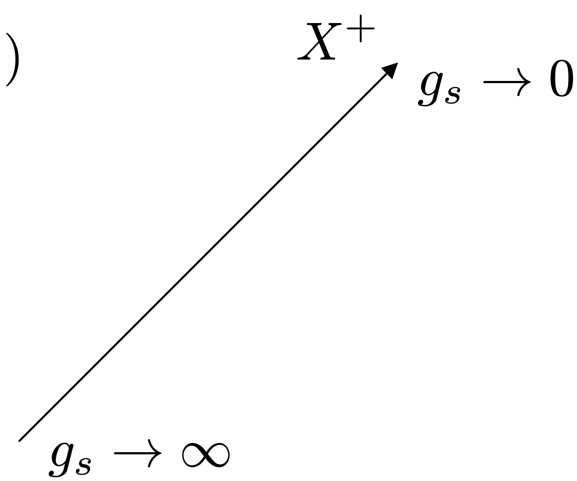
Simple time-dependent solution of 10d (type IIA) string theory: flat space with a light-like linear dilaton (preserves ½ susy):

$$ds^2_{10} = -2dX^+dX^- + (dX^i)^2$$

$$\Phi = -QX^+$$

The dilaton Φ is a scalar field that appears in the low-energy effective action as

$$S \sim \int d^{10}x\,\sqrt{G}\,e^{-2\Phi}\,(R + 4\partial_\mu\Phi\partial^\mu\Phi + \ldots)$$



Therefore, the exponential of the dilaton can be viewed as the string coupling “constant”: $g_s = e^\Phi$

Strong coupling singularity for $X^+ \to -\infty$ corresponds in Einstein frame to curvature singularity at finite affine parameter

BC, Sethi, Verlinde

Matrix description of the light-like linear dilaton: "matrix big bang"

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \text{fermions} \right)$$

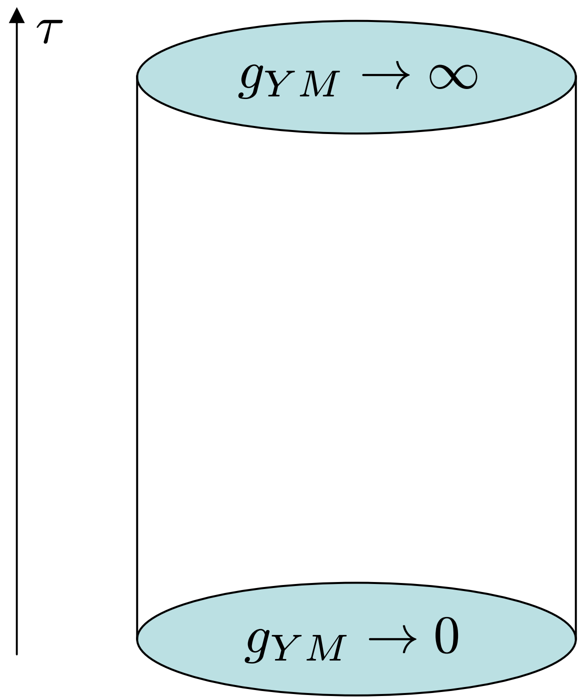
Turns out: τ is related to X^+ by $X^+ = \dfrac{\tau}{R}$, where R is a parameter related to the total light-cone momentum p^+ of the system under consideration: $p^+ = \dfrac{N}{R}$.

Result: simply plug in $g_s = e^{-QX^+} = e^{-\frac{Q\tau}{R}}$, leading to (1+1)-dimensional SYM on the cylinder, with coupling

$$g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q\ell_s\tau}{R}\right)$$

BC, Sethi, Verlinde

Cosmological evolution: emergence of space



[X^i, X^j] = 0: spacetime emerges
(weakly coupled strings)

free SYM: non-commuting matrices
(new light degrees of freedom)

BC, Sethi, Verlinde

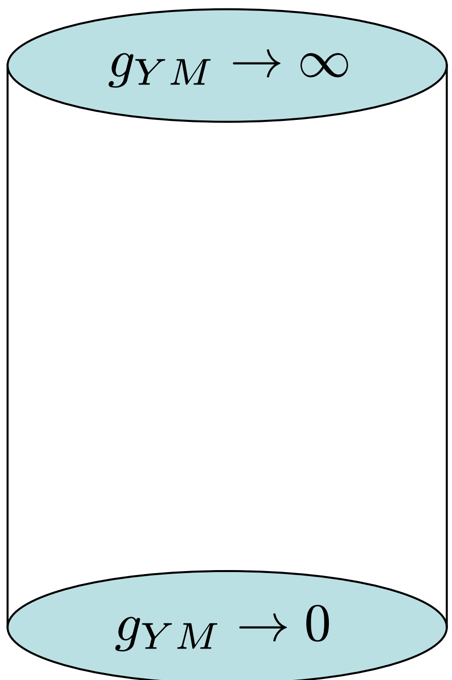
Cosmological evolution: two equivalent pictures

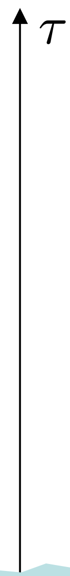
SYM on the cylinder with

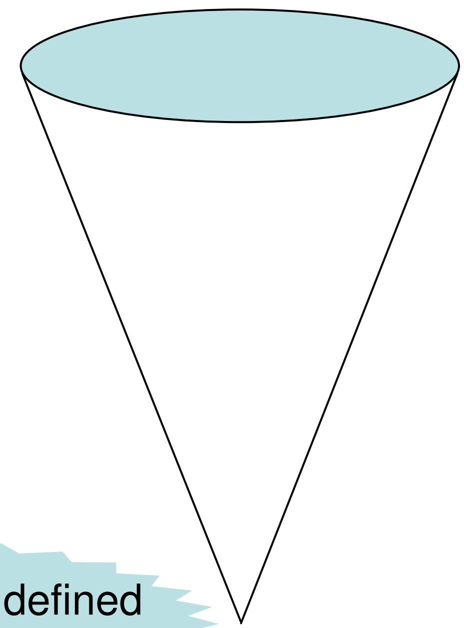
$$g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q\ell_s\tau}{R}\right)$$

SYM with constant coupling on future Milne cone

$$ds^2 = e^{\frac{2Q\tau}{R}}(-d\tau^2 + d\sigma^2)$$







Can time evolution be defined beyond the Milne singularity?

BC, Sethi, Verlinde

Plane wave matrix big bang models

$$ds^2 = -2dX^+dX^- - \frac{k}{(X^+)^2}(X^i)^2(dX^+)^2 + (dX^i)^2 \qquad e^{2\phi} \sim (X^+)^{8k}$$

<u>Result</u>:

$$S = \int d\tau d\sigma \mathrm{Tr} \left(-\frac{1}{4g_{YM}^2} F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{2} D_\alpha X^i D^\alpha X^i + \frac{1}{4} g_{YM}^2 [X^i, X^j][X^i, X^j] - \frac{k}{2\tau^2}(X^i)^2 \right)$$

$$g_{YM}^2 \sim \tau^{-8k}$$

$k < 0$ ←→ large string coupling at singularity ←→ small YM coupling ←→ tachyonic masses

Blau, O'Loughlin. Earlier work on plane wave matrix models by Li; Li, Song; Chen; Chen, Chen; Das, Michelson; Ishino, Ohta;…

Summary [Matrix theory]:

- In matrix big bang models, space-time coordinates are replaced by non-commuting matrices near a (lightlike) singularity.

- It is unclear whether time evolution can be consistently continued beyond the singularity.

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The AdS/CFT correspondence

Quantum gravity in asymptotically AdS_4 bulk spacetime ⟺ CFT on 3d conformal boundary

AdS_4: t, θ, φ, r ⟺ CFT_3: t, θ, φ

Radial position r ⟺ Energy

Radial direction is emergent

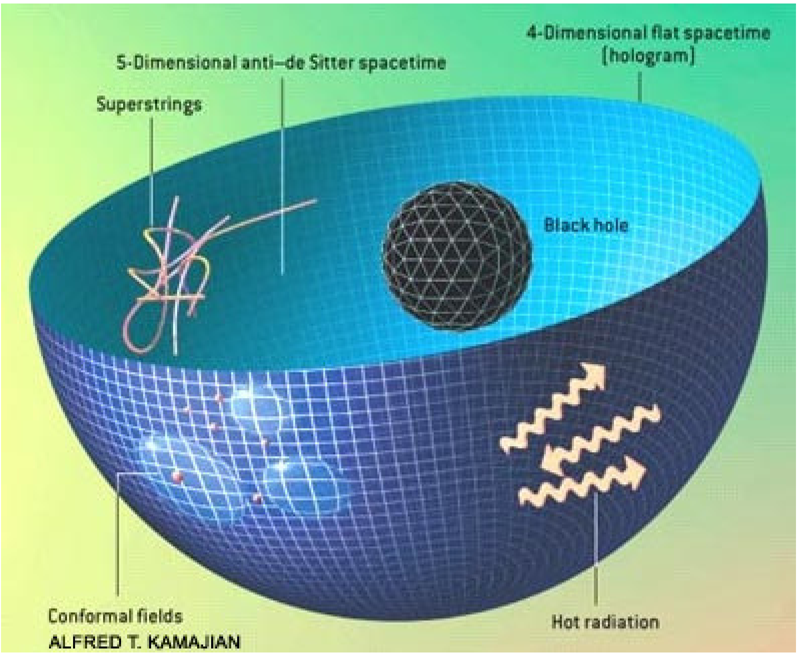
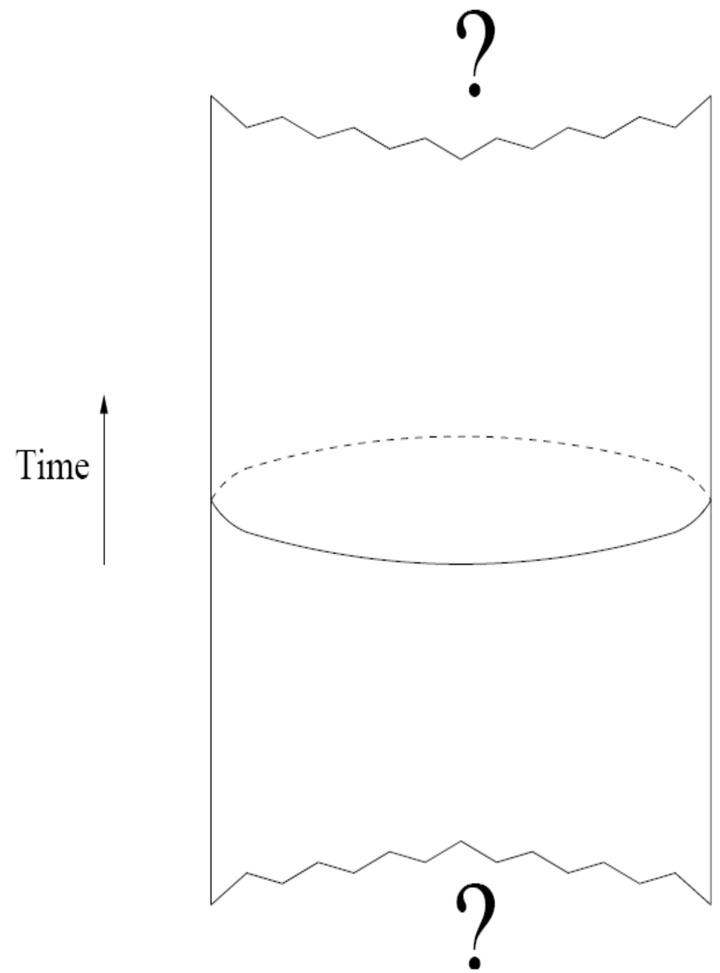


Figure: universe-review

AdS cosmologies: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity in the future (and to a big bang singularity in the past).

Can a dual gauge theory be used to study the singularity in quantum gravity?



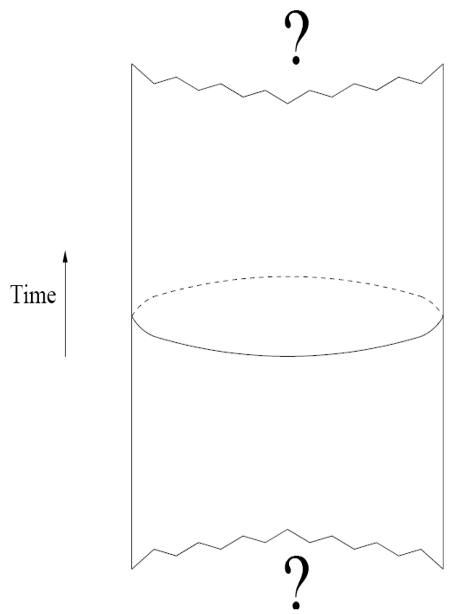
Hertog, Horowitz

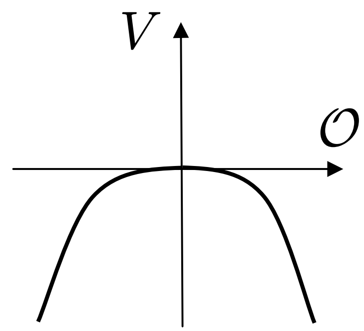
AdS cosmologies: basic idea

Modified (non-susy) boundary conditions: smooth initial data that evolve into big crunch

⟺

Potential unbounded from below in dual field theory; scalar field reaches infinity in finite time

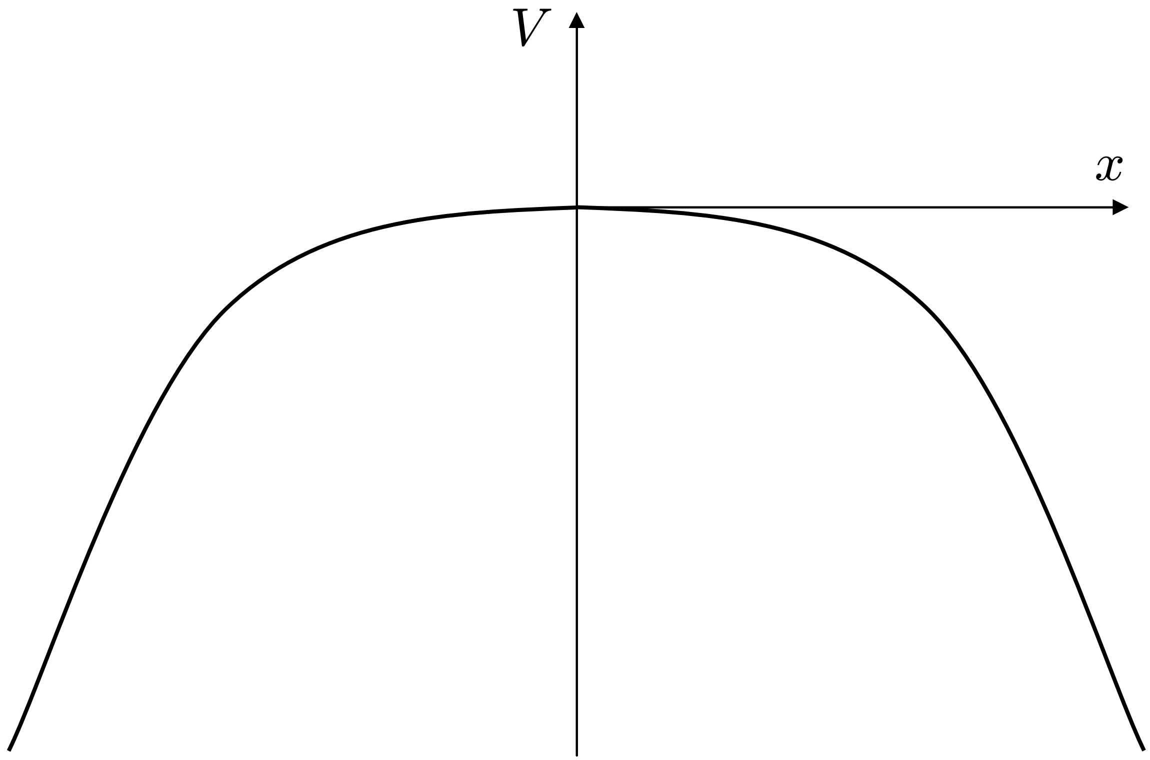




Goal: learn something about cosmological singularities (in the bulk theory) by studying unbounded potentials (in the boundary theory)

Hertog, Horowitz

Beyond the singularity? Self-adjoint extensions



- QM: self-adjoint extension → unitary time evolution
- Possible in QFT?? (Not known yet.)

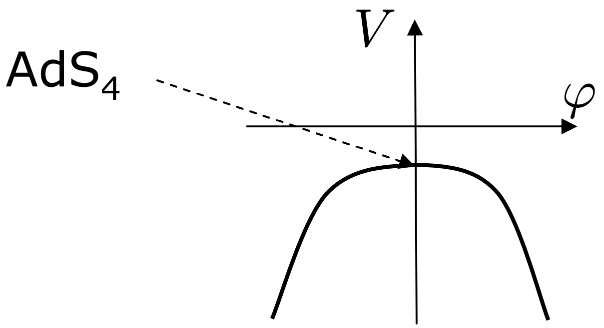
BC, Hertog, Turok

The bulk theory: AdS_4 cosmology

11d sugra on $S^7/\mathbb{Z}_k$

↓ consistent truncation

4d $g_{\mu\nu}$ + scalar φ



$$m_{BF}^2 < m^2 < m_{BF}^2 + 1$$

AAdS: $$ds^2 \sim (1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2^2 \qquad (r \to \infty)$$

$$\varphi(r) \sim \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2} \qquad (r \to \infty)$$

Boundary condition: $\beta = -h\alpha^2$

AdS invariant; crunch from smooth (instanton) initial data

Hertog, Horowitz

The dual field theory: ABJM theory

$\mathcal{N}=6$ superconformal U(N) x U(N) Chern-Simons theory, levels k resp. -k

- gauge fields $A_\mu,\ \hat{A}_\mu$
- scalars $Y^A,\ A=1,\ldots,4$, in $\begin{cases} \text{fundamental of } SU(4)_R \\ (N,\bar{N}) \text{ of } U(N)\times U(N) \end{cases}$

  sextic single trace potential

- describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$: $y^A \to e^{\frac{2\pi i}{k}} y^A$
- 't Hooft limit: $N\to\infty,\ N/k$ fixed (weakly coupled IIA string theory)
- operator dual to bulk scalar φ : $\mathcal{O}\sim\frac{1}{N^2}\mathrm{Tr}(Y_1Y_1^\dagger - Y_2Y_2^\dagger)$ BC, Hertog, Turok

Aharony, Bergman, Jafferis, Maldacena

The dual field theory: modified boundary conditions

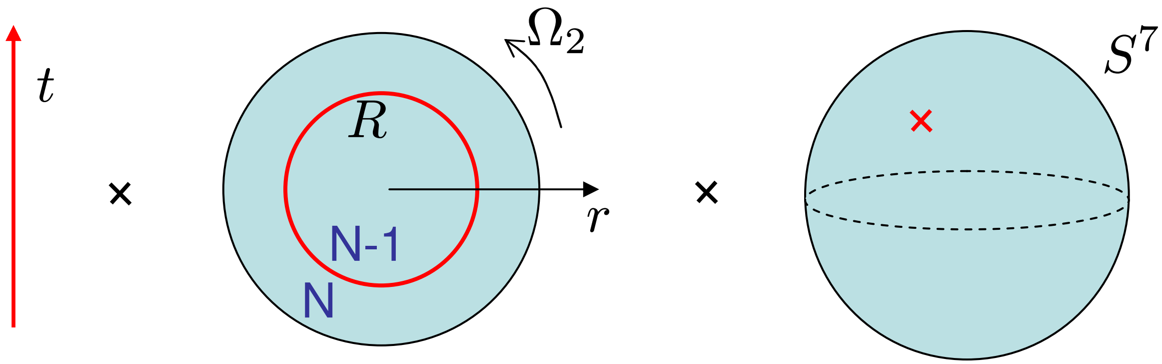
$\beta = -h\alpha^2$

$$\varphi(r) \sim \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2} \qquad (r \to \infty)$$

$$S = S_{ABJM} + \text{ conf. coupl. } + \frac{h}{N^4}\left[\mathrm{Tr}(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger)\right]^3$$

BC, Hertog, Turok

M2-brane interpretation of the instability



Bernamonti, BC

Consider spherical M2-branes in $AdS_4 \times S^7$ (can nucleate or be present in initial state)

Susy b.c. → quadratic potential for radius R of spherical M2 (conformal coupling) → spherical branes shrink and annihilate

Modified b.c. → sextic negative potential for radius R
→ large spherical M2-branes pulled to infinite radius in finite time

M2's are domain walls → 5-form flux at fixed r decreases
→ effective 't Hooft coupling decreases

Summary [AdS/CFT]:

- The AdS/CFT correspondence relates gravitational theories allowing big crunch singularities to field theories with potentials unbounded from below.
- If it were consistent to restrict to homogeneous modes, one could use self-adjoint extensions to describe singularity transition.
- It is unclear, though, whether self-adjoint extensions exist for quantum field theory.

Summary

- Cosmological models are incomplete without an understanding of the cosmological singularity.
- String theory successfully resolves various static singularities.
- In simple time-dependent orbifolds, string perturbation theory is invalidated by divergences due to large backreaction.
- Free strings get infinitely excited when crossing the singularity of a singular homogeneous plane wave (using a geometric resolution prescription).
- In matrix big bang models, space-time coordinates are replaced by non-commuting matrices near a (lightlike) singularity.
- The AdS/CFT correspondence relates gravitational theories allowing big crunch singularities to field theories with potentials unbounded from below.