# THE PRINCIPLE OF RELATIVE LOCALITY

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# Relative Locality

Relative locality is a framework in which we can relax in a controlled manner the notion of locality

We do so by allowing momentum space to be curved

What could be the motivation?

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What could be the motivation?

Quantum gravity

We know that in quantum gravity the notion of spacetime disolve

Locality as we know it at low energy is replaced by something more fundamental, what is it?

# Relative Locality

Relative locality is a framework in which we can relax in a controlled manner the notion of locality

What could be the motivation?

Quantum gravity phenomenology

How is it possible to probe experimentally theories of Q-gravity?

It is presumably impossible to detect a quantum graviton

So is there a regime of quantum gravity that may be accessible to experiment? and show radically new features?

# Quantum gravity limit

Quantum gravity

As a mathematical theory Quantum gravity depends parametrically on 3 parameter $\hbar \quad G_N \quad c$

QFT limit $\hbar \to 0 \quad G_N$ fixed GR limit $G_N \to 0 \quad \hbar$ fixed

Relative Locality limit

We work in a regime where we neglect both quantum mechanics and gravity $\hbar$ and $G_{Newton}$ are neglected while

$\hbar, G \to 0$

is held fixed $\sqrt{\frac{\hbar}{G_{Newton}}} = m_p$ $\ell_P \to 0$

In this limit there is a fundamental energy scale

How does this scale enters physics?

⟶ One expects a deformation of momentum space

# Quantum gravity limit

The limit $\ell_P \to 0 \quad m_P$ fixed can be stated in terms of dimensionless ratio

Given a massive object we can consider its Comptom wave lenght $\lambda$ and its Schwarschild radius $R_S$ and we denote by D the typical distance/ time of observation

QFT limit is a limit in which $\frac{R_S}{D} \to 0 \qquad \frac{\lambda}{D}$ finite

GR limit $\frac{R_S}{D}$ finite $\qquad \frac{\lambda}{D} \to 0$

RL limit is a classical limit in which $\frac{\lambda}{D} \to 0 \qquad \frac{R_S}{\lambda} = \left(\frac{m}{m_P}\right)^2$ finite

How does this scale enters physics?

$\longrightarrow$ One expects a deformation of momentum space

# Born Reciprocity

Max Born 1938

Born reciprocity principle: postulate that there is a fundamental symmetry in Quantum Mechanics between space and momentum space

Gravity however stress the difference, between them, space is curved while momentum space stays flat (a cotangent fiber)

How can we reconcile them?

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space is curved while momentum space stays flat (a cotangent fiber)

How can we reconcile them?

By allowing momentum space to be curved

a momentum scale is needed

This is what happens in 3d gravity !

# Operational locality

We don’t really see spacetime, we see momentum space...

As naive observers we do not directly observe spacetime points

we do not directly observe events macroscopically displaced from us

Our most fundamental measurements are all about energy-momentum quanta we absorb and emit and time of emission/reception

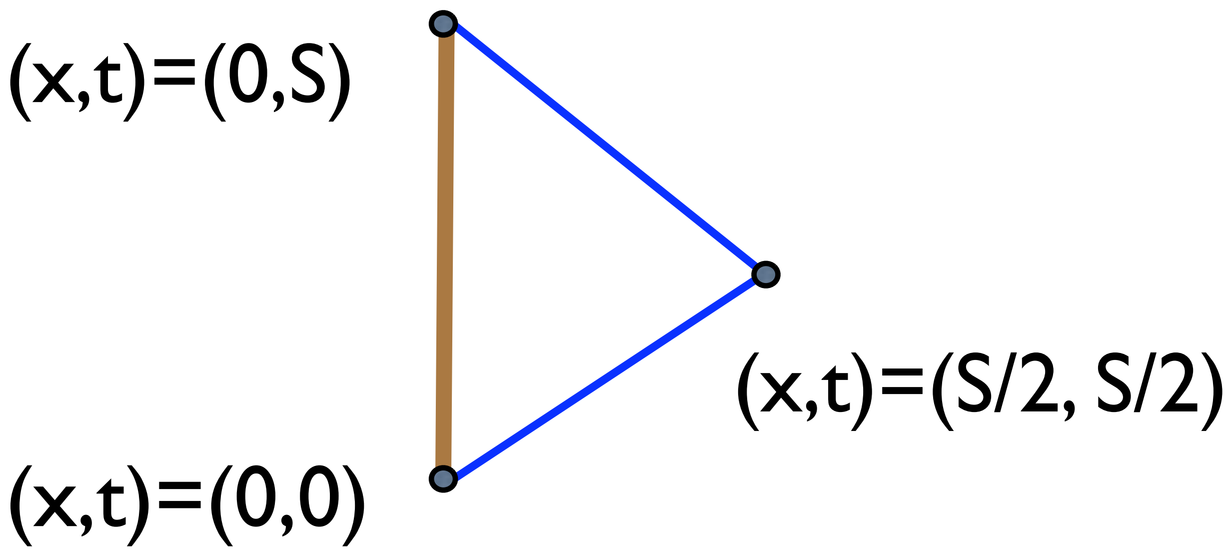
We see photons arriving with different momenta and energies at different angles

Physics happens in phase space. We do not have to assume that the projection from phase space to space time is trivial.

# Absolute locality

Spacetime is constructed by inference from energy and momenta measurement

e.g. Einstein procedure of photon exchange to give coordinates to distant events via momentum space measurements




One fundamental hypothesis is that the energy of the probe we use is inessential.

This is the absolute locality hypothesis.   Why?

Could  it  be  low energy approximation?

# Absolute locality

As we are going to see the absolute locality hypothesis is equivalent to the assumption that momentum space is a linear manifold.

→ The notion of locality is related to hypothesis about the geometry of momentum space

What if momentum space is a non linear manifold?

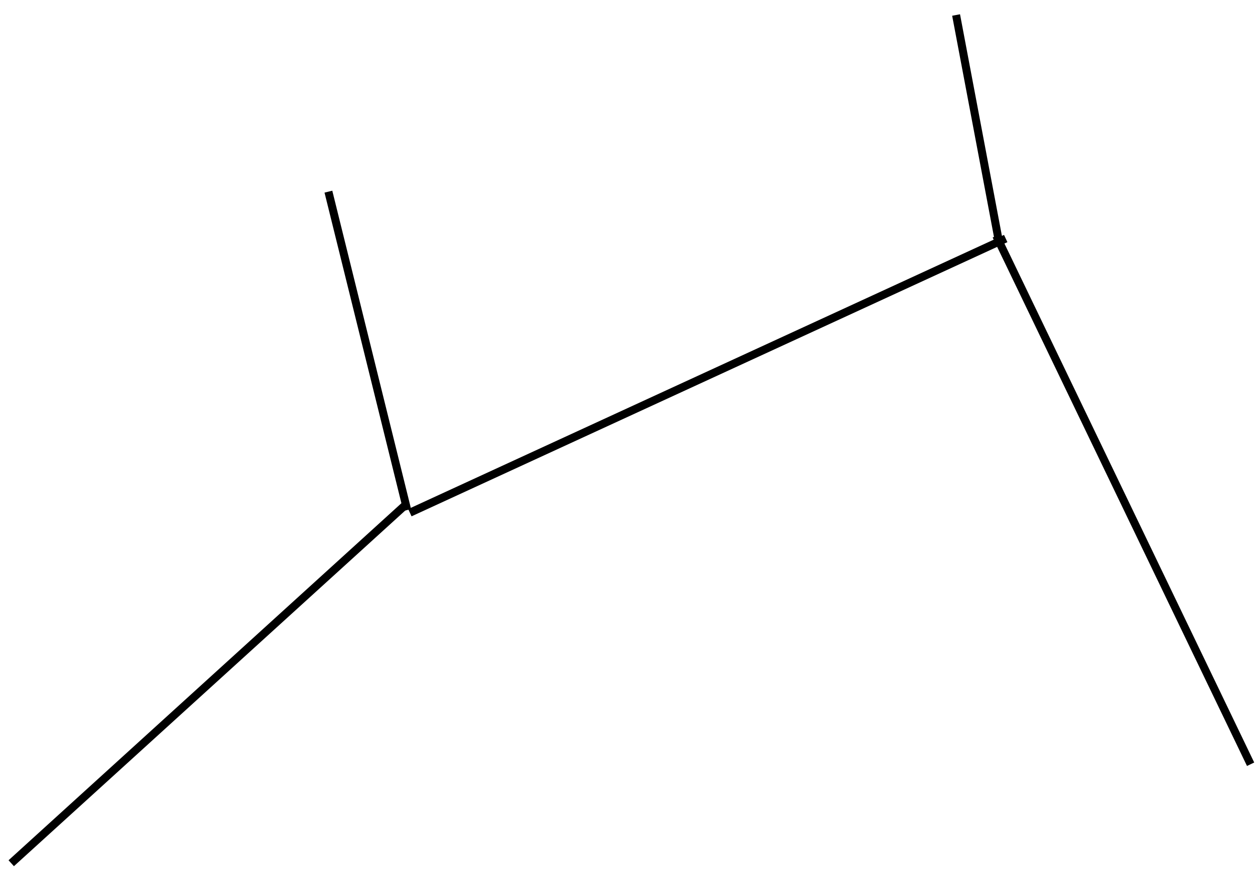
Do we still all infer the same spacetime?
Do we still infer the same spacetime at different energies?

Introducing the possibility for momentum space to be non linear allows us to propose a framework in which locality is relaxed in a controlled manner

# Usual Relativistic Dynamics

- Spacetime emerges from the dynamics on momentum space.
- In our limit, we study first classical particle dynamics
- Each process has an action principle worldline formalism

$$S^{process} = \sum_{trajectories, I} S_I^{free} + \sum_{interactions, \alpha} S_\alpha^{int}$$



# Emergence of space-time

Spacetime is an auxiliary concept that emerges from the dynamics of particles

Free particle dynamics

$$S = \int_{-\infty}^{0} (x^a \dot{k}_a - N\mathcal{C}(k))$$

conjugate part coordinates

mass shell

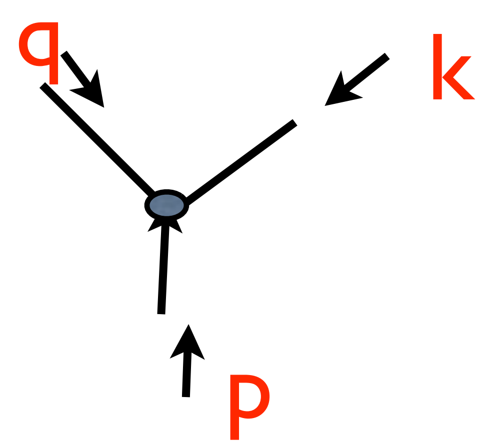
$$\{x_I^a, k_b^J\} = \delta_b^a \delta_I^J$$

$$\mathcal{C}^J(k) \equiv D^2(k) - m_J^2.$$

$$D(k)^2 = k_0^2 - \vec{k}^2$$

*The free particle action makes no reference to a metric for spacetime. Spacetime geometry is inferred from the geometry of momentum space.*

The interaction imposes the conservation law




$$S^{int} = (p + q + k)_\mu z^\mu$$

linear relation

Lagrange multiplier

# Worldline action

The variation of the worldline action gives

bulk eom

$$\dot{k}_\mu^I = 0$$

$$\dot{x}_I^\mu = \mathcal{N}_I k_I^\mu$$

$$k_I^2 = m_I^2$$

boundary eom

$$x_I^\mu = z^\mu$$

particle coordinates

The interaction is local

# Geometry of momentum space

Nowhere in the previous formulation the geometry of spacetime enters! only the properties of momentum space, entered.

We now suppose that Energy-momentum space $\mathcal{P}$ can have a non trivial geometry:

- a non trivial metric
- a non trivial connection

Metric enters the propagation of single particle

The connection enters the definition of interactions

# Geometry of momentum space

One postulates that single particle measurements determine the geometry of $\mathcal{P}$

$\mathcal{P}$ is a lorentzian metric manifold with an origin 0

The mass is interpreted as the timelike distance from the origin

$$D^2(p) \equiv D^2(p,0) = m^2.$$

The kinetic energy defines the geodesic spacelike distance between a particle $p$ at rest and a particle $p'$ of identical mass $D(p) = D(p') = m$

$$D^2(p,p') = -2mK.$$

from these measurements we can reconstruct the metric on $\mathcal{P}$

$$dk^2 = h^{ab}(k)dk_a dk_b$$

# Geometry of momentum space

In the multiple particle case we should have a rule to associate a total momenta to the combination of particles

We postulate that there exists a composition of momenta

$$(p,q) \rightarrow p'_a = (p \oplus q)_a$$

More complicated interaction processes are build up by iteration of this composition e.g $(p \oplus q) \oplus k$

We do not assume that it is linear or commutative or associative

Outgoing momenta can be turned in ingoing momenta: there is an operation $p \rightarrow \ominus p$

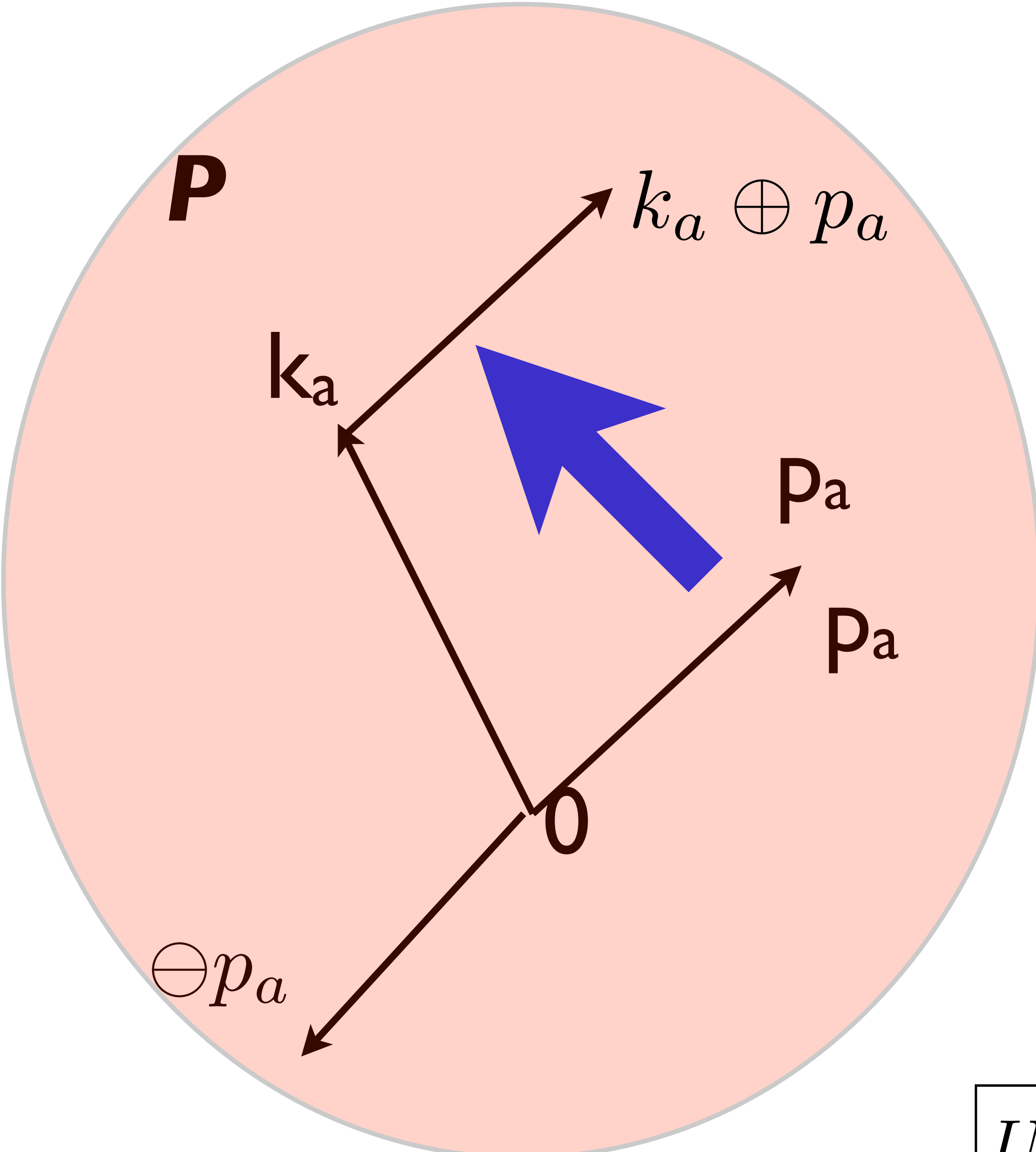
satisfying $(\ominus p) \oplus p = 0$

We also ask that $(\ominus p) \oplus (p \oplus k) = k,$ Left Loop kikkawa, sabinin, L.F

# Geometry of momentum space

Momenta combine into interactions: The rule:

$$(k, q) \to k'_a = k_a \oplus q_a$$




can be thought as a rule for combining geodesics on a curved manifold, so it defines a connection or parallel transport.

$$k_a \oplus dp_a = k_a + U(k)_a^b dp_b$$
$$= k_a + dp_a + \Gamma_a^{bc} k_b dp_c + \ldots$$

$$\boxed{U(p)_a^b \equiv \partial_q^b (p \oplus q)_a|_{q=0}}$$

transforms as a map from $T_0 P$ to $T_p(P)$

# Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p\oplus q)_c|_{q,p=o} = -\Gamma^{ab}_c(0)$$

transforms as an affine connexion

Torsion measures non commutativity

$$-\partial^a_p\partial^b_q[(p\oplus q)_c - (q\oplus p)_c]|_{p,q=0} = T^{ab}_c(0)$$

Curvature measures non associativity

$$2\frac{\partial}{\partial p_{[a}}\frac{\partial}{\partial q_{b]}}\frac{\partial}{\partial k_c}((p\oplus q)\oplus k - p\oplus(q\oplus k))_d|_{q,p,k=o} = R^{abc}{}_d(0)$$

To define the connection away from 0 we "translate" the addition using the left translation operator $L_k(p) \equiv k\oplus p$

# Three aspects of geometry, which can be measured:

$$p_a \oplus q_a = p_a + q_a + \Gamma_a^{bc} p_b q_c + ...$$

- Torsion: measures non-commutativity of interactions.

$$T_a^{bc} = \Gamma_a^{bc} - \Gamma_a^{cb}$$

- Curvature: measures non-associativity of interactions.

$$R^{abc}{}_d = \partial^a \Gamma_d^{bc} - \partial^b \Gamma_d^{ac} + \Gamma\Gamma$$

- Non-metricity: if the connection defined by interactions is not the metric connection defined from propagation.

$$N^{abc} = \nabla^a g^{bc}$$

# Dynamics

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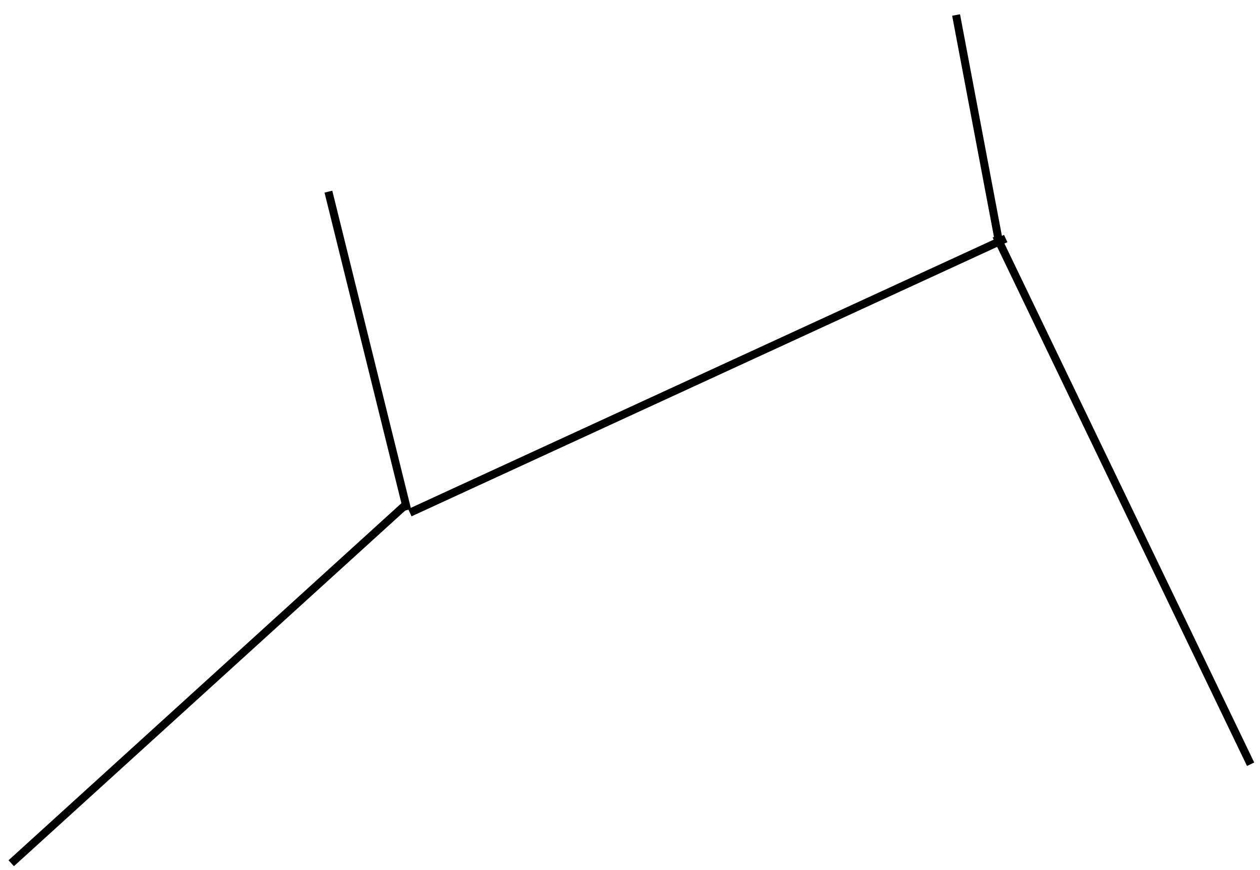

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$$\mathcal{C}^J(k) \equiv D^2(k) - m_J^2 .$$

dist from 0

*The free particle action makes no reference to a metric for spacetime. Spacetime geometry is inferred from the geometry of momentum space.*

in the usual case the metric is flat and $D(k)^2 = k_0^2 - \vec{k}^2$

also the case in Normal coordinates

# Worldline action

The variation of the worldline action gives

bulk eom

$$\dot{k}_a^J = 0$$

$$\dot{x}_J^a = \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k_a^J}$$

$$\mathcal{C}^J(k) = 0$$

← simplified by Riemannian normal coordinates

$$\mathcal{C}(k) = k_0^2 - k_i^2 - m^2$$

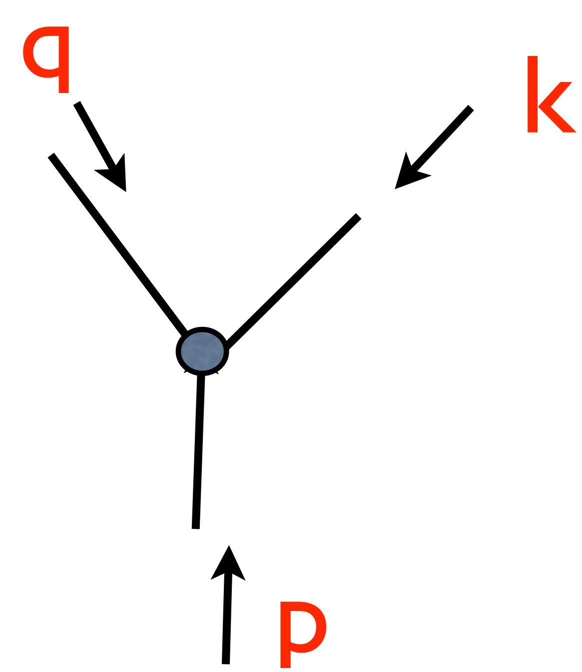
# Emergence of space-time

The interaction imposes the conservation law

$$S^{int} = \mathcal{K}(k(o))_a z^a$$

Lagrange multiplier

z becomes the location of the interaction:
the interaction coordinate




e.g. $\mathcal{K}_a = (p \oplus (q \oplus k))_a$

Choice of interaction vertex.

# Worldline action

The variation of the worldline action gives

boundary eom

$$x_J^a(0) = z^b \frac{\delta \mathcal{K}_b}{\delta k_a^J}$$

particle coordinates interaction coordinates

$$x_J^a(0) = z^a - z^b \sum_{L \in \mathcal{J}(J)} C_{J,L} \Gamma_b^{ac} k_c^L + \dots$$

if z=0 then x=0 : interaction is local for an observer at the origin

For a distant observer there is a dispersion $\Delta x \approx |z||\Gamma|k$

Locality is relative

# Two kinds of spacetime coordinates

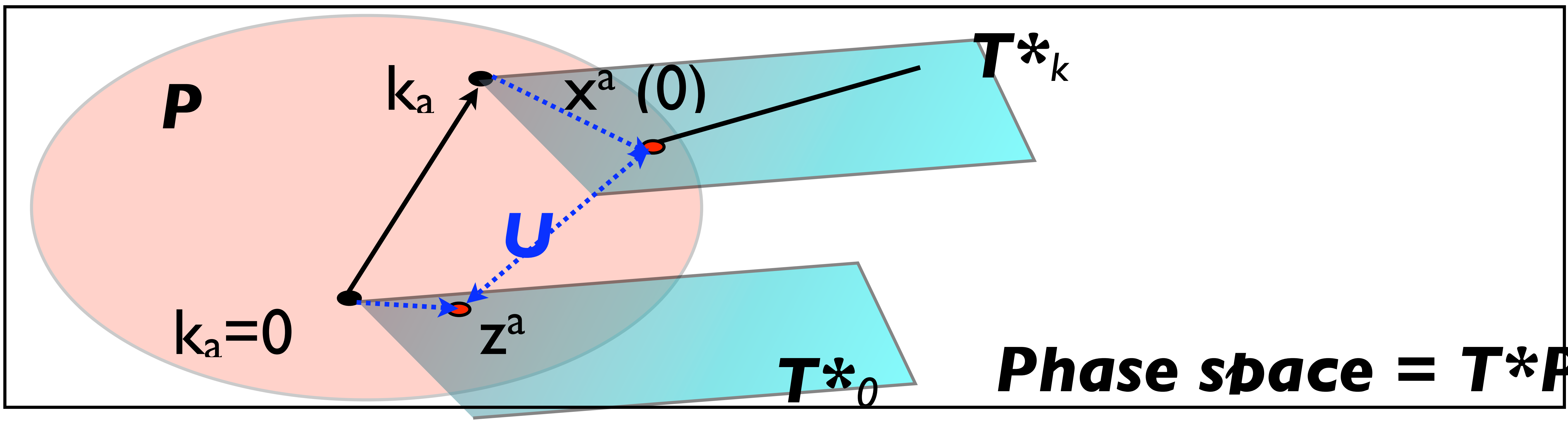
$$x_k^\mu = z^\nu W_\nu^\mu(k)$$

particle coordinates → $x_k^\mu$ interaction coordinates → $z^\nu$

$$W_\mu^\nu(k) = \frac{\delta K_\mu}{\delta k_\nu}$$

Paralell transport operator

$T_k^* P$ $T_0^* P$




No canonical projection from phase space to space time. $T^*P \neq M \times P$

each particle carries its own momentum dependent spacetime related by parallel transport to the interaction spacetime

# Two kinds of spacetime coordinates

$$\boxed{x^\mu_k = z^\nu W^\mu_\nu(k)}$$

$$W^\nu_\mu(k) = \frac{\delta K_\mu}{\delta k_\nu}$$

particle coordinates ↗ $x_k^\mu$ — interaction coordinates ↖ $z^\nu$

Paralell transport operator ↑ $W^\nu_\mu(k)$

$T^*_k P$ $\qquad\qquad$ $T^*_0 P$

The paralell transport operator is determined by the connection and a path from $0$ to $k$ which is determined by the interaction vertex

If the conservation law is linear then W = I and x=z → local int.
Otherwise the interaction is only relatively local i-e x=0 if z=0

x is a commutative coordinate
z is a non commutative coordinate $\qquad z \in T^*_0 P$

$$\{z^a, z^b\} = T^{ab}_d z^d + R^{abc}{}_d p_c z^d + \cdots$$

e.g 3d quantum gravity N=R=0.

# Experimental test

*Theorists propose but experiments decide.*

The geometry of momentum space should be measured rather than assumed

A new phenomenological sets of questions opens up

Two types of search: theoretical or purely phenomelogical

Given the maximally symmetric model: N=T=0 R= cst
find a clean measure of the dual cosmological constant

Test the 4 principles:
Torsion, non metricity, Lorentz invariance, homogeneity

# Experimental tests

•Experiment that follows from modified localisation equation: Gamma Rays experiment: see following.

•Measure of the curvature of momentum space

A thomas precession analogy motivated by Girelli, Livine

A system in orbit ( electron ,part at the LHC)
encloses a loop in momentum space at each period of revolution

The localisation of the orbiting particle will be shifted with respect to the particle at rest, it experiences a boost

$$N_i = \frac{\Delta A_{cd}}{m_P^2} R^{cda}{}_i p_a \approx \frac{\Delta A_{cd}}{m_P^2} m R^{cd0}{}_i$$

small displacement in space and time, cumulative.

It pulls itself by the bootstraps!

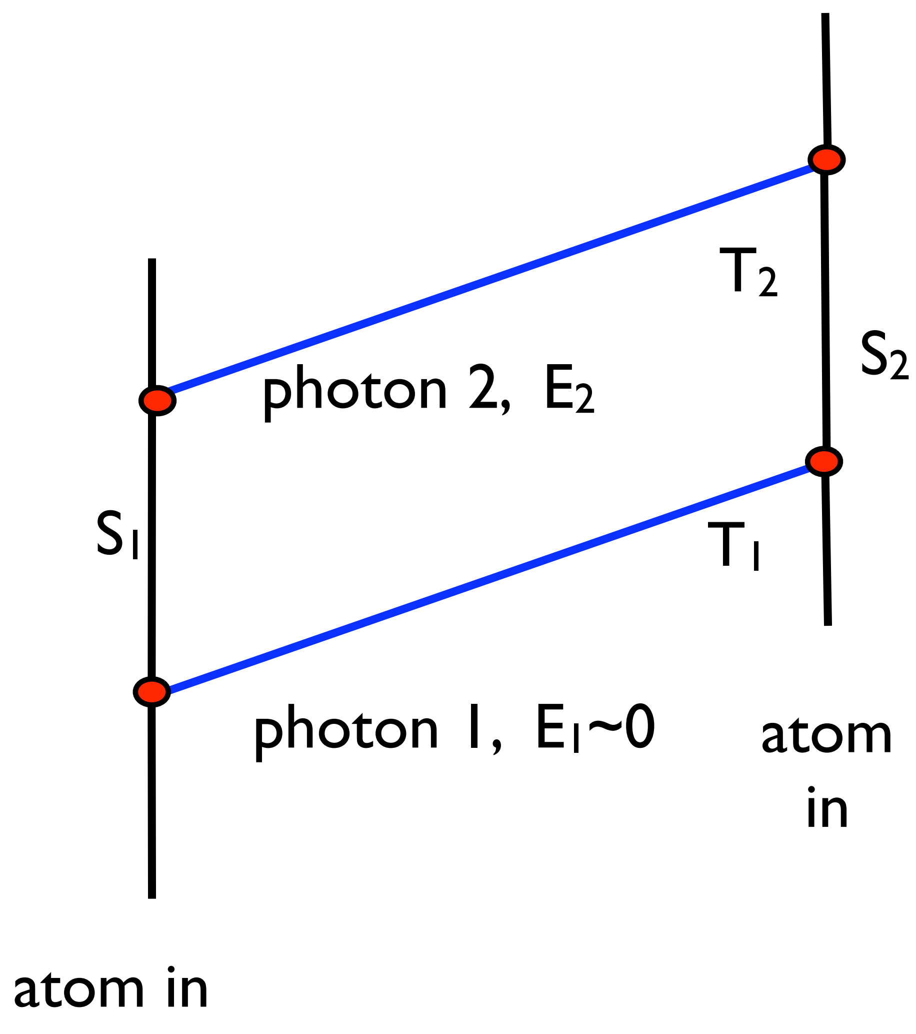
•Interferometry in momentum space

# Gamma ray exp

The process of localising a distant object is momentum dependent

The experiment a distant star emits two photons
One of low energy and one of high energy

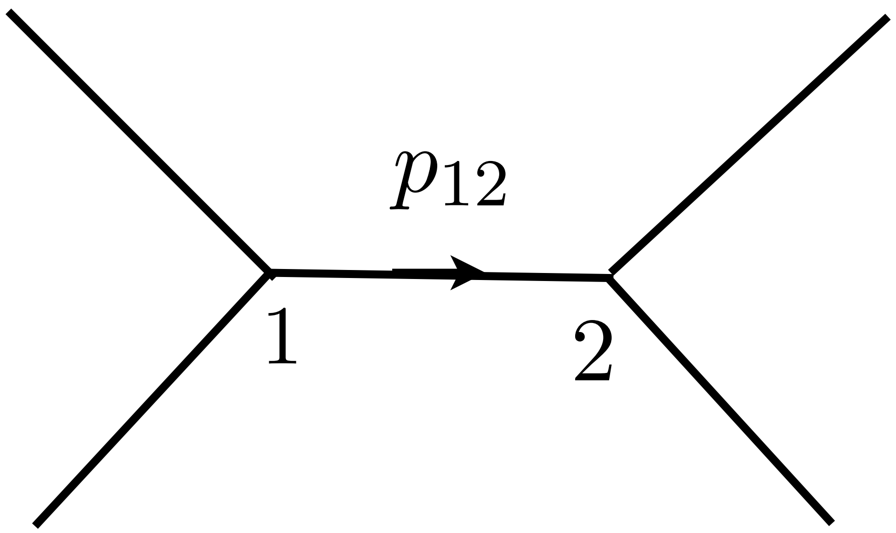
If the photons are emitted at the same time for an observer local to the star are they observed arriving at the same time by us ?




Not necessarily !
even if the photons have the same speed

# Localisation

In SR we use the eom are used to relate the position of the interaction vertex with the momenta and proper time:

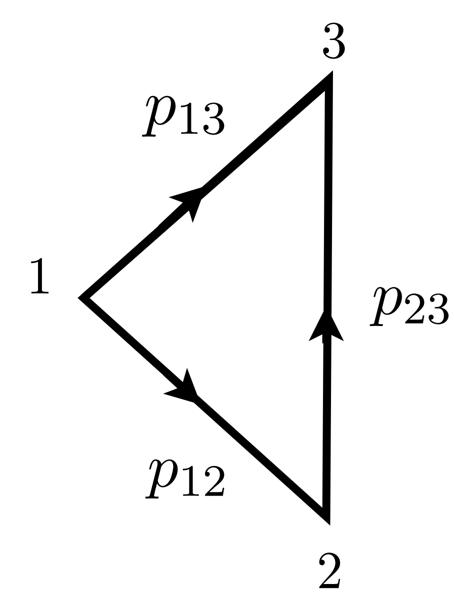



$$z_1 - z_2 = p_{12} T_{12}$$

$$p = m\dot{x}$$

For a closed loop we get a localisation equation of the type

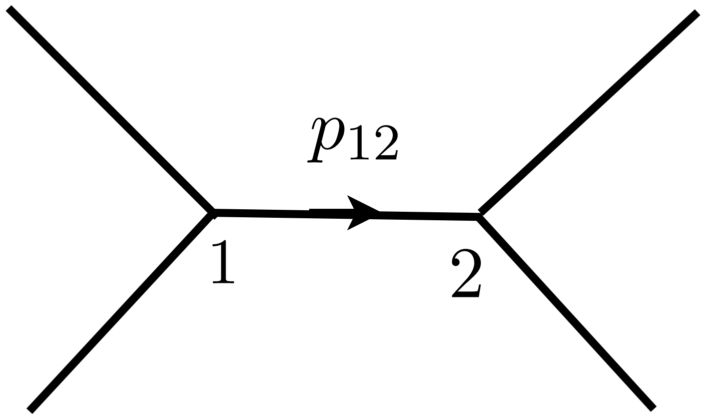
$$p_{12} T_{12} + p_{23} T_{23} = p_{13} T_{13}$$



# Relative Localisation

In RL the eom gives a modified relation between the interaction coordinate with the momenta and proper time:

$$\boxed{x_k^\mu = z^\nu W_\nu^\mu(k)}$$ and $p = m\dot{x}$ →




$$z_2 - z_1 H_{12} = P_{12} T_{12}$$

edge holonomy

$$H_{12} = W_1 W_2^{-1} \quad \in T_0^*\mathcal{P} \to T_0^*\mathcal{P}$$

$$P_{12} = p_{12} W_2^{-1}$$ dressed momenta

H is the composition of particle holonomies

$$(W_1)_\mu^\nu = \frac{\partial \mathcal{K}_\mu^1}{\partial p_\nu^{12}}, \quad (W_2)_\mu^\nu = \frac{\partial \mathcal{K}_\mu^2}{\partial p_\nu^{12}} \qquad \in T_0^*\mathcal{P} \to T_k^*\mathcal{P}$$

# Relative Localisation 2

For a closed loop the relation between momenta is now

$$T_{12}P_{12} + T_{23}P_{23}H_{12} - T_{13}P_{13}H_{12}H_{23} = z_1(1 - H_L)$$

dressed momenta

vanish if the curvature of the connection is 0

The curvature vanish at first order so the first order effect are controlled by non-metricity and Torsion.

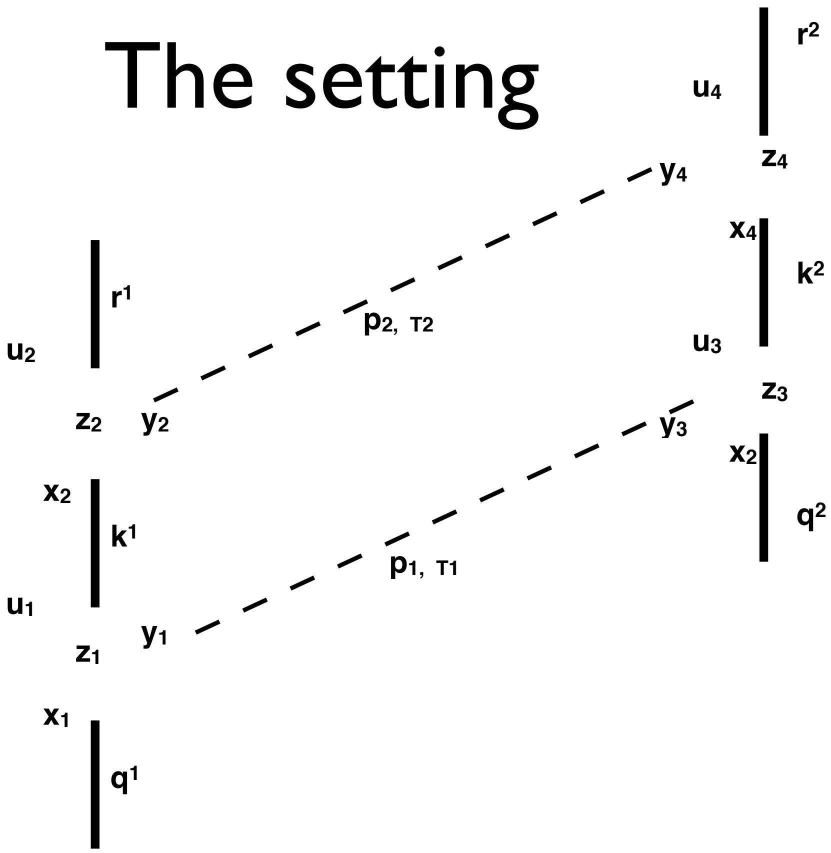
→ time delay and dual gravitational effects

Kappa Poincare and 3d gravity are example of zero curvature, non vanishing torsion geometry...

Is there a way to relate curvature effect in momentum space to curvature effect in space?

# Gamma ray exp

The setting




Results:

The leading order effect is due to non-metricity.

$$S_2 - S_1 = -T \Delta E N^{+++}$$

For a metric connection the next effect is due to the torsion

no time delay $\Delta S = 0$ but

photons of different energies appear to come from different locations

$$\Delta\theta = \frac{1}{2}(E_1 + E_2)\sqrt{T_-^{+a}\eta_{ab}T_-^{+b}}$$

dual gravitational lensing

# Soccer ball issue

If one modifies the law of addition of momenta with a scale $m_P$ how come we do not see strange effects for soccer balls?

$$p_1 \oplus p_2 = p_1 + p_2 + \frac{1}{m_P} T(p_1, p_2) + \cdots$$

The main point is that the effective mass scale for the interaction of two soccer balls of size N is N $m_P$

# Soccer ball issue

The argument is as follows:
Supose that two rigid bodies 1, 2 both composed of N particles interact such that each atom of 1 exchanges a photon with an atom of 2

$$p^1_{in} = p^1_{out} \oplus k \quad k \oplus p^2_{in} = p^2_{out}$$

Lets also assume that all atoms of 1, 2 have the same momenta

$$P_{in} = N p_{in} \qquad P_{out} = N p_{out}$$

$$p \oplus p = 2p$$

normal coord.

$$P_{tot} = N(p^1_{in} \oplus p^2_{pin}) = N(p^1_{out} \oplus p^2_{out})$$

$$= P_1 + P_2 - \frac{1}{N m_P} \Gamma(P_1, P_2) + \cdots$$

$m_P \to N m_p$ There is no soccer ball prob !

# Conclusion

New framework in which we can relax the notion of absolute locality in a controlled manner

Momentum space possess a non trivial geometry (metric, connection) that can and should be probed experimentally

Under general principles a preferred class of momentum space geometries can be proposed

Many interesting and extremely surprising experimental consequences

Interesting new math: connection between algebra and geometry

No soccer ball problem

# Conclusion

Field theory description: Follows from the worldline formalism

Is there a way to prove the Born principle?

Any theory of quantum gravity should imply that momentum space is dynamical.

It is true for 3d gravity. Is it true in string theory? In Loop Quantum gravity?

What is the source of curvature in momentum space?