# Very basic issues concerning quantum mechanics and gravity

Domenico Giulini

ZARM Bremen and University of Hannover

ETH Zürich June 16th 2011

# Equivalence Principle

- **Universality of Free Fall (UFF):** The space-time trajectory of a *test-particle* only depends on its initial position and velocity but not on its other (physical or chemical) attributes. This introduces a path structure on spacetime, though not necessarily that of geodesics with respect to a linear connection.
- **Local Lorentz Invariance (LLI):** The outcome of any local experiment is independent of the instantaneous orientation and velocity of the equipment (laboratory). In particular, there are no preferred-frame effects.
- **Universality of Clock Rates and Gravitational Redshift (UCR/UGR):** The rates of any two *standard clocks* agree if taken along the same worldline. If taken along different worldlines and intercompared, e.g., by means of electromagnetic signals, they differ by the standard ($\alpha = 0$) redshift formula

$$\frac{\nu_2 - \nu_1}{\nu_1} = (1+\alpha)\,\frac{U(\vec{x}_2) - U(\vec{x}_1)}{c^2}$$

# Experimental Status of EP

- **Universality of Free Fall** is tested by measuring the Eötvös factor for two different materials *A* and *B*. Typical modern results on macroscopic test bodies are:

  $$\eta(A,B) = 2 \cdot \frac{|a(A) - a(B)|}{a(A) + a(B)} < 10^{-13}$$

- **Local Lorentz Invariance** is tested by, e.g., modern versions of the Michelson-Morley experiment (isotropy of two-way speed of light). Typical modern results using microwave cavities are:

  $$\frac{\Delta c}{c} < 10^{-16}$$

  Note that Michelson-Morley experiments do not cover preferred lightlike directions, which are much harder to detect (Cohen & Glashow 2006).

- **Universality of Gravitational Redshift.** Still best test to date is 1976 comparison of two maser clocks (Gravity-Probe-A), one of which boosted to an altitude of 10 000 $\mathrm{km}$ by a Scout rocket:

  $$\alpha < 7 \times 10^{-5}$$

  This is clearly the weakest part of EEP. However, recent re-interpretations by Müller, Peters, and Chu (Nature 2010) of some 10-year old gravimeter experiments by Peters, Chung, and Chu using Caesium-based atom interferometers claim improvement by factor $10^4$ (→ more below).

# One-particle Schrödinger wave in homogeneous force-field

**Proposition:**$\psi$ solves the Schrödinger Equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m_i}\Delta - \vec{F}(t)\cdot\vec{x}\right)\psi$$

iff

$$\psi = \left(\exp(i\alpha)\,\psi'\right)\circ\Phi^{-1}\,,$$

where $\psi'$ solves the free Schrödinger equation (i.e. without potential).

$\Phi : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is the following spacetime diffeomorphism (preserving time)

$$\Phi(t,\vec{x}) = \left(t,\vec{x}+\xi(t)\right).$$

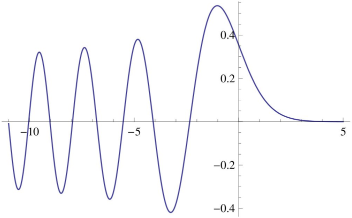
$\xi$ is a solution to

$$\ddot{\vec{\xi}}(t) = \vec{F}(t)/m_i$$

with $\vec{\xi}(0) = \vec{0}$ and $\alpha : \mathbb{R}^4 \rightarrow \mathbb{R}$ is given by

$$\alpha(t,\vec{x}) = \frac{m_i}{\hbar}\left\{\dot{\vec{\xi}}(t)\cdot\left(\vec{x}+\vec{\xi}(t)\right) - \frac{1}{2}\int^t dt'\,\|\dot{\vec{\xi}}(t')\|^2\right\}.$$

# Quantum particle in static homogeneous gravitational field



Airy function

- Return-time for particle with energy $E$ ejected at $z = z_i$ upwards far from classical turning point shows no corrections due to tunnelling, unlike for other potential shapes (corrections $\propto 2d/v$). Have (Davies 2004):

$$T_{\rm ret} = 2 \cdot \left[\frac{m_i}{m_g}\right]^{\frac{1}{2}} \cdot \left[\frac{2h}{g}\right]^{\frac{1}{2}}\,, \quad h = (E/m_g g) - z_i\,.$$

- Eigen-energies (with hard wall at $z = 0$) and penetration depth are:

$$E_n = \left[\frac{m_g^2 g^2 \hbar^2}{2m_i}\right]^{\frac{1}{3}} \cdot (-z_n)\,, \qquad d = \left[\frac{\hbar^2}{2m_i\, m_g\, g}\right]^{\frac{1}{3}}\,.$$

# The NR propagator

- For polynomial Lagrangians of at most quadratic order the propagator has the exact representation

$$K(z_b, t_b \,;\, z_a, t_a) = F(t_b, t_a) \ \exp\left\{\frac{i}{\hbar} S_*(z_b, t_b \,;\, z_a, t_a)\right\}$$

  where $F(t_b, t_a)$ does not depend on the initial and final position and $S_*$ is the action for the extremising path (classical solution).
- Here we take

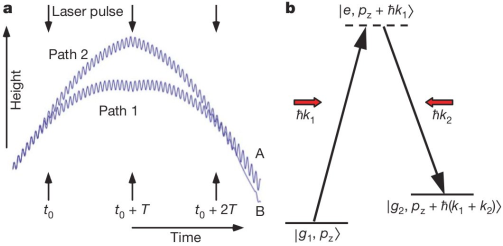
$$L(z, \dot{z}) = \tfrac{1}{2} m_i \dot{z}^2 - m_g g z$$

  and get for parabolic path with downward acceleration $g'$ $(= g m_g / m_i)$

$$\begin{aligned} S_{g'}(z_b, t_b; z_a, t_a) = {} & \frac{m_i}{2} \frac{(z_b - z_a)^2}{t_b - t_a} \\ & - \frac{m_g g}{2} (z_b + z_a)(t_b - t_a) \\ & + \frac{g'}{24} (t_b - t_a)^3 (m_i g' - 2 m_g g) \end{aligned}$$

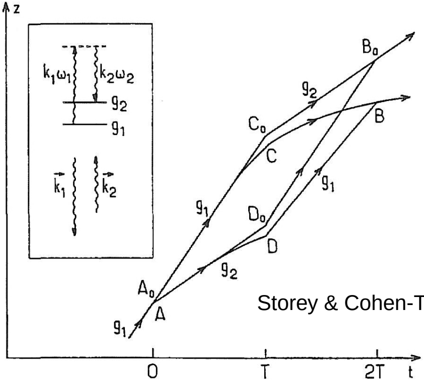
  Terms in red ($\propto m_g$) originate from the potential part, those $\propto m_i$ from the kinetic part.

# 1999 experiment by Peters, Chung, and Chu



Picture taken from Müller, Peters, and Chu (Nature 2010)

# Atom interferometry with Raman beam splitters



Storey & Cohen-Tannoudji (1999)

Figure: Spacetime paths followed by the atoms in the experiment of Kasevich and Chu. Raman pulses occur at times 0, $T$, and $2T$ with four-momenta $p_1 = \hbar(-k_1\vec{e}_z, \omega_1)$ and $p_2 = \hbar(k_2\vec{e}_z, \omega_2)$. The insert shows the atomic level scheme and the directions of the laser beams. Transitions $g_1 \longrightarrow g_2$ and $g_2 \longrightarrow g_1$ are accompanied by four-momentum transfers $\Delta_{12}p = (-\kappa, \omega)$ and $\Delta_{21}p = -\Delta_{12}p$ respectively, where $\kappa = k_1 + k_2 > 0$ and $\omega = \omega_1 - \omega_2 > 0$.

# Total phase shift

- For parabolic trajectories with downward acceleration $g'$ ($g' = (m_g/m_i)g$ on solution paths), we get

$$\Delta\phi = \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{time}}} - \underbrace{\kappa T^2 (m_g/m_i)\, g}_{\Delta\phi_{\text{redshift}}} - \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{light}}}$$

- Here $g$ is not measured. It is eliminated through a nearby reference measurement of the acceleration $\bar{g} = (M_g/M_i)g$ of a corner cube of inertial mass $M_i$ and gravitational mass $M_g$.
- Using the Nordtvedt parameter for the atom-cube pair,

$$\eta := \eta(\text{atom}, \text{cube}) := 2\,\frac{(m_g/m_i) - (M_g/M_i)}{(m_g/m_i) + (M_g/M_i)}\,,$$

we get for the total phase shift :

$$\Delta\phi = -\kappa T^2 \bar{g} \quad \frac{2+\eta}{2-\eta} \approx -\kappa T^2 \bar{g} \quad (1+\eta)\,.$$

# Total phase shift: Interpretation as test of UFF

- The results of Peters, Chung, and Chu (1999) were

$$\begin{aligned}\zeta_{\mathrm{meas}} &:= \frac{-\Delta\phi}{\kappa T^2 c^2} &&= (1.090\,322\,683 \pm 0.000\,000\,003) \times 10^{-16} \cdot \mathrm{m}^{-1}\,,\\ \zeta_{\mathrm{pred}} &:= \tilde{g}/c^2 &&= (1.090\,322\,675 \pm 0.000\,000\,006) \times 10^{-16} \cdot \mathrm{m}^{-1}\,.\end{aligned}$$

- This implies an upper bound on UFF violations of

$$\eta(\text{atom}, \text{cube}) = \frac{\zeta_{\mathrm{meas}}}{\zeta_{\mathrm{pred}}} - 1 < (7 \pm 7) \times 10^{-9}\,,$$

  which is more than four orders of magnitude worse (higher) than the lower bounds obtained by more conventional methods using classical bodies.
- But Müller, Peters, and Chu (2010) interpret the findings of Peters, Chung, and Chu (1999) quite differently ...

# Total phase shift: Interpretation as UGR

- Since $\Delta\phi_{\rm time} + \Delta\phi_{\rm light} = 0$, Müller, Peters, and Chu say

$$-\kappa T^2 g = \Delta\phi = \Delta\phi_{\rm redshift}\,.$$

- Then have

$$\Delta\phi = \omega\,\Delta T = -\omega T\,\frac{\Delta U}{c^2} = -\omega T\,\frac{g\,\Delta h}{c^2} = -\omega T\,\frac{g\,\hbar\kappa}{m_i c^2}\,T = -\frac{\omega}{\omega_C}\,\kappa T^2 g$$

- Hence clocks must tick at Compton frequency!!!
- Being at the $10^{-9}$ level this interpretation - if tenable - would imply an improvement of UGR tests by a factor of $10^4$.
- This would be achieved by measuring the redshift in the gravitational field of the Earth over a distance of $0.12\,\mathrm{mm}$.
- “The experiment thus confirms local position invariance by excluding anomalous variations of more than 7 parts in $10^{28}$ in the frequency of the Compton clocks. This corresponds to comparing the elapsed times to $\approx 10^{-29}\,\mathrm{sec}$.”
- Exercise: Can that be?

# Self-Gravity of Matter Waves?

- The Schrödinger-Poisson system (as approximation to semi-classical Einstein equation)

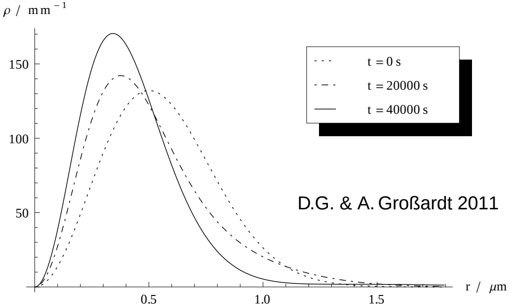
$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\Delta + m\Phi\right)\Psi$$
$$\Delta\Phi = 4\pi G m\,|\Psi|^2$$

- A stable ground state exists (Lieb 1977), which is of energy (Tod et al. 1998-2003)

$$E_0 = -0.163\,\frac{G^2m^5}{\hbar^2} = -0.163\cdot mc^2\cdot\left(\frac{m}{m_P}\right)^4$$

- For $m < m_P = 10^{19}\,\mathrm{u}$ this is well bounded away from black hole formation and the Newtonian approximation can be trusted.

# The time-dependent SN-Equation



D.G. & A. Großardt 2011

- Time evolution of rotationally symmetric Gauß packet of initial width 500 nm. Collapse sets in for masses $m \gtrsim 4 \times 10^9\,\mathrm{u}$, but collapse times are still very long indeed.
- This is a $10^6$ correction to earlier simulations by Carlip and Salzman (2006), though not outrageously beyond experimental reach.