Renormalizability of noncommutative quantum field theories

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Université Catholique de Louvain

Quantum Theory and Gravitation, ETH Zürich, June 2011

Plan

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1. Introduction: Noncommutative Geometry and Physics
2. Noncommutative scalar theory
3. Noncommutative gauge theory
4. Review and perspectives

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Essence of NCG

- Correspondence between spaces and commutative algebras (fields)
- Gelfand-Naimark thm: topological spaces $X \rightleftarrows$ commutative C*-algebras $C(X) = \{X \to \mathbb{C} \text{ continuous}\}$
- Noncommutative algebras: viewed as associated to a "noncommutative space"
- NCG: generalize geometrical constructions and properties to noncommutative algebras
- NCG could be adapted to the unification of quantum physics with gravitation as an extension of differential geometry (classical QFT, general relativity) within an operator algebras' framework (quantum physics)

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- classical Standard Model coupled with gravitation: spectral action principle on $C^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{H} \oplus \mathbb{M}_3(\mathbb{C}))$ (Chamseddine Connes '07) (non-abelianity)
- Quantum space-time at Planck scale (Doplicher *et. al.* '94) (fuzzy structure)
- Emergent gravity from nc gauge theory (Steinacker '07, Yang '09)
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Quantum spaces

- Important class of nc algebras: deformations of (Poisson) manifolds M: $(C^\infty(M), \star_\theta)$
  such that $f \star_\theta g \to f \cdot g$ and $\frac{1}{\theta}[f, g]_\star \to \{f, g\}_{PB}$
- QFT: construct an action by replacing commutative product with $\star_\theta$
- Renormalizability of QFT: consistency of the theory for a change of scale
- various examples of lorentzian and riemannian deformed spaces
- In this talk: Moyal space in the euclidean framework, deformation quantization of $\mathbb{R}^4$

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Presentation of the Moyal space

- Algebra of functions $f : \mathbb{R}^4 \to \mathbb{C}$ (fields)
- Deformed Moyal product:

$$(f \star g)(x) = \frac{1}{\pi^4 \theta^4} \int d^4y d^4z \, f(x+y) g(x+z) e^{-2iy\Theta^{-1}z}$$

$$\Theta = \theta\Sigma, \qquad \Sigma = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- Limit $\theta = 0$: $(f \star g)(x) = f(x) \cdot g(x)$
- The Moyal space is the nc space associated to this algebra. Scalar fields are elements of this deformed algebra and will be multiplied by using $\star$.
- Tracial property: $\int d^4x \, (f \star g)(x) = \int d^4x \, f(x) g(x)$

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Construction of the nc scalar theory

- Real scalar field theory on the euclidean Moyal space:

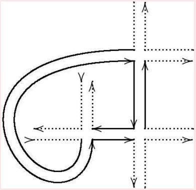
$$S[\phi] = \int d^D x \Big(\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \lambda \, \phi \star \phi \star \phi \star \phi \Big)$$

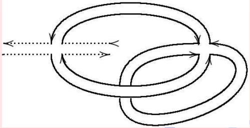
- Feynman rules in impulsions (Filk '96):

$$\lambda \delta \Big(\sum_{i=1}^{4} p_i \Big) \quad \mapsto \quad \lambda e^{i \frac{\theta^2}{2} (p_1 \Theta^{-1} p_2 + p_1 \Theta^{-1} p_3 + p_2 \Theta^{-1} p_3)} \delta \Big(\sum_{i=1}^{4} p_i \Big)$$

- Vertex symmetries: only cyclic permutations

$\Rightarrow$ Ribbon Feynman graphs (genus: $2 - 2g = \frac{N}{2} - V + F$)





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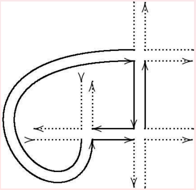

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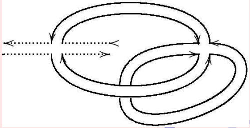

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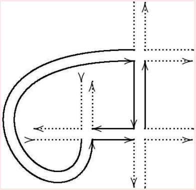

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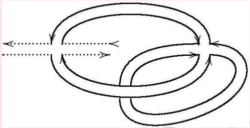

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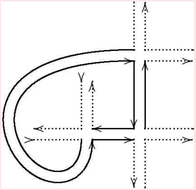
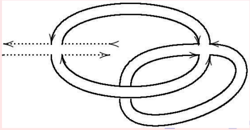
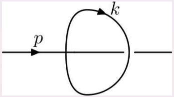
$g = 0$   $g = 1$

UV-IR mixing

(Minwalla *et al.* '00)

- Tadpole ($F = B = 2$, $g = 0$): 2 broken faces



$$\lambda \int d^4k \frac{e^{ik\Theta p}}{k^2 + m^2} \propto_{|p| \to 0} \frac{1}{\theta^2 p^2}$$

- Amplitude is finite but singular for $p \to 0$
- Generic behavior for graphs with $B \geq 2$
- When inserted many times in higher order graphs, p is an internal impulsion $\Rightarrow$ IR divergence in p (coming from the UV sector of k)

$\to$ Non-renormalizability of the theory

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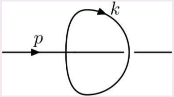

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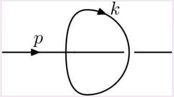

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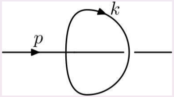

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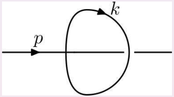

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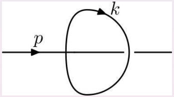

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Harmonic theory

- Addition of a harmonic term to the action:

$$S[\phi] = \int d^4x \Big(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\Omega^2}{2}\widetilde{x}^2\phi^2 + \frac{m^2}{2}\phi^2 + \lambda\, \phi \star \phi \star \phi \star \phi \Big)$$

where $\widetilde{x} = 2\Theta^{-1}x$

- Propagator: Mehler kernel (Gurau Rivasseau Vignes '06)
- Power counting

$$\omega = 4 - N - 4(2g + B - 1)$$

- Primitively divergent graphs: planar, $B = 1$, $N = 2, 4$

$\Rightarrow$ Renormalizability of the theory to all orders (Grosse Wulkenhaar '05)

- Loss of translation-invariance: essential here for the removing of the UV-IR mixing

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$$S[\phi] = \int d^4x \Big(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{\Omega^2}{2}\widetilde{x}^2\phi^2 + \frac{m^2}{2}\phi^2 + \lambda\,\phi \star \phi \star \phi \star \phi \Big)$$

where $\widetilde{x} = 2\Theta^{-1}x$

- Propagator: Mehler kernel (Gurau Rivasseau Vignes '06)
- Power counting

$$\omega = 4 - N - 4(2g + B - 1)$$

- Primitively divergent graphs: planar, $B = 1$, $N = 2, 4$

$\Rightarrow$ Renormalizability of the theory to all orders (Grosse Wulkenhaar '05)

- Loss of translation-invariance: essential here for the removing of the UV-IR mixing

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- Covariant under Langmann-Szabo duality (Langmann Szabo '02):

$$S[\phi, m, \lambda, \Omega] = \Omega^2 \, S\left[\hat{\phi}, \frac{m}{\Omega}, \frac{\lambda}{\Omega^2}, \frac{1}{\Omega}\right]$$

where $\hat{\phi}$ is a symplectic Fourier transformation

- New properties of the flow at the fixed point $\Omega = 1$: $\beta_\lambda = 0$

(Grosse Wulkenhaar '04, Disertori Gurau Magnen Rivasseau '06)

→ Towards a constructive version of the nc ϕ^4 harmonic model

(Grosse Wulkenhaar '09, Rivasseau Wang '11)

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Supergeometrical interpretation

(Bieliavsky A.G. Tuynman '10)

- Non-formal deformation of $\mathbb{R}^{p|q}$: Moyal $\otimes$ Clifford $Cl(q,\mathbb{C})$
- Product on $\mathbb{R}^{4|1}$: if $\Phi(x,\xi)=\phi_0(x)+\phi_1(x)\xi$,

$$(\Phi\star\Psi)(x,\xi)=\Big(\phi_0\star\psi_0+\frac{i\theta}{4}\phi_1\star\psi_1+(\phi_0\star\psi_1+\phi_1\star\psi_0)\xi\Big)(x)$$

- Trace: $\mathrm{Tr}(\Phi)=\int \mathrm{d}^4x\,\Phi(x,0)$
- Reduction: $\Phi(x,\xi)=(1+b\xi)\phi(x)$ with $b\in\mathbb{R}$
- Standard action on the deformed $\mathbb{R}^{4|1}$:

$$\mathrm{tr}\Big(\frac{1}{2}|[-\frac{i}{2}\widetilde{x}_\mu(1+b\xi),\Phi]_\star|^2+\frac{m^2}{2}\Phi^{\star 2}+\lambda\Phi^{\star 4}\Big)$$

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Construction of the nc gauge theory

- Nc gauge potential: A_μ (real)
- Gauge transformation: $g^\dagger \star g = g \star g^\dagger = 1$

$$A_\mu \quad \mapsto \quad g \star A_\mu \star g^\dagger + ig \star \partial_\mu g^\dagger$$

- Field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_\star$
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- Coupled to the Grosse-Wulkenhaar model at the 1 loop order

  (A.G. Wallet Wulkenhaar '07, Grosse Wohlgenannt '07)

- Quadratic part: $\int \left(\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{\beta}{2}\widetilde{x}^2 A_\nu A_\nu + \frac{\kappa}{2} A_\nu A_\nu \right)$.

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- Mass term for gauge fields without Higgs mechanism
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$$S[A] = \int d^4x \Big(\frac{1}{4} F_{\mu\nu} \star F_{\mu\nu} + \frac{\beta}{4} \{\mathcal{A}_\mu, \mathcal{A}_\nu\}_\star^2 + \frac{\kappa}{2} \mathcal{A}_\mu \star \mathcal{A}_\mu \Big)$$

- Coupled to the Grosse-Wulkenhaar model at the 1 loop order

  (A.G. Wallet Wulkenhaar '07, Grosse Wohlgenannt '07)
- Quadratic part: $\int \left(\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{\beta}{2}\widetilde{x}^2 A_\nu A_\nu + \frac{\kappa}{2} A_\nu A_\nu \right)$.

$\Rightarrow$ Good candidate to renormalizability

- Mass term for gauge fields without Higgs mechanism
- Not Langmann-Szabo covariant
- Problem: no trivial vacuum

Vacua of the theory

- Equation of motion: ($\mathcal{A}_\mu = A_\mu + \frac{1}{2}\widetilde{x}_\mu$)

$$[[\mathcal{A}_\mu,\mathcal{A}_\nu]_\star,\mathcal{A}_\nu]_\star + \beta\{\{\mathcal{A}_\mu,\mathcal{A}_\nu\}_\star,\mathcal{A}_\nu\}_\star + \kappa\mathcal{A}_\mu = 0$$

→ No trivial vacuum $A_\mu = 0$

- Solution $A_\mu = -\frac{1}{2}\widetilde{x}_\mu$: matrix model without dynamic
- Computation of the other solutions (product of Bessel, gaussian and hypergeometric functions) (A.G. Wallet Wulkenhaar '08)

→ Too complicated for an analytic expansion of the theory

- A particular gauge fixing permits to give a trivial vacuum to the theory (in progress)
- Ghost sector remains to be fully understood (Blaschke Grosse Kronberger Schweda Wohlgenannt '09)

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Supergeometrical interpretation

(A.G. Masson Wallet '08)

- Reduced gauge potential on the deformed $\mathbb{R}^{4|1}$: $(1+b\xi)A_\mu(x)$
- Same gauge transformations
- Graded curvature:

$$\mathcal{F}^{(1)}_{\mu\nu} = F_{\mu\nu}(x), \quad \mathcal{F}^{(2)}_{\mu\nu} = 2bF_{\mu\nu}(x)\xi, \quad \mathcal{F}^{(3)}_{\mu\nu} = \frac{b^2\theta}{4}\{\mathcal{A}_\mu, \mathcal{A}_\nu\}_\star(x)$$

$+$ other part

- Standard Yang-Mills action:

$$\mathrm{Tr}\Big(\sum_i (\mathcal{F}^{(i)}_{\mu\nu})^2\Big) = \int \mathrm{d}^4x \Big(\frac{1}{4}(1+\frac{b^2\theta}{2})(F_{\mu\nu})^2 + \frac{b^4\theta^2}{16}\{\mathcal{A}_\mu, \mathcal{A}_\nu\}^2_\star + (\frac{80}{\theta} + 20b^2 + \frac{\theta}{2})(\mathcal{A}_\mu)^2\Big)$$

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Another renormalizable scalar theory

- Amplitude of the tadpole: singular in $\frac{1}{p^2}$
- Addition to the standard ϕ^4 action the term

$$\frac{a}{2\theta^2}\int d^4p\,\frac{1}{p^2}\hat{\phi}(-p)\hat{\phi}(p)$$

- Renormalizable to all orders (Gurau Magnen Rivasseau Tanasa '09)
- Translation invariant
- Same flow as in the commutative theory, finite renormalization for a
- Study of the commutative limit (Magnen Rivasseau Tanasa '09)
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Other theories

- Another interpretation of the harmonic term: noncommutative scalar curvature (Buric Wohlgenannt '10)
- Complex ϕ^4-orientable scalar theory: renormalizable

$$S[\phi] = \int d^D x \Big(\frac{1}{2} |\partial_\mu \phi|^2 + \frac{m^2}{2} |\phi|^2 + \lambda\, \phi^\dagger \star \phi \star \phi^\dagger \star \phi \Big)$$

- ϕ^3 scalar theories in 2,4,6 dimensions (Grosse Steinacker '06)
- Fermionic theories of type Gross-Neveu in 2 dimensions (Vignes-Tourneret '07)
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Conclusion

About the theory with harmonic term

- Renormalizable theory
- New properties of the flow
- Interpretation in terms of a nc supergeometry
- Good candidate for the gauge theory
- Understand BRST formalism
- Relation between renormalizability and nc supergeometry
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