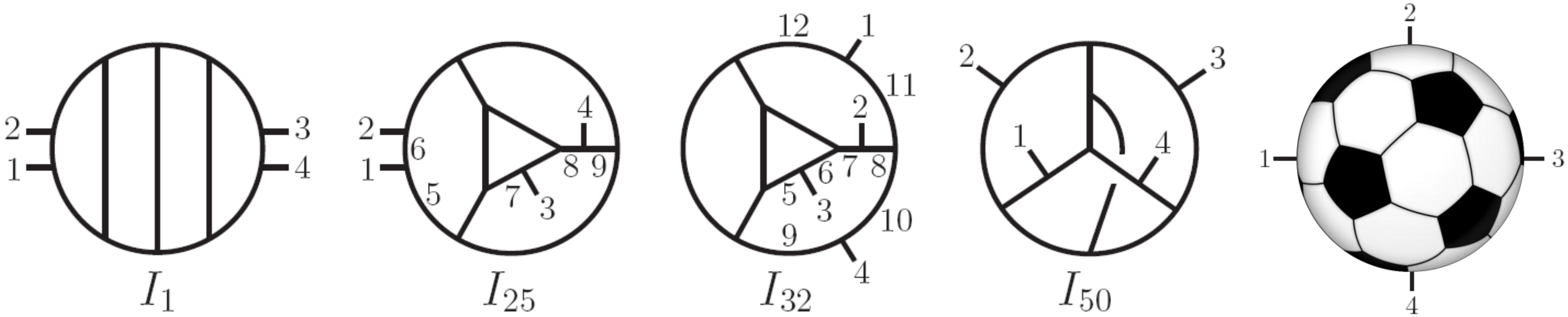
Ultraviolet behavior of quantum (super)gravity through four loops



Lance Dixon (CERN & SLAC)

ETH Zürich Conference on

Quantum Theory and Gravitation

based on work with

Z. Bern, J.J. Carrasco, H. Johansson, R. Roiban

0905.2326, 1008.3327, 110?.????

Introduction

• Quantum gravity is nonrenormalizable by power counting: the coupling, Newton's constant, $G_N = 1/M_{Pl}^2$ is dimensionful

• String theory cures the divergences of quantum gravity by introducing a new length scale, the string tension, at which particles are no longer pointlike.

• Is this necessary? Or could enough symmetry, e.g. **N=8** supersymmetry, allow a point particle theory of quantum gravity to be perturbatively ultraviolet finite?

• N=8 supergravity (ungauged) DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)

• Other point-like proposals include flow to (conjectured?) nontrivial fixed points:

- asymptotic safety program

Weinberg (1977); ...; Niedermaier, Reuter, Liv. Rev. Rel. **9**, 5 (2006)

- UV theory could be Lorentz asymmetric, but renormalizable

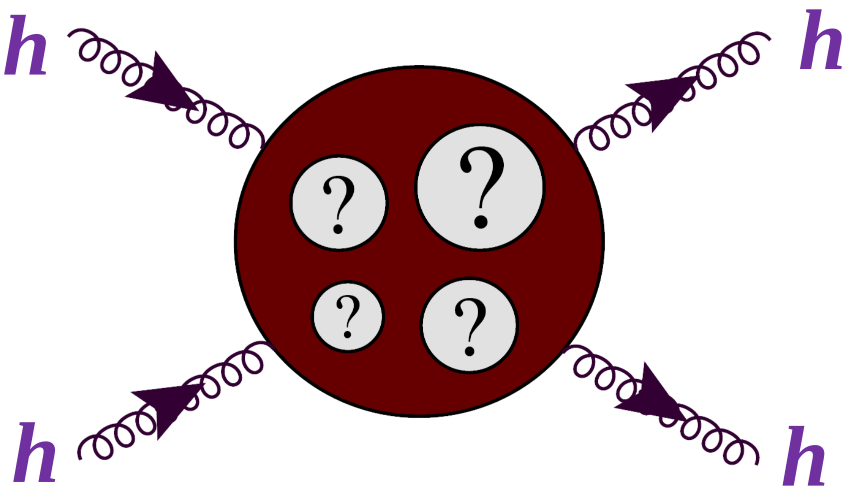
Hořava, 0812.4287, 0901.3775

• Here we will perturb around a (conjectured?) Gaussian fixed point

Graviton Scattering: a Gedanken Experiment

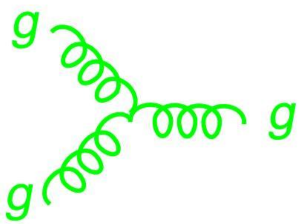
“Mathematics is the part of physics where experiments are cheap”

– V.I. Arnold



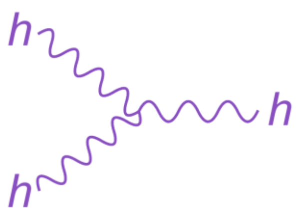
Why gravity should behave badly

gauge theory (spin 1) renormalizable



$$\supset \; \ell^{\mu} \, \eta^{\nu\rho} \; + \; \cdots$$

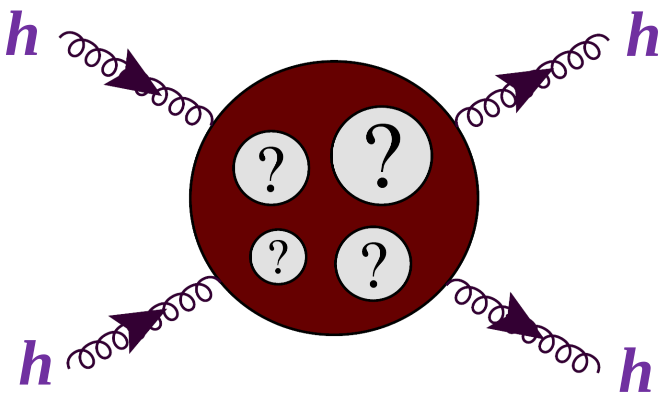
gravity (spin 2) nonrenormalizable



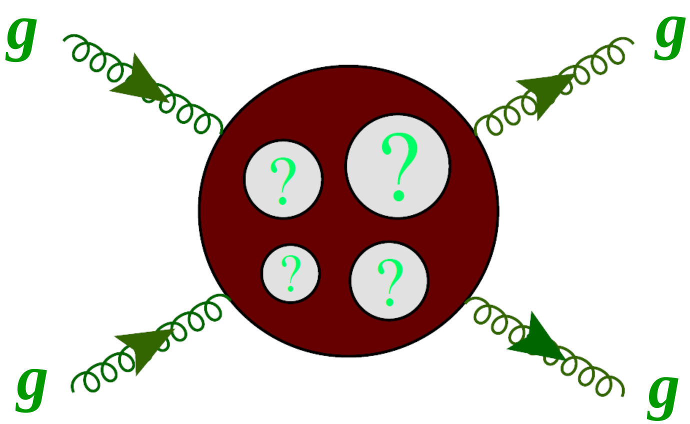
$$\supset \; \ell^{\mu_1} \, \ell^{\mu_2} \, \eta^{\nu_1\rho_1} \, \eta^{\nu_2\rho_2} \; + \; \cdots$$

Extra $\dfrac{\ell^2}{M_{\rm Pl}^2}$ per loop

Strategy for Assessing N=8 Supergravity



vs.



N=8 SUGRA

N=4 Super-Yang-Mills

How does N=8 SUGRA compare to N=4 SYM? What is the critical dimension $D_c(L)$ in which it first diverges?

A "mere" gauge theory. UV finite in $D = 4$.
Strong evidence that it's also finite at L loops for

$$D < 4 + \frac{6}{L}$$

Counterterm Basics

- Divergences associated with local counterterms
- On-shell counterterms are generally covariant, built out of products of Riemann tensor $R_{\mu\nu\sigma\rho}$ (& derivatives $\mathcal{D}_\mu$)
- Terms containing Ricci tensor $R_{\mu\nu}$ and scalar R removable by nonlinear field redefinition in Einstein action

$R^{\mu}_{\nu\sigma\rho} \sim \partial_\rho \Gamma^{\mu}_{\nu\sigma} \sim g^{\mu\kappa}\partial_\rho\partial_\nu g_{\kappa\sigma}$ has mass dimension 2

$G_N = 1/M_{Pl}^2$ has mass dimension -2

Each additional $R_{\mu\nu\sigma\rho}$ or $\mathcal{D}^2$ ←→ 1 more loop (in D=4)

One-loop → $R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

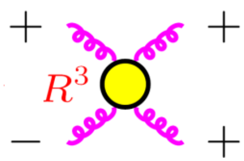
However, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

is Gauss-Bonnet term, total derivative in four dimensions.

So pure gravity is UV finite at one loop (but not with matter)

't Hooft, Veltman (1974)

Pure supergravity ($\mathcal{N} \geq 1$):
Divergences deferred to at least three loops



$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu} R^{\mu\nu}{}_{\sigma\tau} R^{\sigma\tau}{}_{\lambda\rho}$ cannot be supersymmetrized

produces helicity amplitude (-+++) incompatible with SUSY Ward identitites

Grisaru (1977); Deser, Kay, Stelle (1977); Tomboulis (1977)

However, at three loops, there is an N=8 supersymmetric counterterm, abbreviated R^4 ,
plus (many) other terms containing other fields in N=8 multiplet.

Deser, Kay, Stelle (1977); Howe, Lindstrom (1981); Kallosh (1981); Howe, Stelle, Townsend (1981)

R^4 produces first subleading term in low-energy limit of 4-graviton scattering in type II string theory:

$$\alpha'^3 R^4 \;\Rightarrow\; \alpha'^3 stu\, \underbrace{M_4^{\text{tree}}(1,2,3,4)}_{\text{4-graviton amplitude in (super)gravity}}$$

Gross, Witten (1986)

Bose symmetric polynomial (stu)

$\mathcal{N} = 8$ Constraints on Counterterms

Elvang, Freedman, Kiermaier (2010)

- Use locality of on-shell amplitudes + powerful N=8 SUSY Ward identitites
  Also related work by Kallosh
- N=8 SWI for maximally helicity violating (MHV) amplitudes:

$$\frac{M_n(++\cdots+-_i+\cdots+-_j+\cdots+)}{\langle i\,j\rangle^8} = \text{Bose symmetric}$$

- N=8 SWI for non-MHV amplitude – solved recently
  Elvang, Freedman, Kiermaier, 0911.3169

$\mathcal{D}^{2k}R^4$ → 4-point → MHV
→ amounts to classifying Bose-symmetric polynomials $P(s,t,u)$

$\mathcal{D}^{2k}R^5$ → still MHV → can still use Bose-symmetry

$\mathcal{D}^{2k}R^{6,7}$ → next-to-MHV analysis required

N=8 allowed

Chart of potential counterterms

Elvang, Freedman, Kiermaier (2010)

Analytic proofs:

- $D^{2k}R^n$ MHV ∄ for $n > 4$ and $k < 4$.
- $D^{2k}R^n$ NMHV ∄ for $n > 5$ and $k < 2$.

Drummond, Heslop, Howe, Kerstan, th/0305202; Kallosh, 0906.3495

L							
3	R^4 MHV ∃!						
4	D^2R^4 MHV ∄	R^5 MHV ∄					
5	D^4R^4 MHV ∃!	D^2R^5 MHV ∄	R^6 (N)MHV ∄				
6	D^6R^4 MHV ∃!	D^4R^5 MHV ∄	D^2R^6 (N)MHV ∄	R^7 (N)MHV ∄			
7	D^8R^4 MHV ∃!	D^6R^5 MHV ∄	D^4R^6 MHV ∄ NMHV	D^2R^7 (N)MHV ∄	R^8 (N)MHV ∄ N^2MHV?		
8	$D^{10}R^4$ MHV ∃!	D^8R^5 MHV ∃!	D^6R^6 MHV ∄ NMHV?	D^4R^7 MHV ∄ NMHV?	D^2R^8 (N)MHV ∄ N^2MHV?	R^9 (N)MHV ∄ N^2MHV?	
9	$D^{12}R^4$ 2×MHV	$D^{10}R^5$?×MHV	D^8R^6 2×MHV NMHV?	D^6R^7 MHV ∄ NMHV?	D^4R^8 MHV ∄ N or N^2MHV?	D^2R^9 (N)MHV ∄ N^2MHV?	R^{10} (N)MHV ∄ N^2 or N^3MHV?

Until 7 loops, any divergences show up in 4-point amplitude!

- red: not excluded
- green: ?
- gray: excluded

$E_{7(7)}$ Constraints on Counterterms

- N=8 SUGRA has continuous symmetries: noncompact form of E_7.
- 70 scalars → coset $E_{7(7)}/SU(8)$. Non-$SU(8)$ part realized nonlinearly.

Cremmer, Julia (1978,1979) quantum level: Bossard, Hillmann, Nicolai, 1007.5472

- $E_{7(7)}$ also implies amplitude Ward identities, associated with limits as one or two scalars become soft Bianchi, Elvang, Freedman, 0805.0757; Arkani-Hamed, Cachazo, Kaplan, 0808.1446; Kallosh, Kugo, 0811.3414
- Single-soft limit of NMHV 6-point matrix element of R^4 doesn't vanish; violates $E_{7(7)}$ Elvang, Kiermaier, 1007.4813
- Similar arguments also rule out $\mathcal{D}^4R^4$ and $\mathcal{D}^6R^4$
- However, $\mathcal{D}^8R^4$ is allowed (L=7 for D=4) Beisert et al., 1009.1643
- Same conclusions reached by other methods Bossard, Howe, Stelle, 1009.0743
- Volume of full N=8 superspace is same dimension as $\mathcal{D}^8R^4$ – but it vanishes! Invariant candidate $\mathcal{D}^8R^4$ counterterm exists, but not full superspace integral. Bossard, Howe, Stelle, Vanhove, 1105.6087

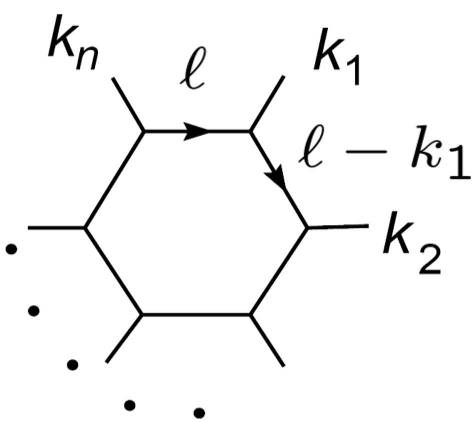
One-loop multi-leg “no triangle” property

Bjerrum-Bohr et al., hep-th/0610043; Bern, Carrasco, Forde, Ita, Johansson, 0707.1035 (pure gravity) ; Kallosh, 0711.2108; Bjerrum-Bohr, Vanhove, 0802.0868

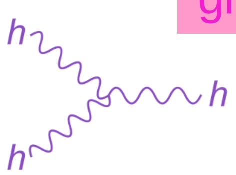
Proofs: Bjerrum-Bohr, Vanhove, 0805.3682; Arkani-Hamed, Cachazo, Kaplan, 0808.1446

- Statement about UV behavior of N=8 SUGRA amplitudes at **one loop** but with **arbitrarily many external legs:**

  “N=8 UV behavior no worse than N=4 SYM at one loop”
- Samples arbitrarily many powers of loop momenta
- Necessary but not sufficient for excellent **multi-loop** behavior
- Implies specific **multi-loop** cancellations Bern, LD, Roiban, th/0611086

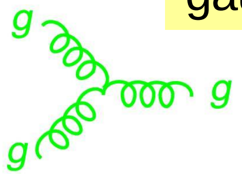


gravity (spin 2)



$$\supset \ \ell^{\mu_1}\,\ell^{\mu_2}\,\eta^{\nu_1\rho_1}\,\eta^{\nu_2\rho_2} + \cdots$$

gauge theory (spin 1)



$$\supset \ \ell^{\mu}\,\eta^{\nu\rho} + \cdots$$

$\mathcal{N}=8$ vs. $\mathcal{N}=4$ SYM

DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)

$2^8 = 256$ massless states, ~ expansion of $(x+y)^8$

$$\mathcal{N}=8: \quad 1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$$

$$\text{helicity}: \quad -2 \quad -\tfrac{3}{2} \quad -1 \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

SUSY $\longleftrightarrow$

$$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$$

$$\mathcal{N}=4 \text{ SYM}: \quad 1 \quad 4 \quad 6 \quad 4 \quad 1$$

$2^4 = 16$ states ~ expansion of $(x+y)^4$

$$g^- \quad \lambda_A^- \quad \phi_{AB} \quad \lambda_A^+ \quad g^+$$

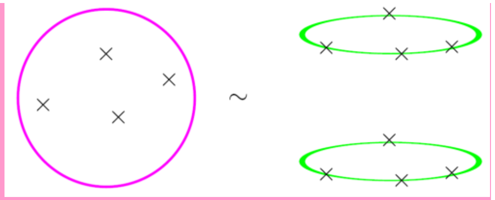
all in adjoint representation

$$\Rightarrow \quad [\mathcal{N}=8] = [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

Kawai-Lewellen-Tye relations

KLT, 1986

Derived from relation between open & closed string amplitudes.



Low-energy limit gives N=8 supergravity amplitudes $M_n^{\rm tree}$ as quadratic combinations of N=4 SYM amplitudes $A_n^{\rm tree}$, consistent with product structure of Fock space,

$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

$$M_3^{\rm tree}(1,2,3) \;=\; [A_3^{\rm tree}(1,2,3)]^2$$

$$M_4^{\rm tree}(1,2,3,4) \;=\; -i\, s_{12}\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

$$M_5^{\rm tree}(1,2,3,4,5) \;=\; i\, s_{12} s_{34}\, A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) + (2 \leftrightarrow 3)$$

$$M_6^{\rm tree}(1,2,3,4,5,6) \;=\; \cdots$$

AdS/CFT vs. KLT

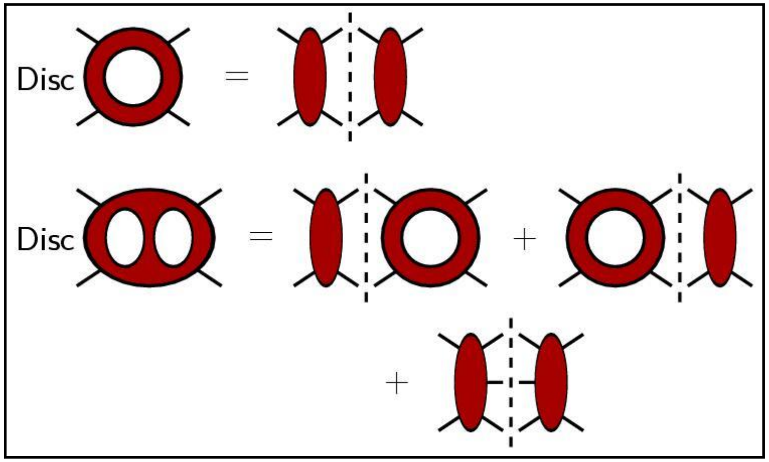
AdS	=	CFT
gravity	=	gauge theory
weak		strong

KLT

gravity	=	(gauge theory)^2
weak		weak

KLT and perturbative unitarity

• *S*-matrix a unitary operator between in and out states
→ unitarity relations (cutting rules) for amplitudes



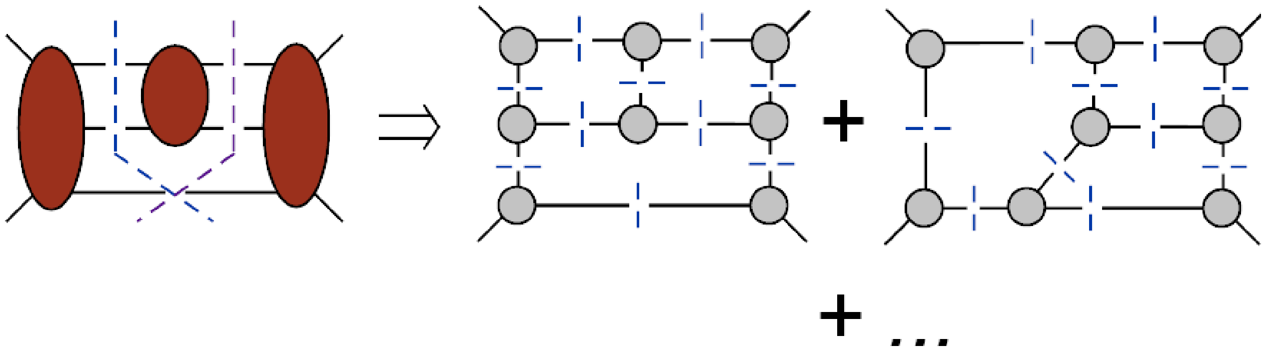
• Reconstruction of multi-loop amplitudes from cuts very efficient, due to simple structure of tree and lower-loop helicity amplitudes

• Generalized unitarity (more propagators open) necessary to reduce everything to trees (in order to apply KLT relations)

Method of maximal cuts

Complex cut momenta make sense out of all-massless 3-point kinematics – can chop an amplitude entirely into 3-point trees

→ **maximal cuts**



Maximal cuts are maximally simple,
yet give excellent starting point for constructing full answer

For example, in planar (leading in N_c) N=4 SYM
they find all terms in the complete answer for 1, 2 and 3 loops

Remaining terms found **systematically**: Let 1 or 2 propagators collapse from each maximal cut → **near-maximal cuts**

Multi-loop "KLT copying"

Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

- N=8 SUGRA cuts are products of N=8 SUGRA trees, summed over all internal states.
- KLT relations let us write N=8 cuts very simply as:

sums of products of two copies of N=4 SYM cuts

$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4] \quad \Longrightarrow \quad \sum_{\mathcal{N}=8} \;=\; \sum_{\mathcal{N}=4}\sum_{\mathcal{N}=4}$$

- Need both **planar** (large N_c) and **non-planar** terms in corresponding multi-loop N=4 SYM amplitude

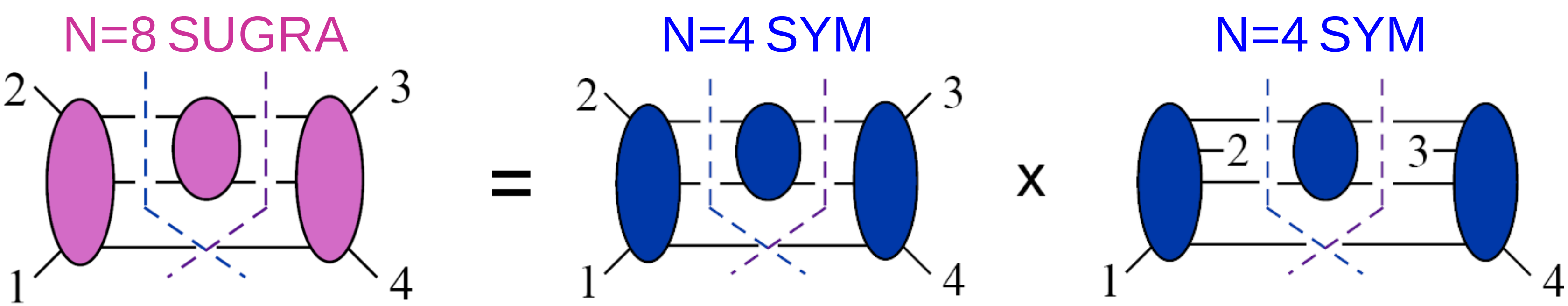
KLT copying at 3 loops

Using

$$M_4^{\text{tree}}(1,2,3,4) = -i\,\frac{st}{u}[A_4^{\text{tree}}(1,2,3,4)]^2$$

$$M_5^{\text{tree}}(1,2,3,4,5) = -i\,s_{51}s_{23}\,A_5^{\text{tree}}(1,2,3,4,5)\,A_5^{\text{tree}}(1,4,2,3,5) + (1 \leftrightarrow 2)$$

it is easy to see that



rational function of Lorentz products of external and cut momenta; **all state sums already performed**

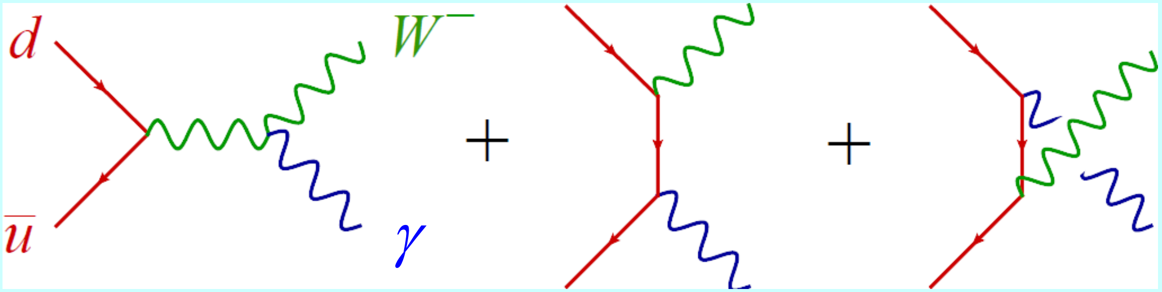
$+$ permutations $(1 \leftrightarrow 2,\ 3 \leftrightarrow 4)$

New gravity = gauge^2 relations

- KLT relations involve permuted products of gauge amplitudes. Makes it non-trivial to reconstruct the N=8 SUGRA integrand from the information provided by the twisted double copy of the gauge theory cuts.
- Also, ambiguities in this reconstruction beyond two loops.
- Initial integrands we found for the 3 and 4 loop 4-graviton amplitudes in N=8 SUGRA were not UV-optimal: individual integrals were worse-behaved in the UV than the full amplitude (sum of all terms).
- A new way to write gravity amplitudes, as sums of squares of gauge theory terms, provides a much more efficient way to construct the supergravity amplitudes.
- Relies on the existence of a color-kinematic duality for gauge theory amplitudes Bern, Carrasco, Johansson, 0805.3993
- Also simplifies construction of N=4 SYM amplitude.

Radiation Zeroes

- In 1979, Mikaelian, Samuel and Sahdev computed $\frac{d\sigma(d\bar{u}\to W^-\gamma)}{d\cos\theta}$



- They found a "radiation zero" at

$$\cos\theta = -(1+2Q_d) = -1/3$$

- Held independent of (W,γ) helicities
- Implies a connection between "color" (here ~ electric charge Q_d) and kinematics ($\cos\theta$)

Radiation Zeroes and Color-Kinematic Duality

- MSS result generalized to other 4-point non-Abelian gauge theory amplitudes by Zhu (1980), Goebel, Halzen, Leveille (1981).
- Massless adjoint gauge theory result:

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s C_s}{s} + \frac{n_t C_t}{t} + \frac{n_u C_u}{u}$$

- Group theory → 3 terms are not independent (Jacobi identity):

$$C_t - C_u = C_s$$

- In a suitable "gauge", one finds: $n_t - n_u = n_s$

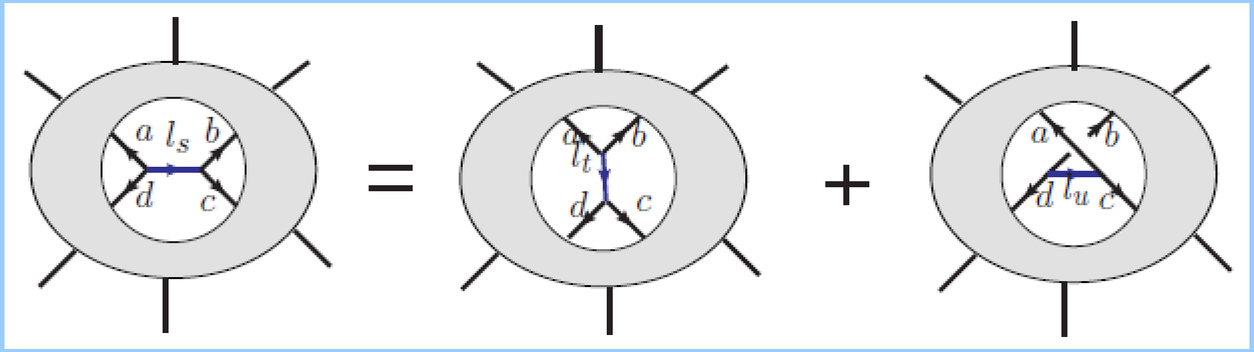
  Same structure can be extended to an arbitrary number of legs and provides a new "KLT-like" relation to gravity:

$$M_4^{\text{tree}} = \frac{n_s^{(L)} n_s^{(R)}}{s} + \frac{n_t^{(L)} n_t^{(R)}}{t} + \frac{n_u^{(L)} n_u^{(R)}}{u}$$

Bern, Carrasco, Johansson, 0805.3993

Color-Kinematic Duality at loop level

- Consider any 3 graphs connected by a Jacobi identity



- Color factors obey

$$C_s = C_t - C_u$$

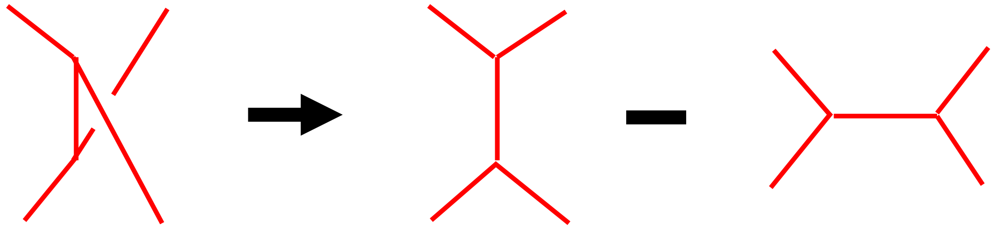
- Duality requires

$$n_s = n_t - n_u$$

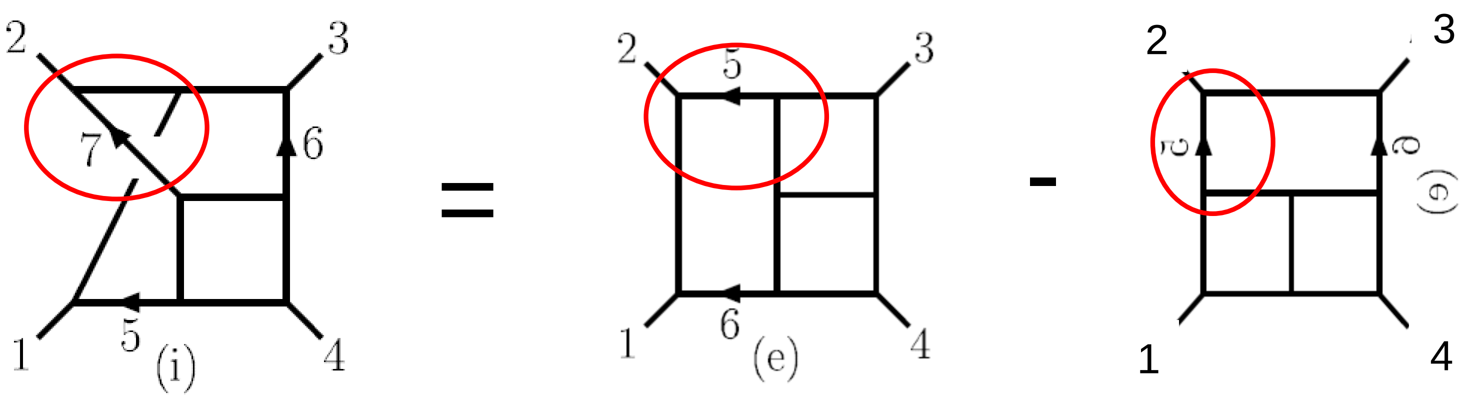
- Very strong constraint on structure of integrands; only a handful of independent integral numerators left after imposing it.

A simple 3 loop example

Using



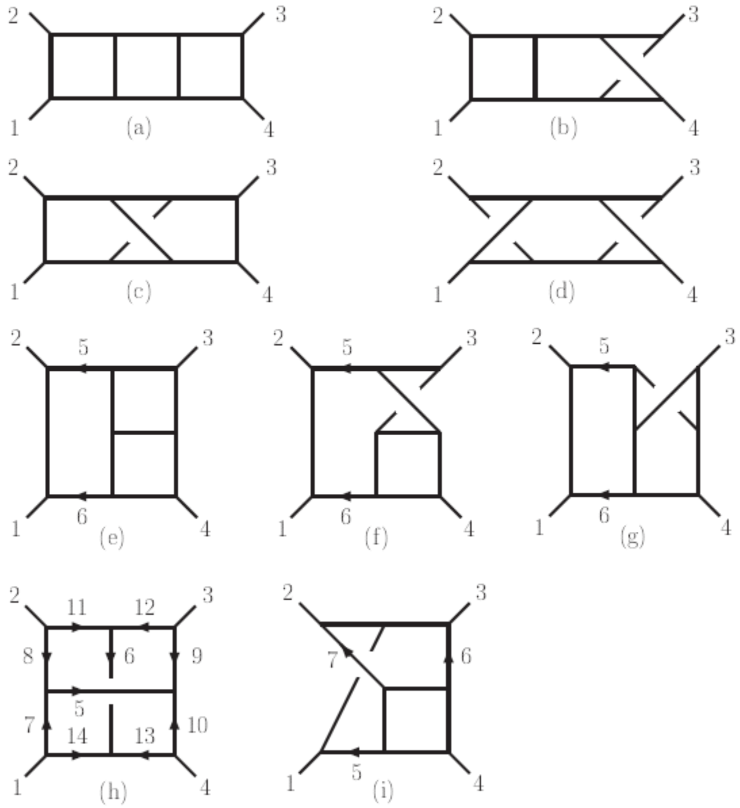
we can relate non-planar topologies to planar ones



3 loop amplitude before color-kinematics duality

Bern, Carrasco, LD, Johansson, Kosower, Roiban, th/0702112
Bern, Carrasco, LD, Johansson, Roiban, 0808.4112

Nine basic integral topologies

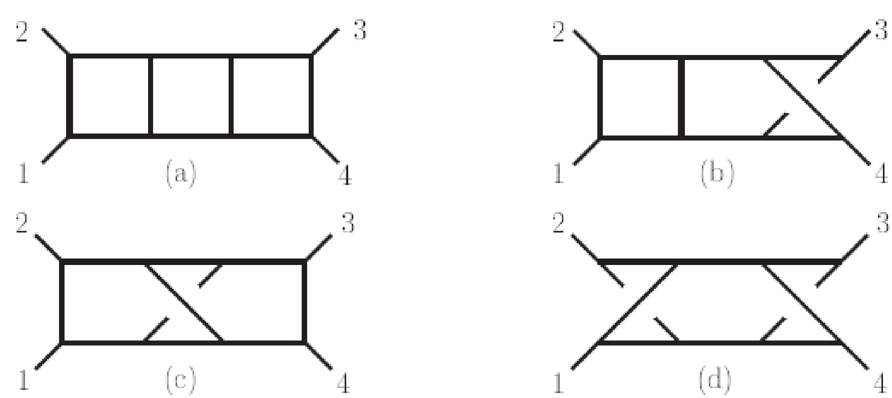


Old N=4 numerators at 3 loops

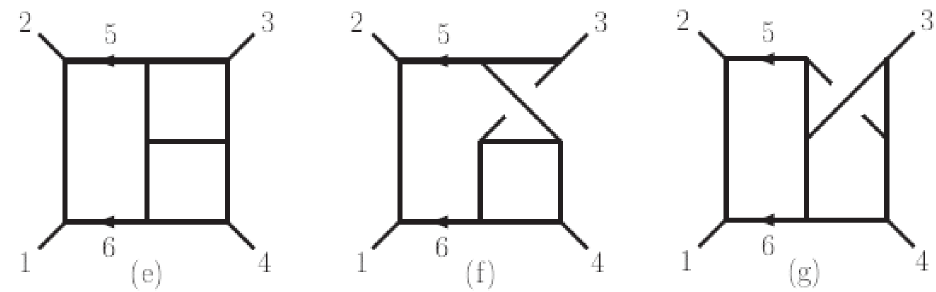
Overall $st\,A_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

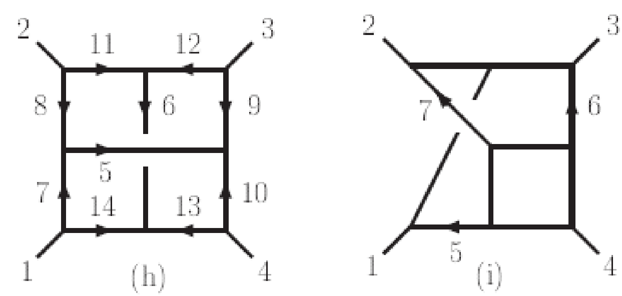
s_{12}^2



$s_{12}\, s_{46}$



$$s_{12}(\tau_{26} + \tau_{36}) + s_{14}(\tau_{15} + \tau_{25}) + s_{12}s_{14}$$



$$s_{12}\, s_{45} - s_{14}\, s_{46} - \frac{1}{3}(s_{12} - s_{14})\, \ell_7^2$$

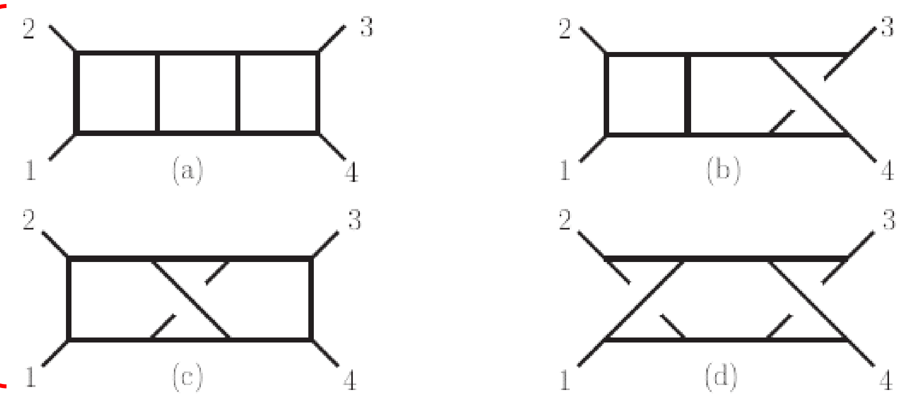
manifestly quadratic in loop momentum ℓ_M

Old N=8 numerators at 3 loops

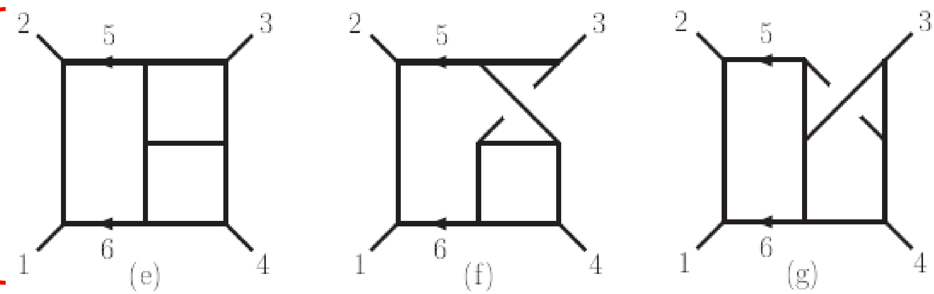
Overall
$(stA_4^{\text{tree}})^2 = stu\, M_4^{\text{tree}}$

$s_{iM} = (k_i + \ell_M)^2$
$\tau_{iM} = 2k_i \cdot \ell_M$

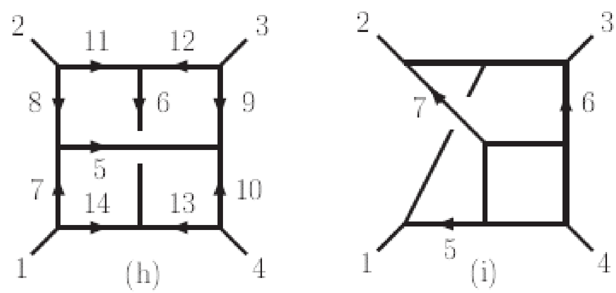
s_{12}^4



$s_{12}^2\, \tau_{35}\, \tau_{46}$



$$(s_{12}(\tau_{26}+\tau_{36}) + s_{14}(\tau_{15}+\tau_{25}) + s_{12}s_{14})^2 + (s_{12}^2(\tau_{26}+\tau_{36}) - s_{14}^2(\tau_{15}+\tau_{25})) \times (\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10}) + s_{12}^2(\tau_{17}\tau_{28}+\tau_{39}\tau_{4,10}) + s_{14}^2(\tau_{28}\tau_{39}+\tau_{17}\tau_{4,10}) + s_{13}^2(\tau_{17}\tau_{39}+\tau_{28}\tau_{4,10})$$



$$(s_{12}\tau_{45} - s_{14}\tau_{46})^2 - \tau_{27}(s_{12}^2\tau_{45} + s_{14}^2\tau_{46}) - \tau_{15}(s_{12}^2\tau_{47} + s_{13}^2\tau_{46}) - \tau_{36}(s_{14}^2\tau_{47} + s_{13}^2\tau_{45}) + l_5^2 s_{12}^2 s_{14} + l_6^2 s_{12} s_{14}^2 - \frac{1}{3} l_7^2 s_{12} s_{13} s_{14}$$

Had to work hard to make manifestly quadratic in ℓ_M

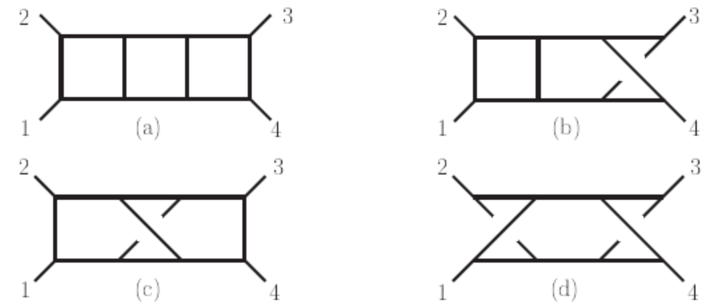
BCDJR (2008)

3 loop amplitude after color-kinematics duality

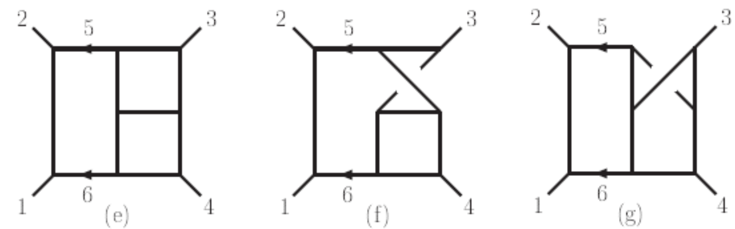
BCJ, 1004.0476

N=8 SUGRA

$$[\, s^2 \,]^2$$

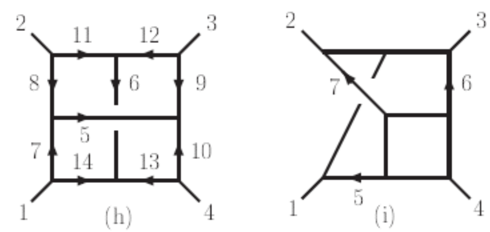


$$\left[\ \begin{array}{ll} \frac{1}{3} & [s(t-\tau_{36}-\tau_{46}) \\ - & t(\tau_{26}+\tau_{46}) \\ + & u(\tau_{26}+\tau_{36})-s^2] \end{array} \right]^2$$



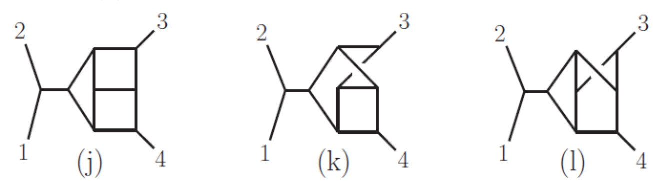
Linear in ℓ_M →

$$[N^{(h)}(\tau_{ij})]^2$$



$$[N^{(i)}(\tau_{ij})]^2$$

$$[\frac{1}{3}s(t-u)]^2$$



N=8 no worse than N=4 SYM in UV

Manifest quadratic representation at 3 loops – same behavior as N=4 SYM – implies same critical dimension (as for $L = 2$):

$$D_c \leq 4 + \frac{6}{L} = 6$$

- Evaluate UV poles in integrals
→ no further cancellation
- At 3 loops, $D_c = 6$ for N=8 SUGRA as well as N=4 SYM:

$$M_4^{(3),D=6-2\epsilon}\Big|_{\rm pole} = \frac{1}{\epsilon}\,\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^8 (s_{12}s_{13}s_{14})^2 M_4^{\rm tree}$$

$\mathcal{D}^6 R^4$ counterterm

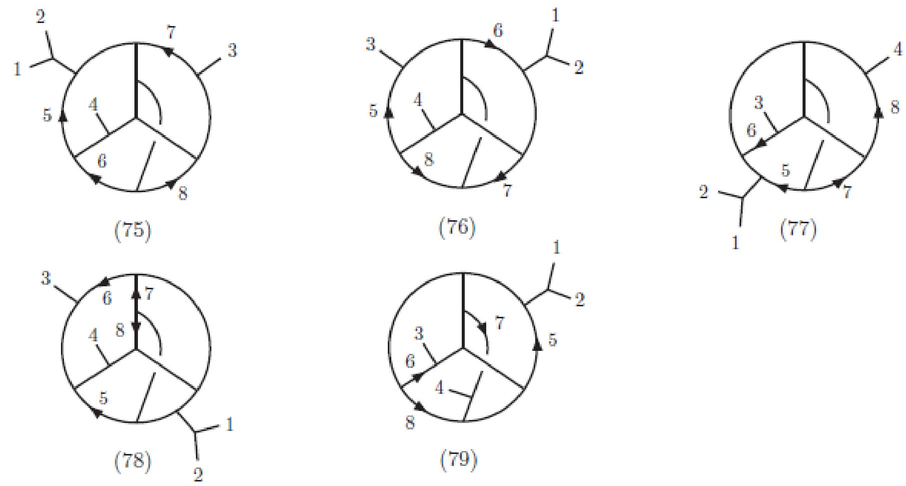
Also recovered via string theory (up to factor of 9?)

Green, Russo, Vanhove,1002.3805

4 loops

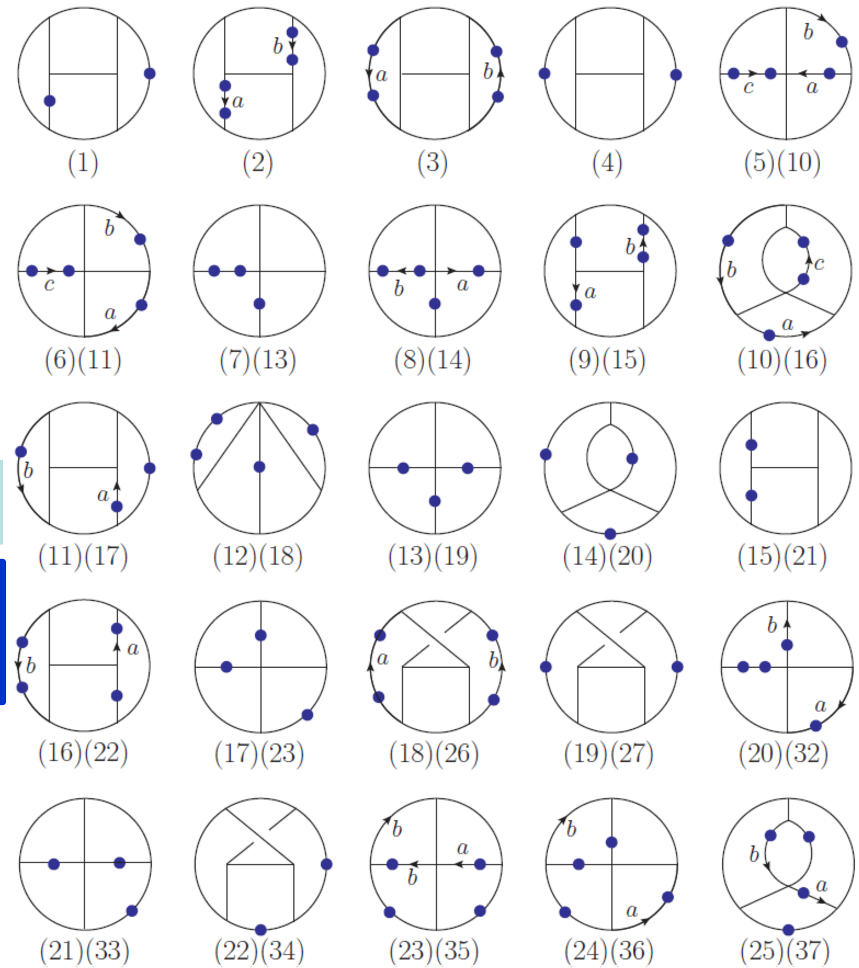
- Computed first without using color-kinematics duality
→50 nonvanishing cubic 4-point graphs   BCDJR, 0905.2326, 1008.3327
- From this form we could show N=8 SUGRA no worse behaved than N=4 SYM using this representation.
- But actually extracting the numerical coefficient of the $\mathcal{D}^8 R^4$ divergence in the critical dimension, $D_c(4) = 4 + 6/4 = 11/2$, was too difficult.
- Now we have a dual form for the N=4 SYM amplitude, from which we can compute the divergence.

35 new, 1-particle-reducible cubic graphs, analogs of (j), (k), (l) at 3 loops



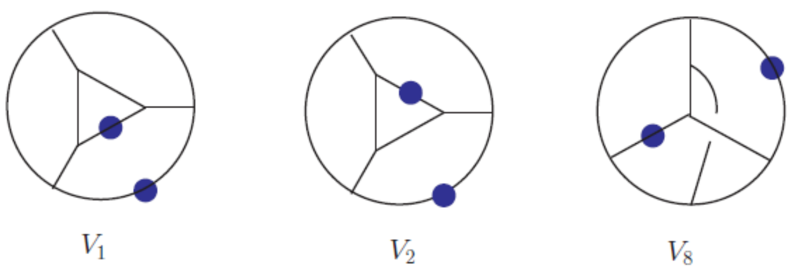
4 loop UV divergence in D=11/2

- In new form, all integrals are at worst log-divergent in D=11/2.
- Can just set external momenta to zero to obtain a (pretty big) set of 4 loop vacuum integrals. (25 out of 69 are shown here)



Remarkably, final answer is simply:

$$\propto \frac{23}{8}(s^2+t^2+u^2)^2 \times (V_1 + 2V_2 + V_8)$$



N=4 SYM in UV at 4 loops

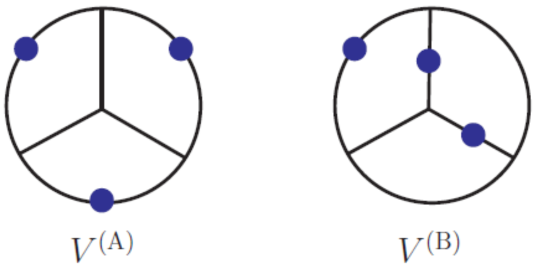
- Remarkable because subleading-color part of UV divergence of N=4 SYM depends on precisely the same linear combination of vacuum integrals:

$$\mathcal{A}_4^{(4)}(1,2,3,4)\Big|_{\text{pole}}^{SU(N_c)} = -6\,g^{10}\,\mathcal{K}\,N_c^2\Big(N_c^2\,V_1 + 12\,(V_1 + 2\,V_2 + V_8)\Big) \times \Big(s_{12}\,(\mathrm{Tr}_{1324} + \mathrm{Tr}_{1423}) + s_{23}\,(\mathrm{Tr}_{1243} + \mathrm{Tr}_{1342}) + s_{13}\,(\mathrm{Tr}_{1234} + \mathrm{Tr}_{1432})\Big)$$

$$\mathrm{Tr}_{ijkl} \equiv \mathrm{Tr}(T^{a_i}T^{a_j}T^{a_k}T^{a_l})$$

- Same property holds at 3 loops in D=6 – both divergences are proportional to

$$V^{(\mathrm{A})} + 3V^{(\mathrm{B})} \propto \zeta(3)$$



What about $L = 5$?

• Motivation: Various arguments point to 7 loops as the possible first divergence for N=8 SUGRA in D=4, associated with a D^8R^4 counterterm:

Howe, Lindstrom, NPB181, 487 (1981); Bossard, Howe, Stelle, 0908.3883; Kallosh, 0903.4630; Green, Russo, Vanhove, 1002.3805; Bjornsson, Green, 1004.2692; Bossard, Howe, Stelle, 1009.0743; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger,1009.1643

• Same D^8R^4 counterterm shows up at $L = 4$ in $D = 5.5$

• Does 5 loops → $D^{10}R^4$ (same UV as N=4 SYM)?
or → D^8R^4 (worse UV as N=4 SYM)?

• 5 loops would be a very strong indicator for 7 loops

• Now 100s of nonvanishing cubic 4-point graphs!

Implications if N=8 perturbatively finite

- Suppose N=8 SUGRA is finite to all loop orders.
- Does this mean it is a nonperturbatively consistent theory of quantum gravity?
- No!
- At least two reasons it might need a nonperturbative completion:
  - Likely $L!$ or worse growth of the order L coefficients,

    $$\sim \; L! \; (s/M_{Pl}^2)^L$$
  - **Different** $E_{7(7)}$ behavior of the perturbative series (invariant!) compared with the $E_{7(7)}$ behavior of the mass spectrum of black holes (non-invariant!)

Is N=8 SUGRA "only" as good as QED?

- QED is renormalizable, but its perturbation series has zero radius of convergence in α: $\sim L!\ \alpha^L$
- UV renormalons associated with UV Landau pole
- But for small α it works pretty well:
$g_e - 2$ agrees with experiment to 10 digits
- Also, tree-level (super)gravity works well for $s \ll M_{Pl}^2$
- Many pointlike nonperturbative UV completions for QED: asymptotically free GUTs
- What is/are nonperturbative UV completion(s) for N=8 SUGRA? Is the only possibility superstring theory? Or could some be pointlike too?

Outlook

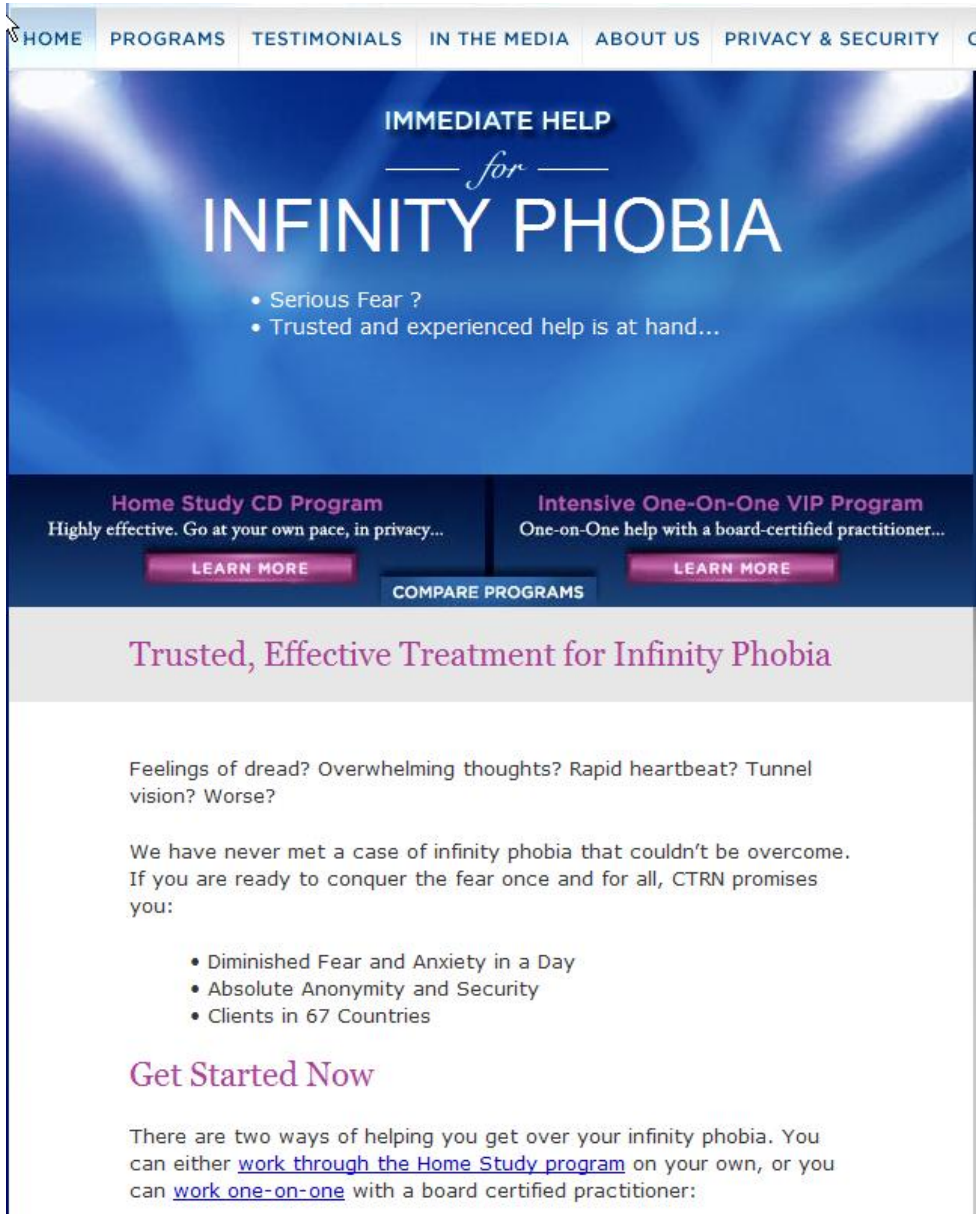
• Through 4 loops, the 4-graviton scattering amplitude of N=8 supergravity has UV behavior no worse than the corresponding 4-gluon amplitude of N=4 SYM.

• Will the same continue to happen at higher loops? 5 loops will provide a strong test! If so, then N=8 supergravity would be a finite, point-like theory of quantum gravity.

• We need a new way to look at the problem, rather than loop by loop! Is there a deep symmetry responsible?

• N=8 supergravity is still only a "toy model" for quantum gravity – we don't see any way to use it to describe the strong and weak interactions.

• Still, could it point the way to other, more realistic, finite point-like theories? (A big challenge, but maybe N=8 gaugings → N<8 can be a first step…)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