Renormalisation group and the Planck scale

Daniel F Litim
Department of Physics and Astronomy
University of Sussex

Quantum Theory and Gravitation
ETHZ, June 23, 2011

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\text{kg}\, s^2}$

classical action

$$S_{\text{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}(-R(g_{\mu\nu}) + 2\Lambda)$$

long distances

gravity not tested beyond 10^{28}cm

short distances

gravity not tested below 10^{-2}cm

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

expect **quantum modifications** at energy scales M_{Pl}

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

effective expansion parameter: $g_{\mathrm{eff}} \equiv G_N\, \mu^2 \approx \frac{\mu^2}{M_{\mathrm{Pl}}^2}$

$[G] > 0:$ superrenormalisable

$[G] = 0:$ renormalisable

$[G] < 0:$ **dangerous** interactions $[G_N] = 2 - d$

perturbative non-renormalisability

gravity with matter interactions

pure gravity (Goroff-Sagnotti term)

quantum field theory

running couplings

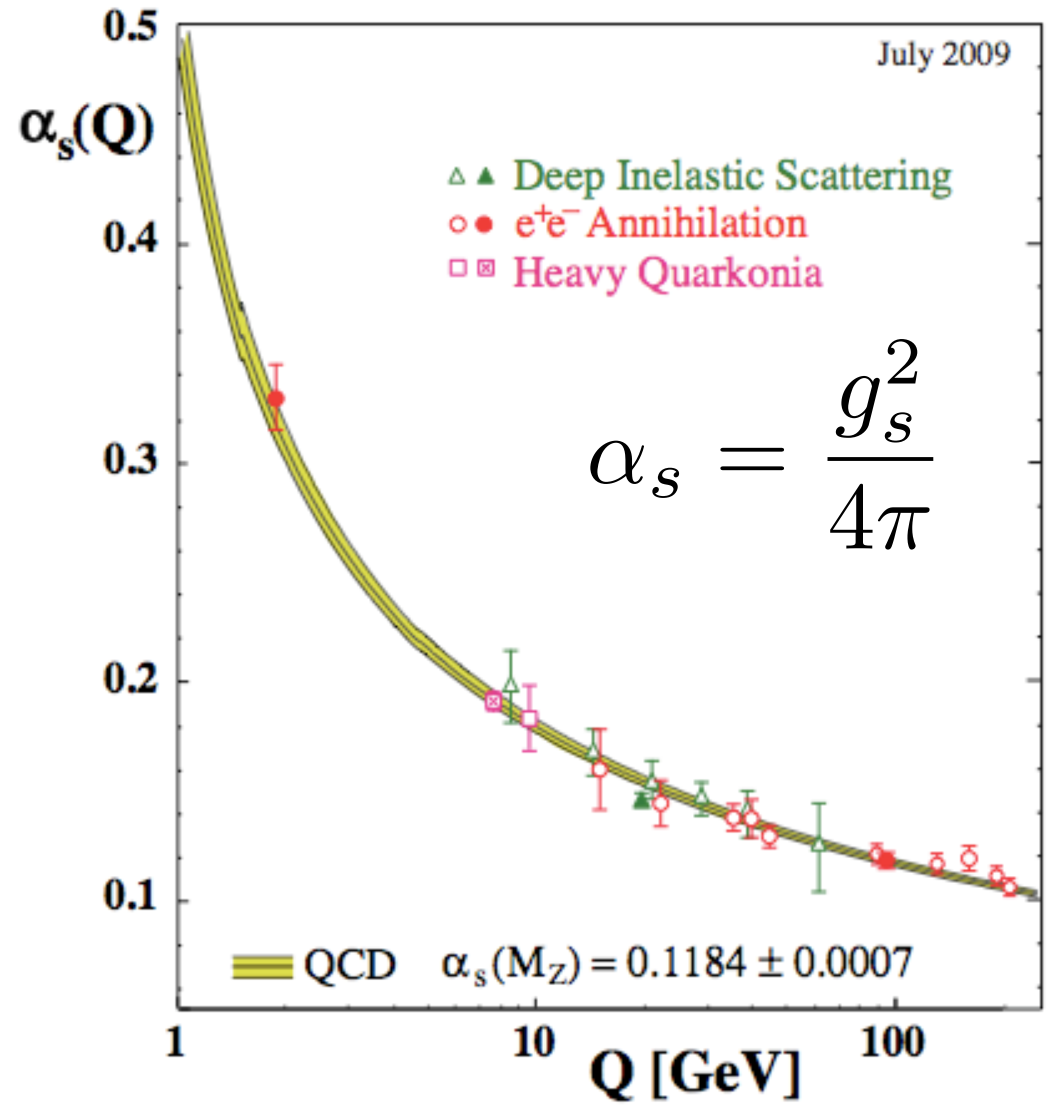
quantum fluctuations modify interactions

couplings depend on eg. energy or distance

asymptotic freedom of the strong force

(taken from PDG)

$$S_{\mathrm{YM}} = \frac{1}{4g_s^2}\int F^2$$



$$\alpha_s = \frac{g_s^2}{4\pi}$$

quantum field theory

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

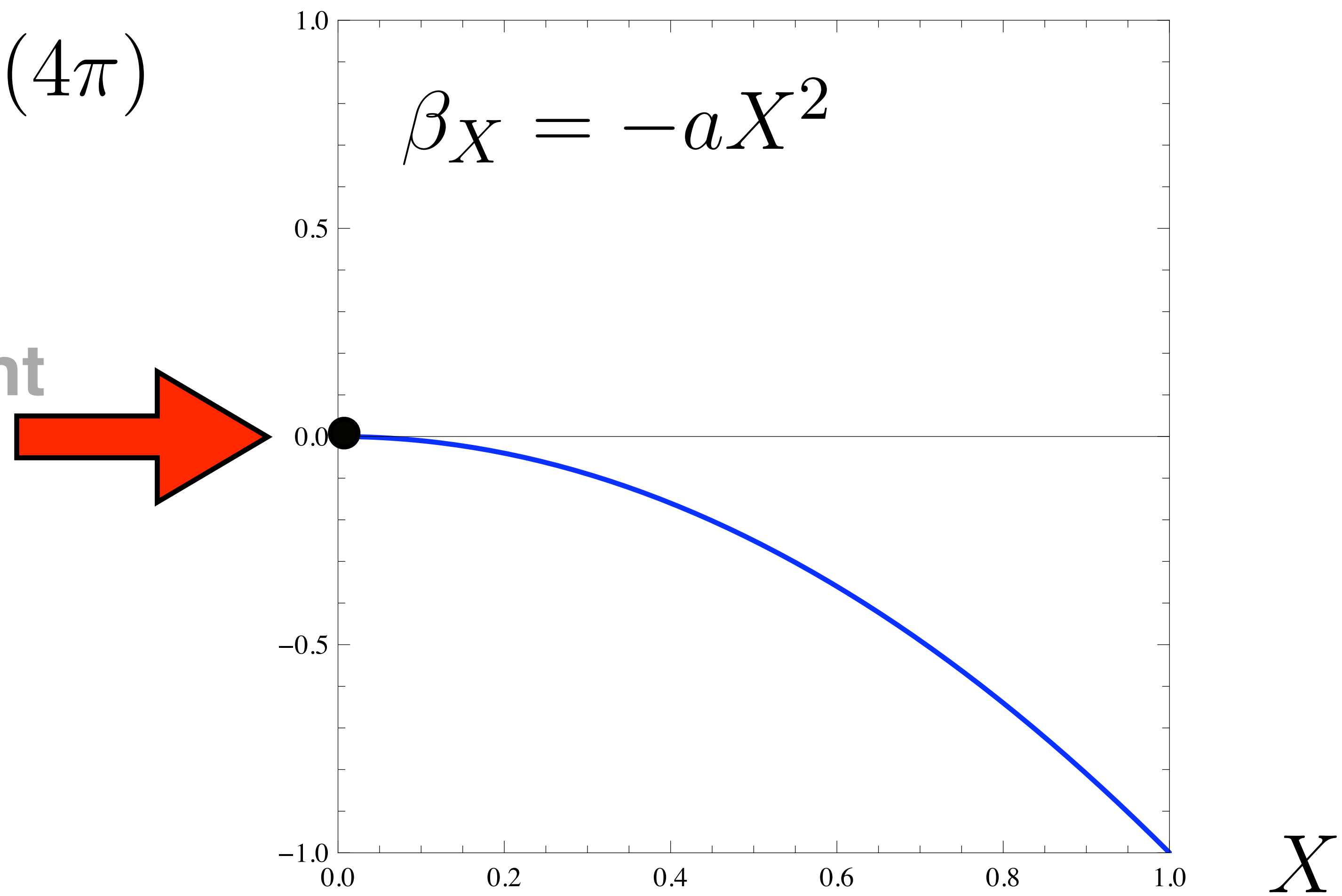
asymptotic freedom of the strong force

coupling $X = g_s^2/(4\pi)$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

trivial UV fixed point

$$X_* = 0$$



quantum field theory

running couplings

quantum fluctuations modify interactions

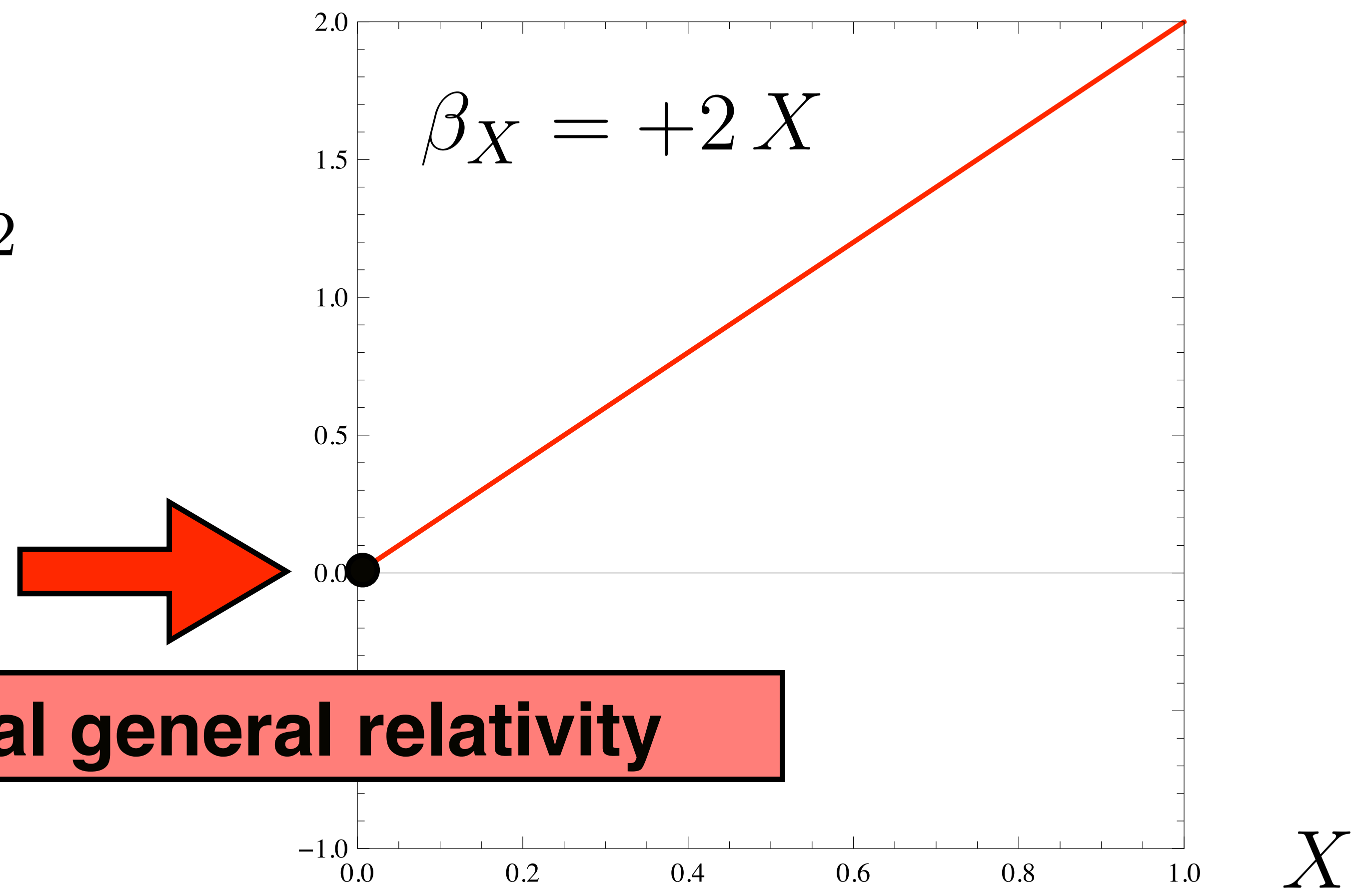
couplings depend on eg. energy or distance

gravitation

coupling $X = G_N \mu^2$

$$\beta_X \equiv \frac{dX}{d \ln \mu}$$

trivial IR fixed point



quantum field theory

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

gravitation

replace

$$G_N \to G(\mu)$$

$$g_{\rm eff} = G_N\,\mu^2 \to g(\mu) \equiv G(\mu)\,\mu^2$$

quantum field theory

running couplings

quantum fluctuations modify interactions

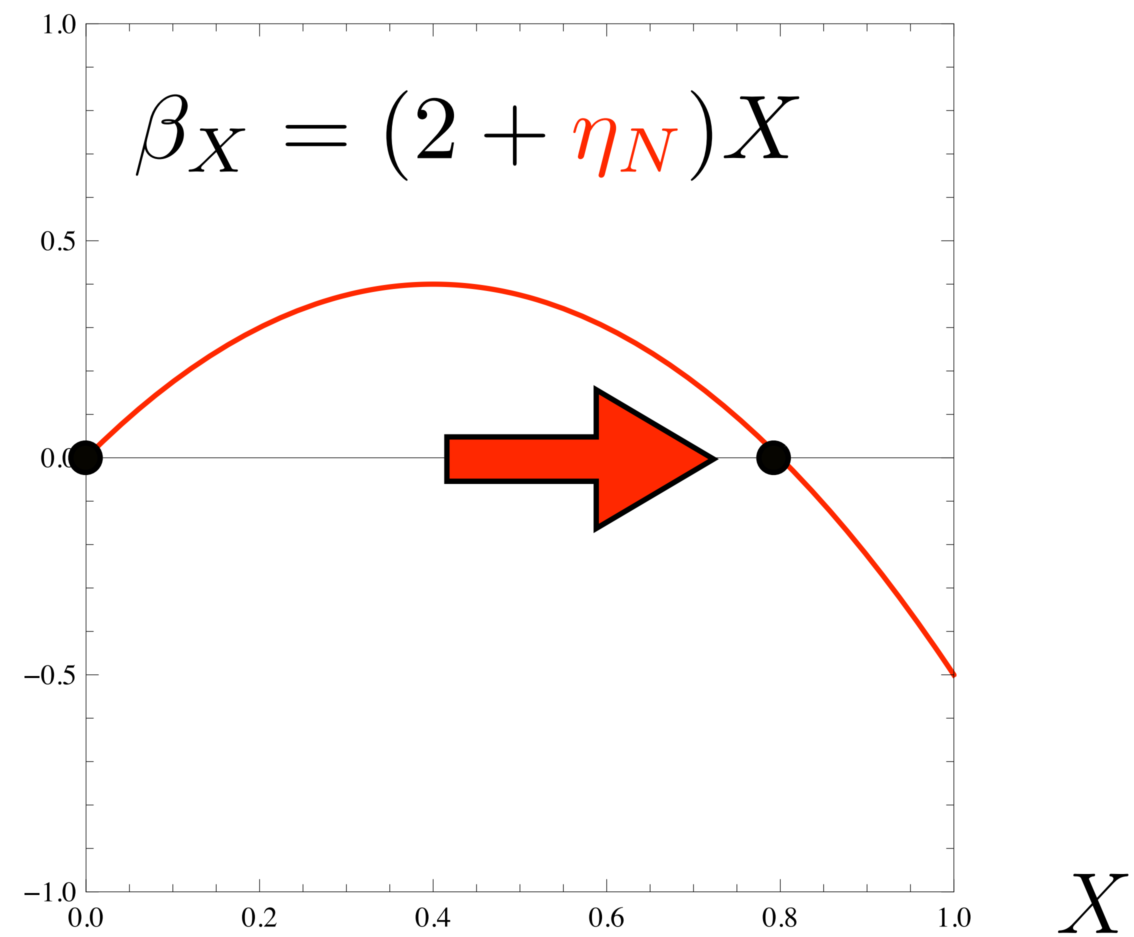
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



Weinberg ('79)
Reuter ('96)
DL ('06), Niedermaier ('06)

quantum field theory

running couplings

quantum fluctuations modify interactions

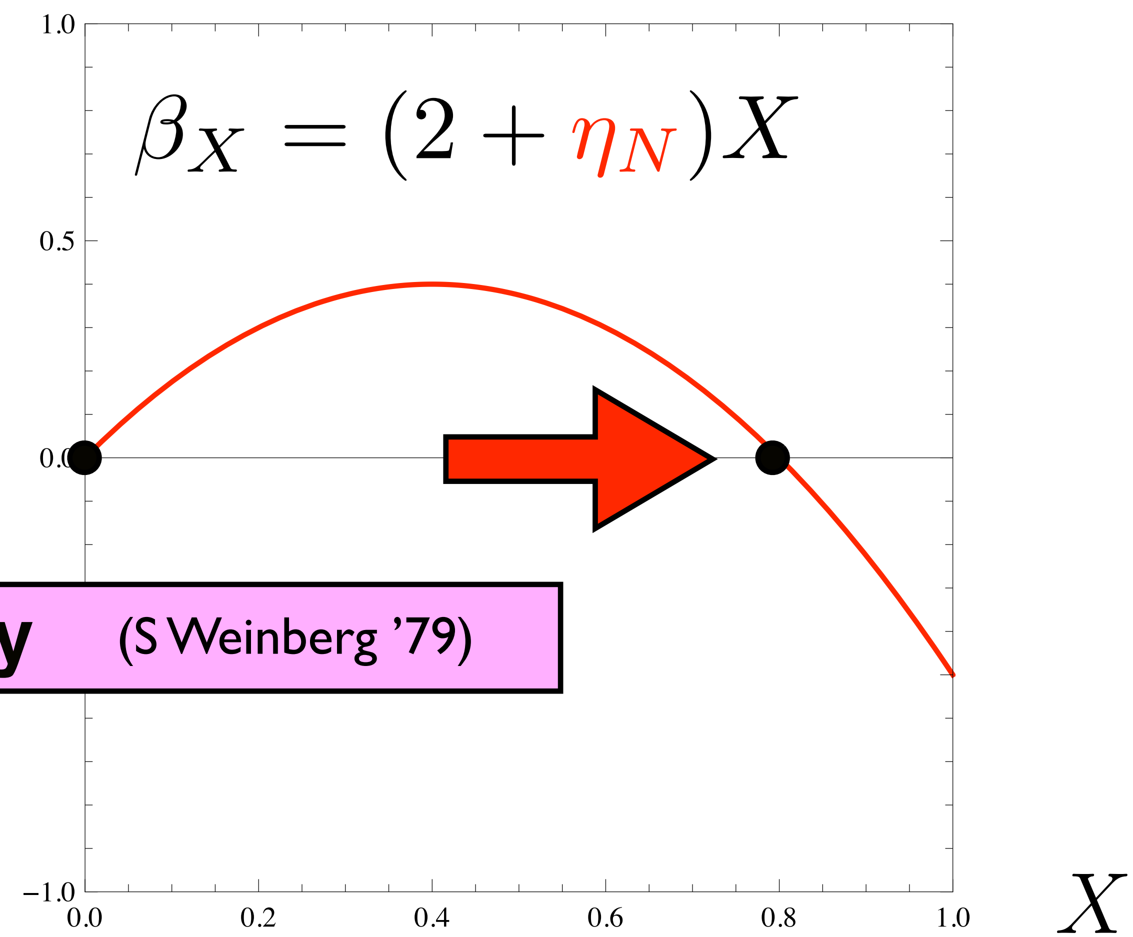
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



asymptotic safety (S Weinberg ’79)

quantum field theory

running couplings

quantum fluctuations modify interactions

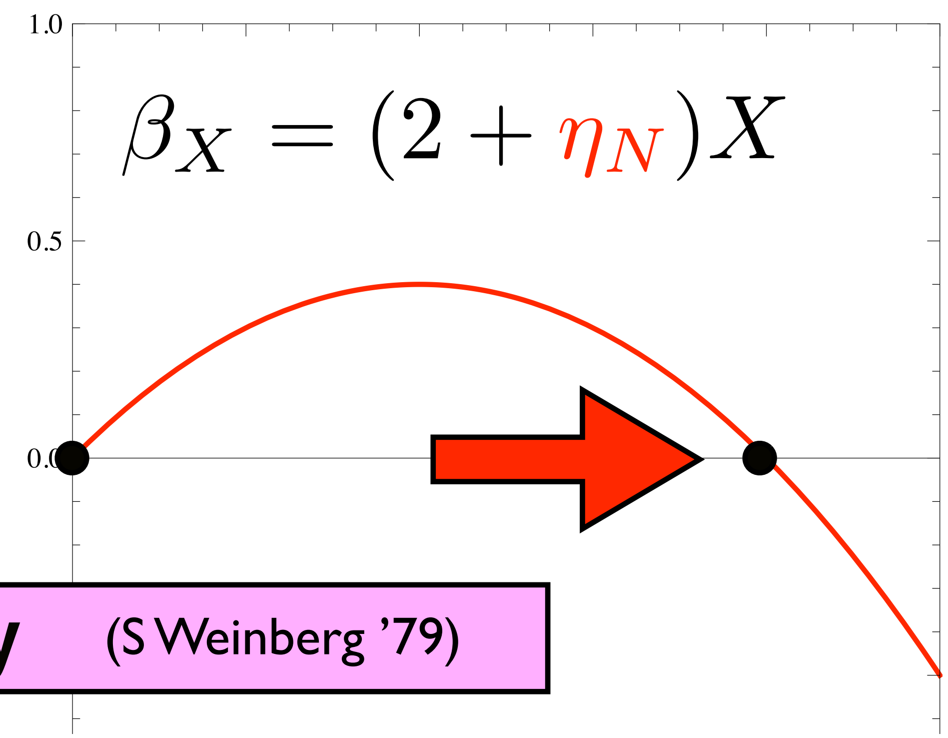
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



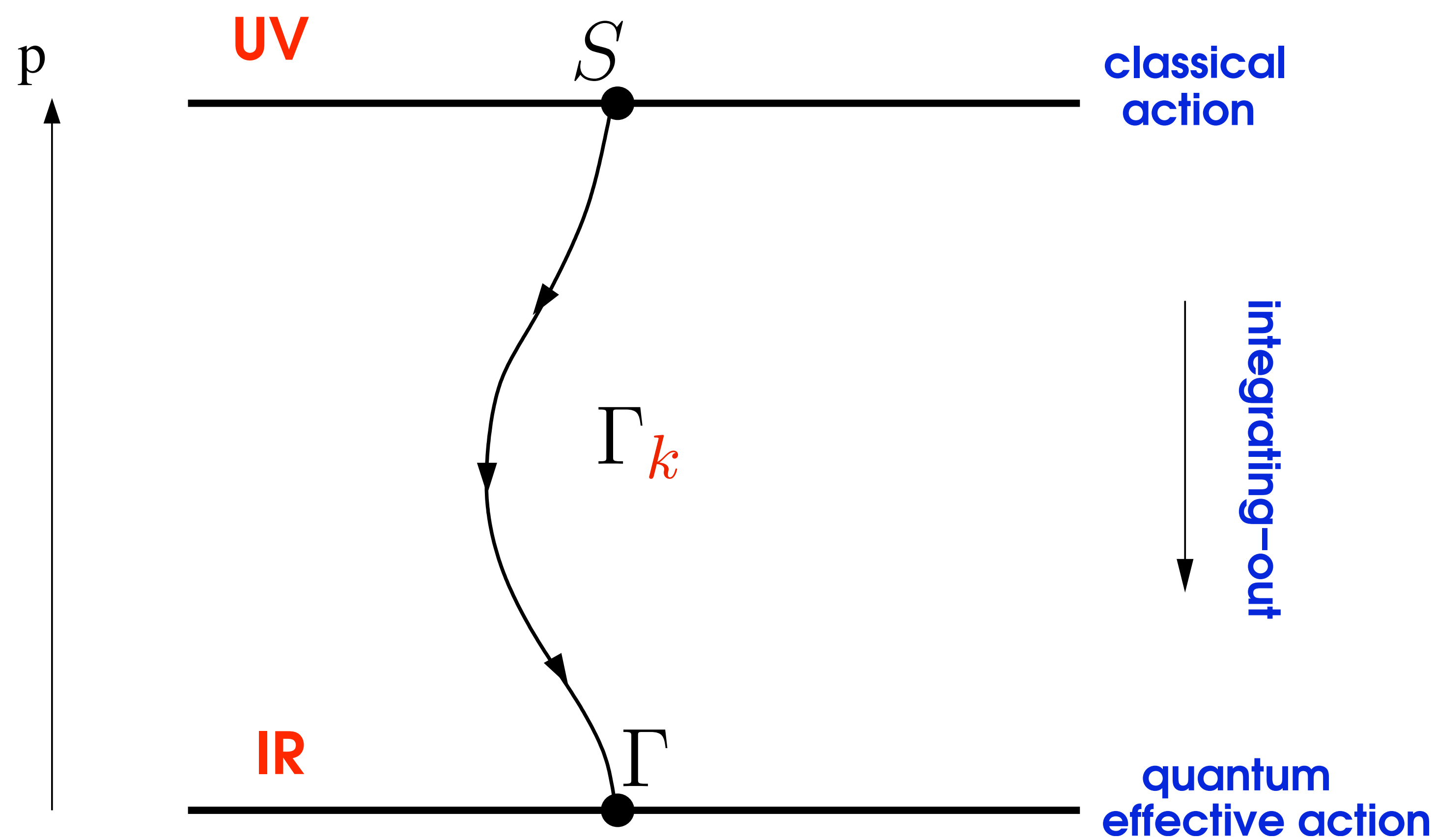
asymptotic safety (S Weinberg '79)

UV fixed point implies **weakly coupled gravity** at **high energies**

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

renormalisation group

integrating-out momentum degrees of freedom: "top-down" (Wilson '71)

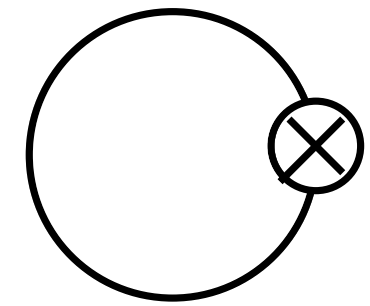


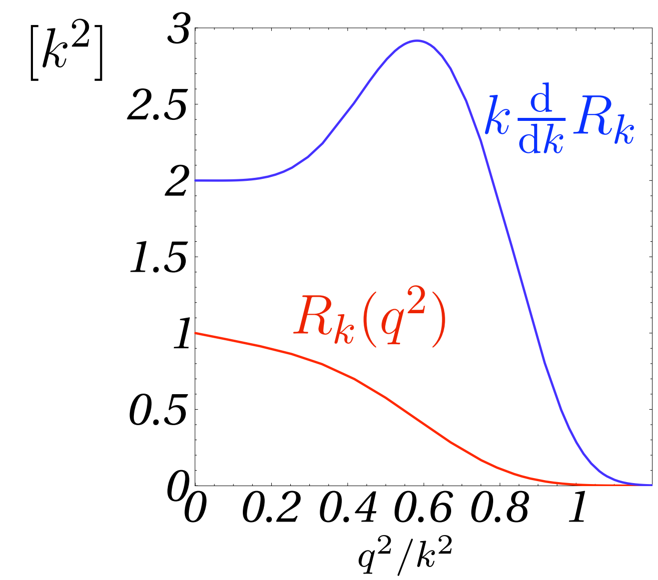
`coarse-graining' of quantum fields

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$





`all-in-one'
`exact'
finite
systematic
Callan-Symanzik type

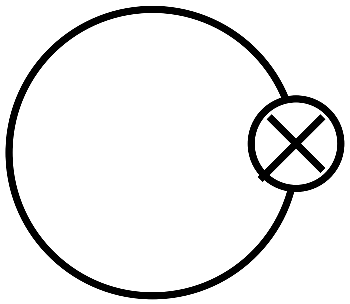
`optimised' choices of R_k

→ stability, analyticity, convergence (DL '01,'02)

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] \quad = \frac{1}{2}$$



QCD

signatures of confinement (Pawlowski, DL, Nedelko, Smekal '03)

Ising-type universality

phase transitions, high precision exponents (DL '02, Bervillier, Juettner, DL '07)

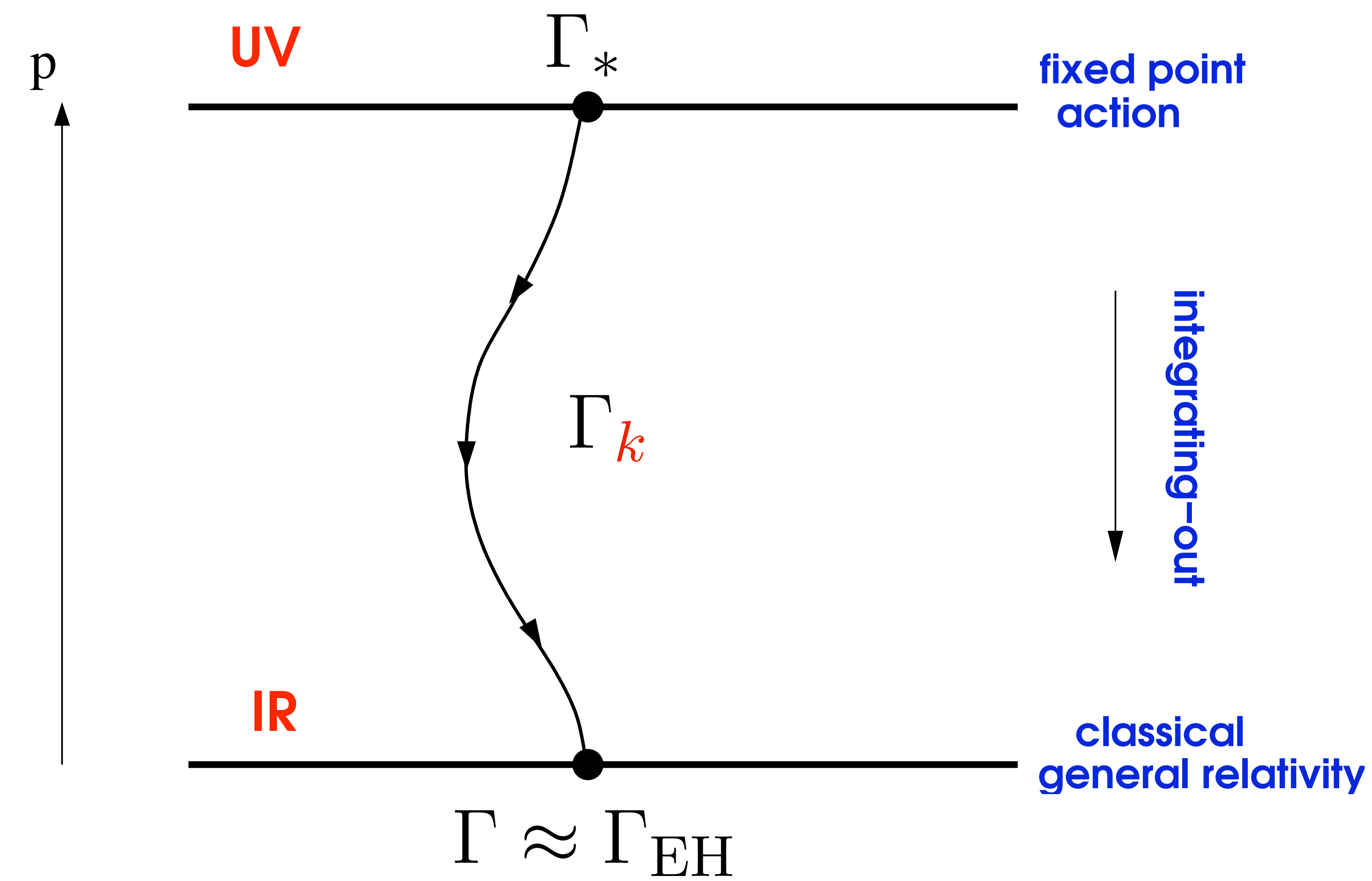
quality control, systematic uncertainties

	η	ν	ω
resummed PT	0.0335(25)	0.6304(13)	0.799(11)
ϵ-expansion	0.0360(50)	0.6290(25)	0.814(18)
world average	0.0364(5)	0.6301(4)	0.84(4)
Monte Carlo	0.03627(10)	0.63002(10)	0.832(6)
functional RGs	0.034(5)	0.630(5)	0.82(4)

(DL, Zappala '10)

renormalisation group

for quantum gravity: **"bottom-up"** (Reuter '96)



`coarse-graining' of quantum fields

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g}\left(\frac{-R+2\Lambda_k}{16\pi G_k}+\cdots\right) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

Einstein-Hilbert theory

$$\beta_g = (D-2+\eta)\,g \qquad g_k = G_k\,k^{D-2} \qquad \eta = \frac{g\,b_1(\lambda)}{1+g\,b_2(\lambda)}$$

$$\beta_\lambda = (-2+\eta)\lambda + g(a_1-\eta\,a_2) \qquad \lambda_k = \Lambda_k/k^2$$

$$a_1 = \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2)$$

$$a_2 = \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda}$$

$$\begin{aligned} b_1 = {} & -\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) \\ & -\frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \end{aligned}$$

$$\begin{aligned} b_2 = {} & -\frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & -\frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)} \end{aligned}$$

(DL′03)

quantum gravity

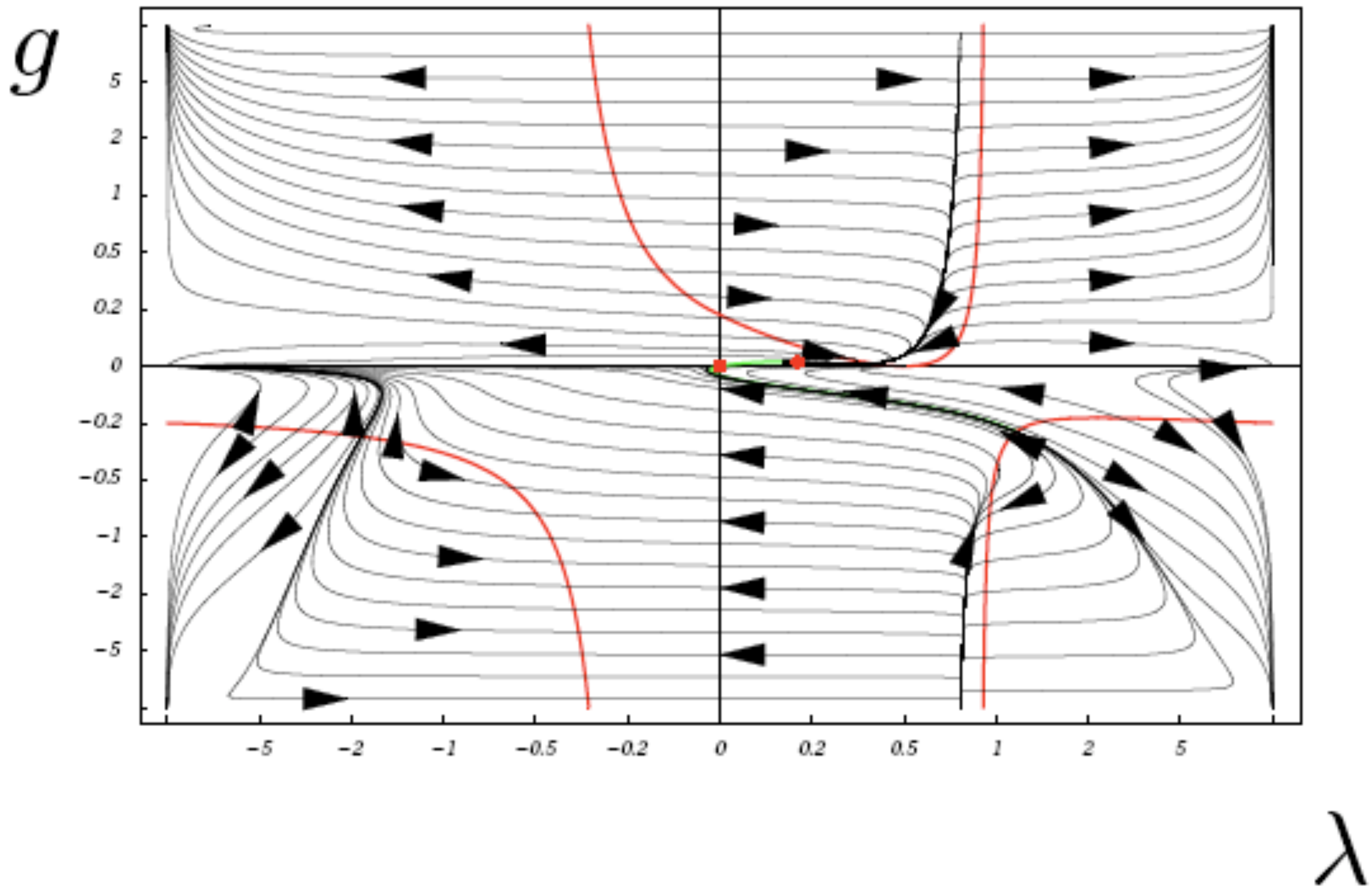
effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$



(DL '03)

quantum gravity

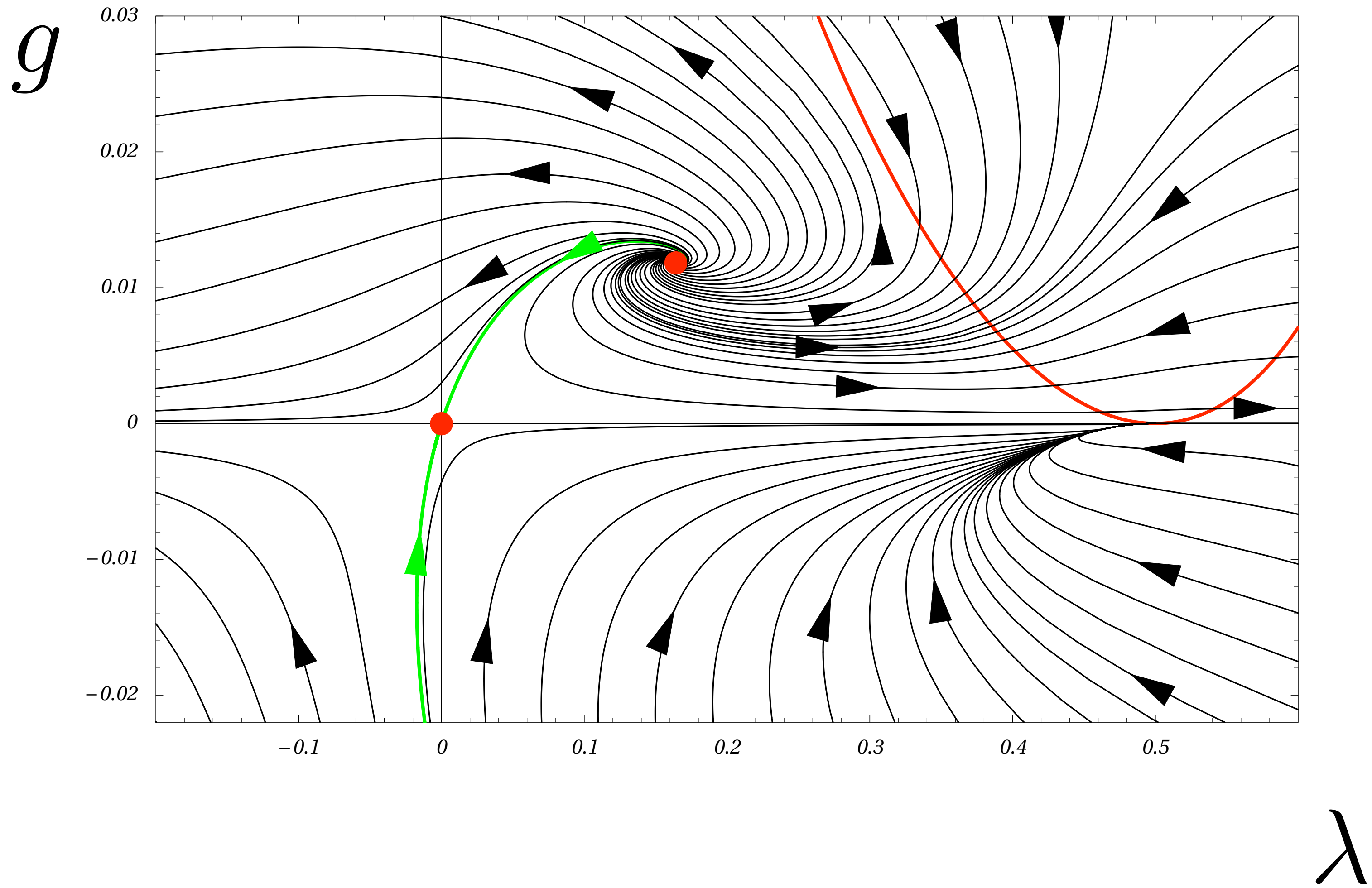
effective action

$$\Gamma_k = \int \sqrt{g}\left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots\right) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$



(DL '03)

quantum gravity

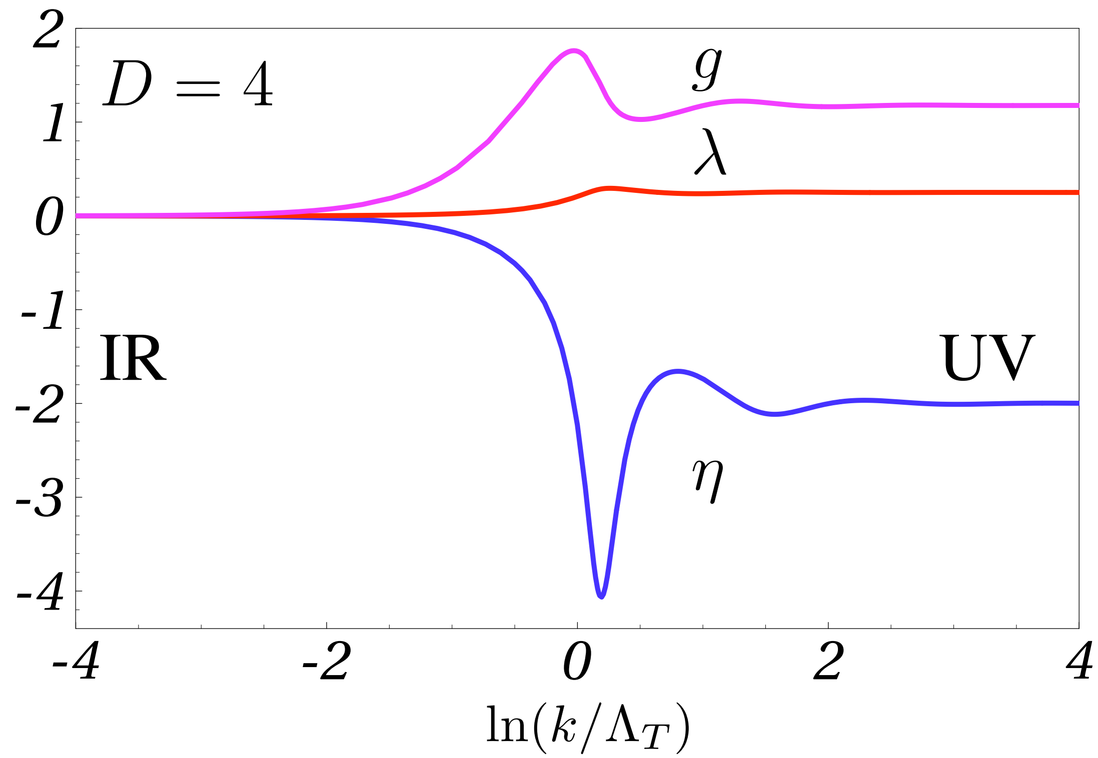
effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

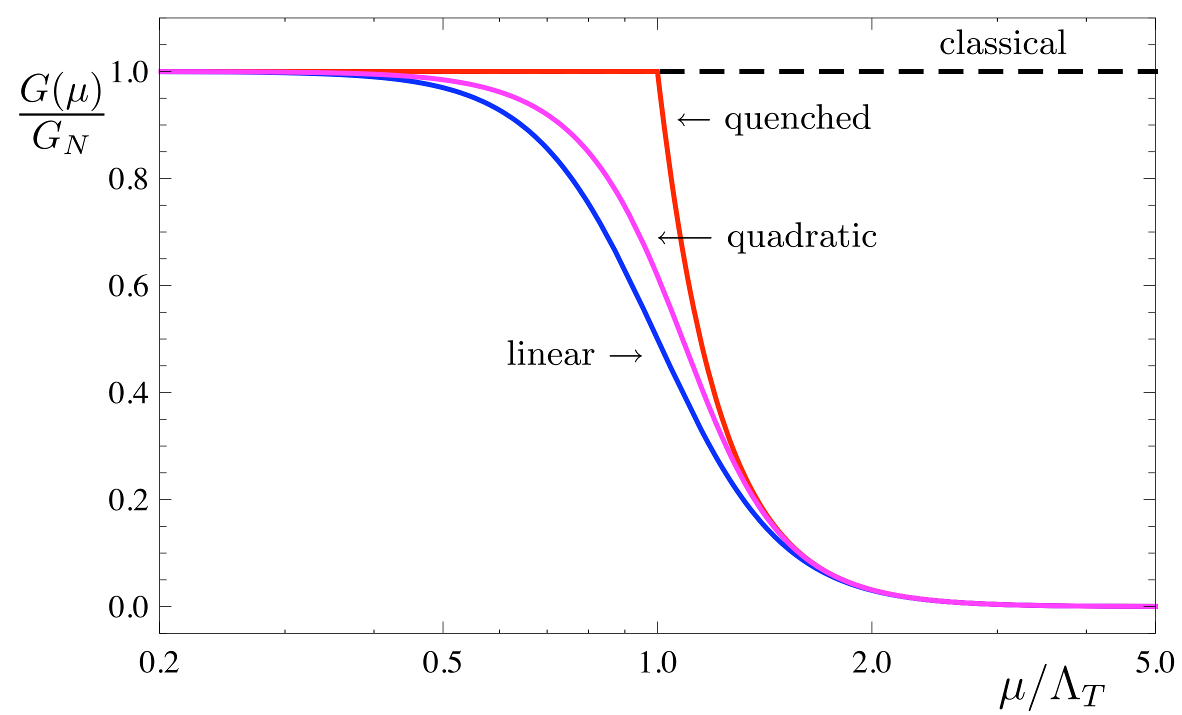
Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$



(DL ’03)



(Gerwick,DL, Plehn ’11)

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

higher dimensions

$$\Lambda_k = \lambda\, k^2$$

Einstein-Hilbert, extensions (DL '03, Fischer, DL '05)

$$G_k = g/k^2$$

$$\lambda_* = \frac{D^2 - D - 4 - \sqrt{2D(D^2 - D - 4)}}{2(D-4)(D+1)}$$

$$g_* = \Gamma(\tfrac{D}{2} + 2)(4\pi)^{D/2-1} \frac{(\sqrt{D^2 - D - 4} - \sqrt{2D})^2}{2(D-4)^2(D+1)^2}$$

RG connected with perturbative infrared regime

quantum gravity and Yang-Mills

(Folkerts, DL, Pawlowski '11)

effective action

$$\Gamma_k = \int \sqrt{g} \left(Z_{N,k} \frac{-R + 2\bar{\Lambda}_k}{16\pi G_N} + \frac{Z_{A,k}}{4g_s^2} F^a_{\mu\Box} F_a^{\mu\Box} \right)$$

does asymptotic freedom persist?

1-loop and effective theory:
asymptotic freedom persists

$$\beta_{\rm YM}\big|_{\rm grav} = -\frac{3\,I}{2\pi}\, g_s^2\, G_N\, E^2 < 0$$

Robinson, Wilczek ('05)
Pietrykowski ('06)
Toms ('07, '10)
Ebert, Plefka, Rodigast ('07)
Daum, Harst, Reuter ('09)
Folkerts, DL, Pawlowski ('11)

quantum gravity and Yang-Mills

(Folkerts, DL, Pawlowski '11)

effective action

$$\Gamma_k = \int \sqrt{g} \left(Z_{N,k} \frac{-R + 2\bar{\Lambda}_k}{16\pi G_N} + \frac{Z_{A,k}}{4 g_s^2} F^a_{\mu\Box} F_a^{\mu\Box} \right)$$

background field flow

$$R_k[\bar{\phi}] = \Gamma_k^{(2)}[\bar{\phi}]\, r[\bar{\phi}]$$

$$r^{gg} = r^{gg}(-\Delta_{\bar{g}}) \quad r^{\bar{\eta}\eta} = -r^{\eta\bar{\eta}} = r^{\bar{\eta}\eta}(-\Delta_{\bar{g}})$$

$$r^{AA} = r^{AA}(-\Delta_{\bar{g}}(A) \quad r^{\bar{C}C} = -r^{C\bar{C}} = r^{\bar{C}C}(-\Delta_{\bar{g}}(A))$$

result: no graviton contribution at one-loop

$$\beta_g\big|_{1-\text{loop}} = \beta_{g,\text{YM}}\big|_{1-\text{loop}}$$

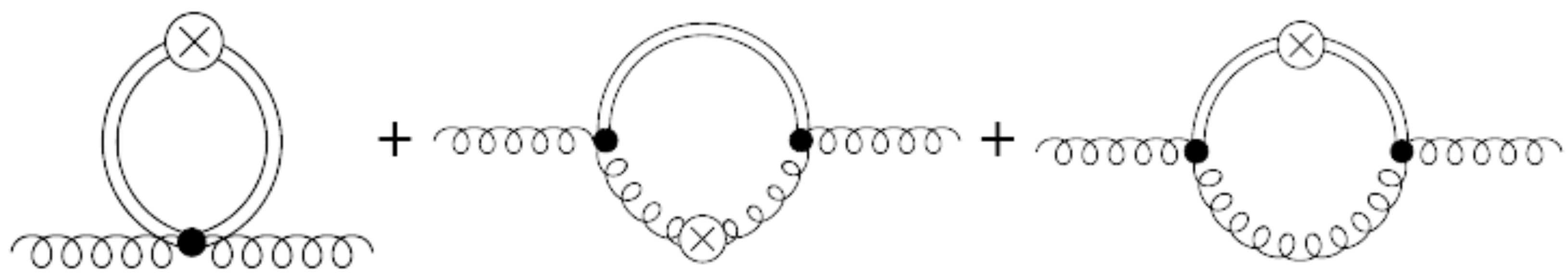
quantum gravity and Yang-Mills

(Folkerts, DL, Pawlowski '11)

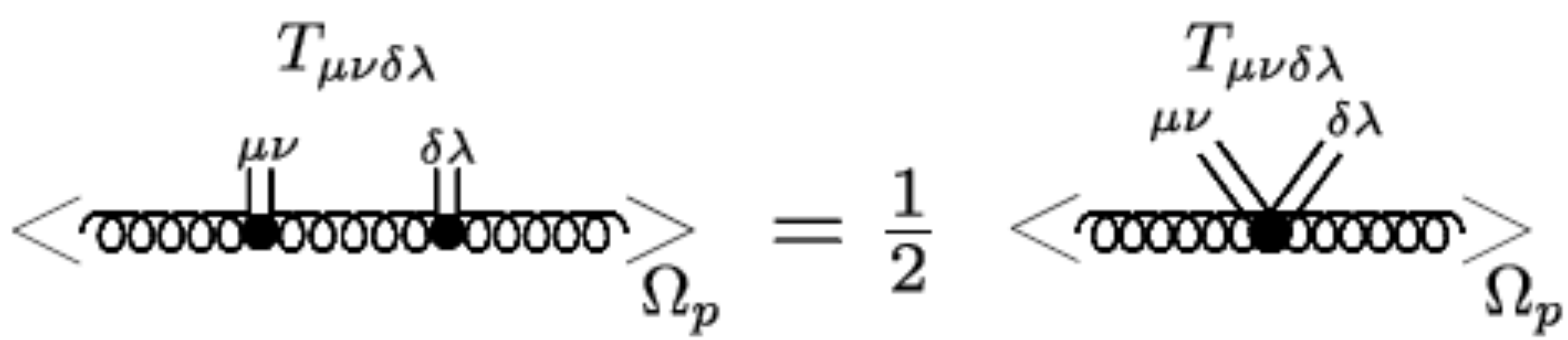
effective action

$$\Gamma_k = \int \sqrt{g}\left(Z_{N,k}\frac{-R+2\bar{\Lambda}_k}{16\pi G_N} + \frac{Z_{A,k}}{4g_s^2} F^a_{\mu\Box}F_a^{\mu\Box}\right)$$

flat background



kinematical identity



result

$$I = \int_0^\infty dx\, \frac{1+\alpha}{1+r_g(x)}\left(1 - \frac{1}{1+r_A(x)}\right) \geq 0$$

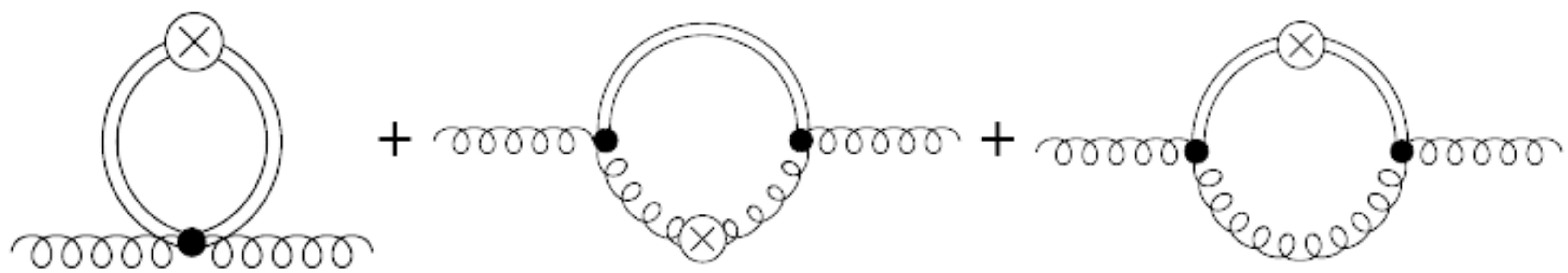
quantum gravity and Yang-Mills

(Folkerts, DL, Pawlowski '11)

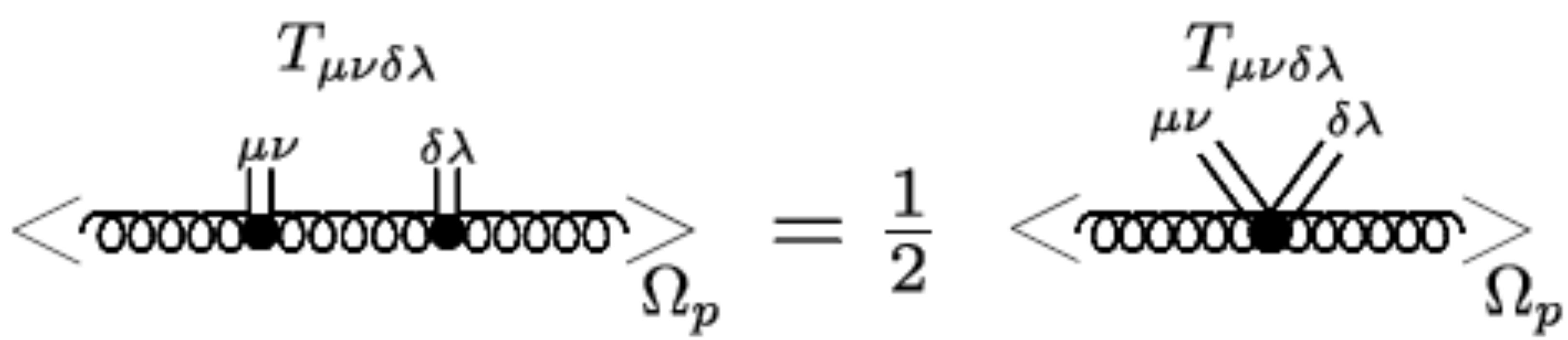
effective action

$$\Gamma_k = \int \sqrt{g} \left(Z_{N,k} \frac{-R + 2\bar{\Lambda}_k}{16\pi G_N} + \frac{Z_{A,k}}{4g_s^2} F^a_{\mu\square} F_a^{\mu\square} \right)$$

flat background



kinematical identity



beyond 1-loop + CC

$$\beta_{\rm YM}\big|_{\rm grav} \leq 0$$

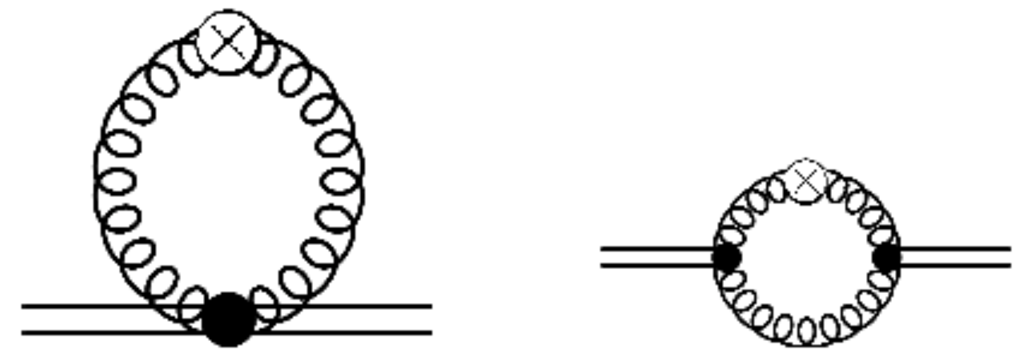
quantum gravity and Yang-Mills

(Folkerts, DL, Pawlowski '11)

effective action

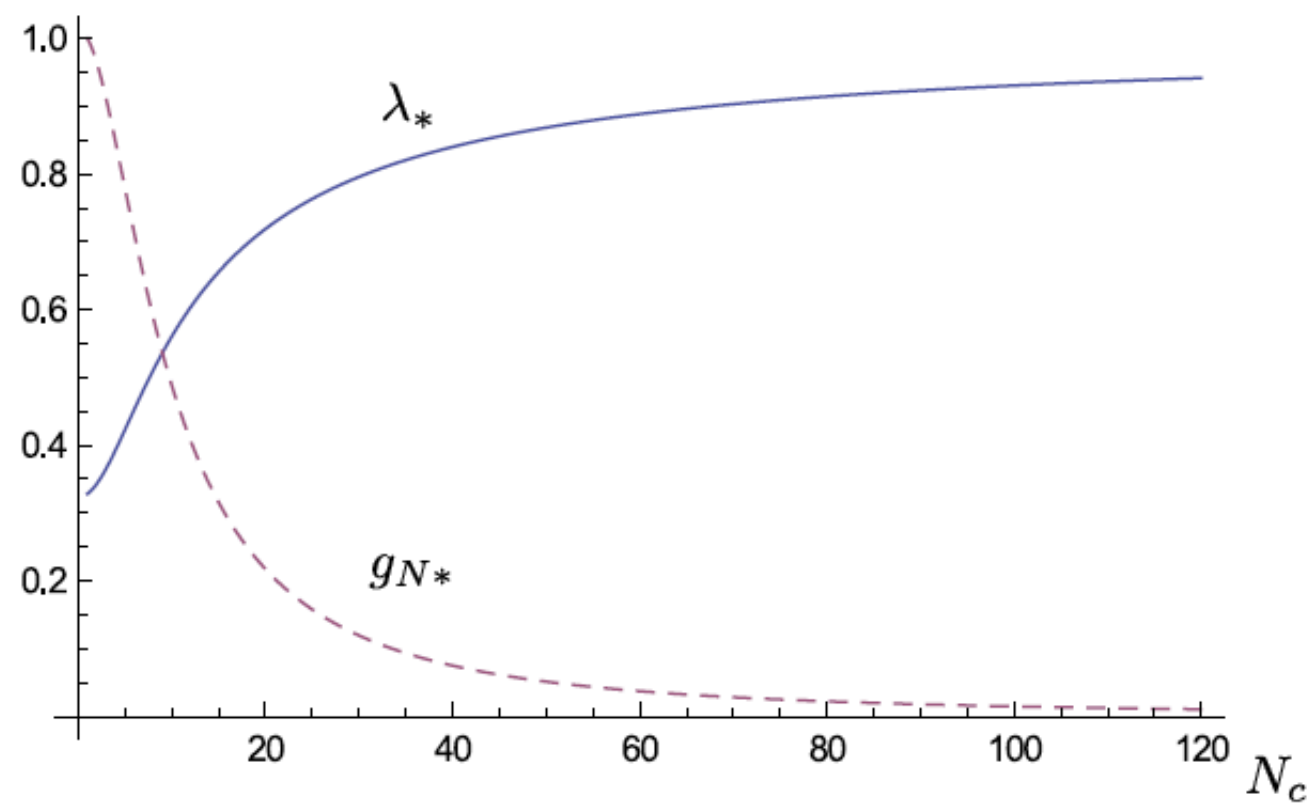
$$\Gamma_k = \int \sqrt{g}\left(Z_{N,k}\frac{-R+2\bar{\Lambda}_k}{16\pi G_N} + \frac{Z_{A,k}}{4g_s^2} F^a_{\mu\square} F_a^{\mu\square}\right)$$

YM contribution to gravity



UV fixed point ...

... in the fully coupled system:
asymptotic freedom and
asymptotic safety persist



further directions

higher derivative gravity

1-loop — Codello, Percacci ('05) Niedermaier ('09)

1-loop and beyond — Benedetti, Machado, Saueressig ('09)

conformal symmetry

Weyl coupling

$$\frac{1}{\sigma}\int d^4x\sqrt{g}\,C_{\mu\nu\rho\tau}\,C^{\mu\nu\rho\tau}$$

asymptotically `free' fixed point $\sigma_* = 0$ — DL, Rahmede ('11)

entails $g_* > 0 \quad \lambda_* \neq 0$

$$\beta_{\rm YM}\big|_{\rm grav} \leq 0$$

conclusions and outlook

quantum theory and gravitation

increasing evidence for asymptotically safe gravity
extended approximations
quantitative and structural insights

particle physics

towards a Standard Model including quantum gravity

challenges

lattice $\leftrightarrow$ RG $\leftrightarrow$ loops $\longleftrightarrow$ strings $\leftrightarrow$ other

cosmology

late-time acceleration, IR fixed points
very early universe, inflation
asymptotically safe cosmology

phenomenology

low-scale quantum gravity: signatures at colliders
black hole physics