Nonperturbative Highlights on Quantum Gravity from Causal Dynamical Triangulations

Zürich,
22 Jun 2011

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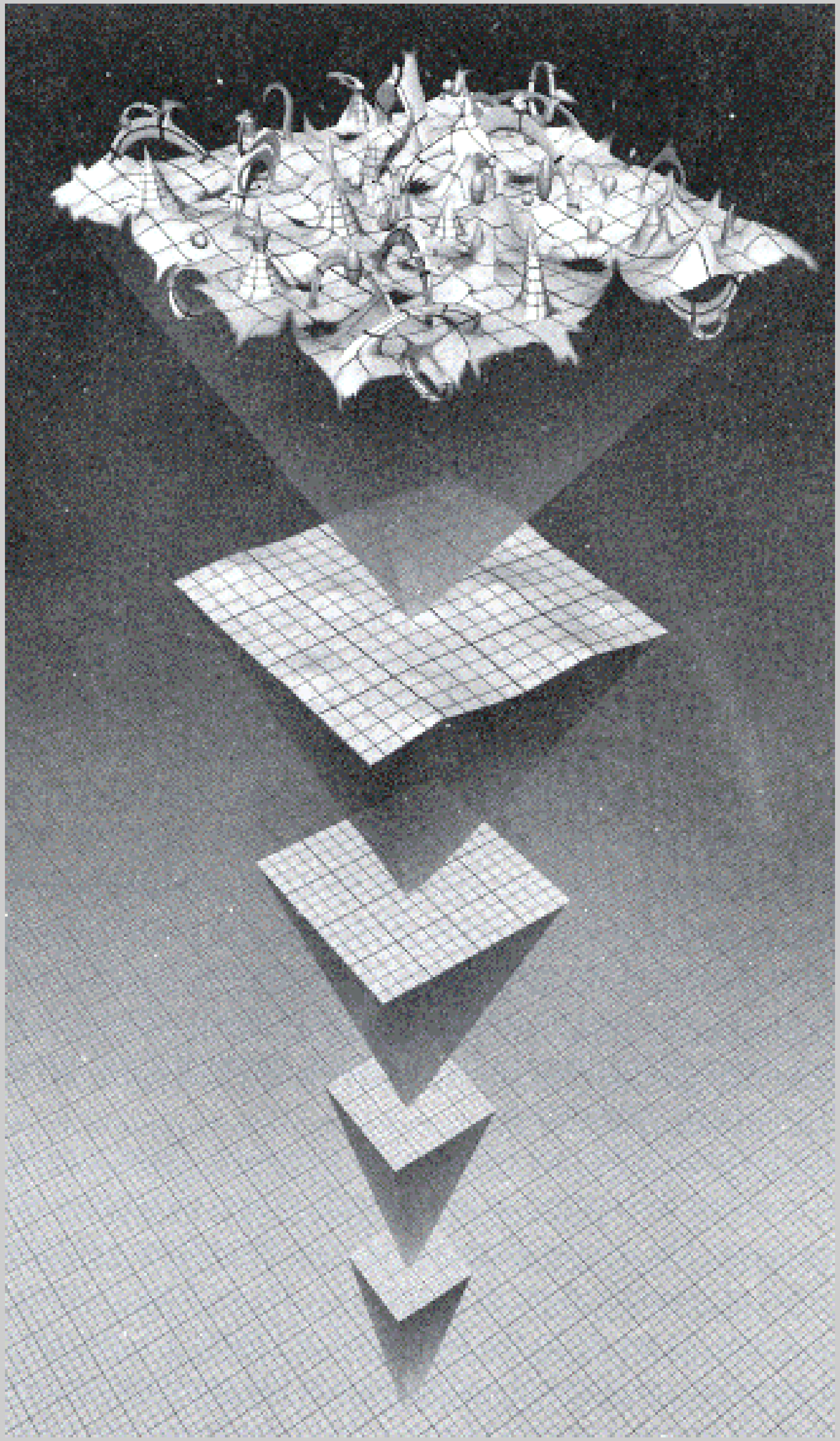
Mission Statement

To understand and describe quantitatively the structure and dynamics of spacetime on all scales.

In other words, to find a theory of Quantum Gravity.

Some unresolved fundamental questions

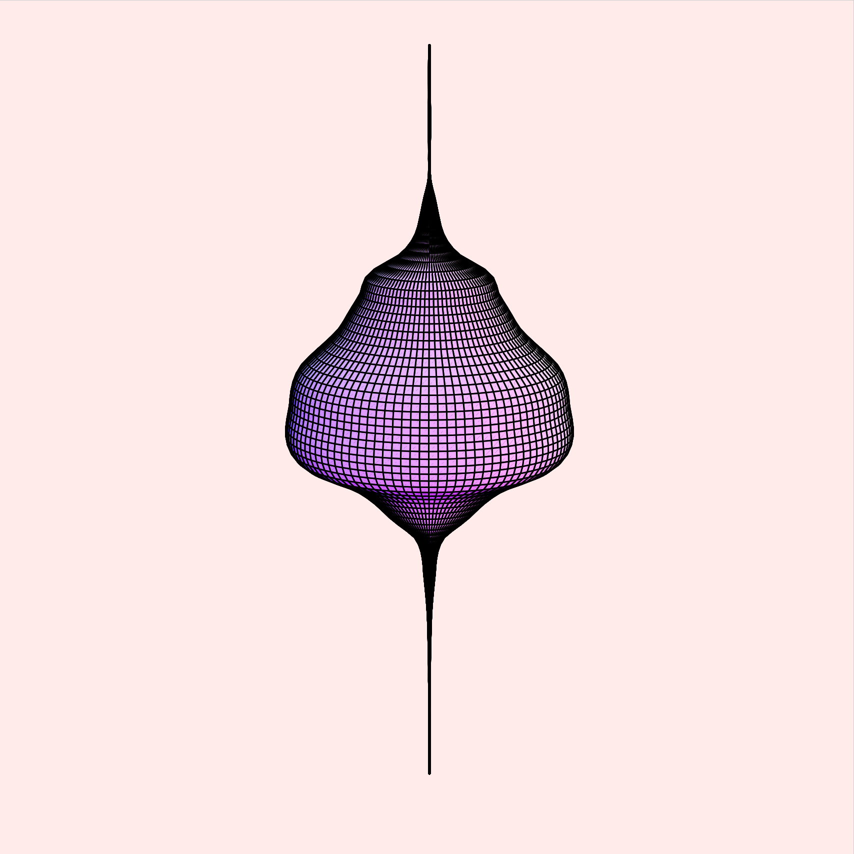
- What are the (quantum) origins of space, time and our universe?
- What is the microstructure of spacetime?
- What are the relevant degrees of freedom at the Planck scale?
- Can their dynamics *explain* the observed large-scale structure of our universe, that of an approximate ~~Minkowski~~ de Sitter space?
- Which aspects of spacetime are dynamical at the Planck scale: geometry? topology? dimensionality?
- Are "space", "time", and "causality" fundamental or emergent?



Quantum Gravity from Causal Dynamical Triangulation (QG from CDT)*

CDT is a no-frills nonperturbative implementation of the gravitational path integral, much in the spirit of lattice quantum field theory, but based on *dynamical* lattices, reflecting the dynamical nature of spacetime geometry.

A key result that puts QG from CDT on the map as a possible quantum theory of gravity is the fact that it can generate dynamically a background geometry with semiclassical properties from pure quantum excitations, in an a priori background-independent formulation. (C)DT has also given us crucial *new* insights into nonperturbative dynamics and pitfalls.



(PRL 93 (2004) 131301, PRD 72 (2005) 064014, PLB 607 (2005) 205)

* my presentation is mainly based on joint work with J. Ambjørn, J. Jurkiewicz, T. Budd, A. Görlich and S. Jordan

recall Jan Ambjørn's talk from last week:

- how "quantum gravity" may exist as a theory despite the perturbative non-renormalizability of gravity
- how we can find evidence for this from investigating the continuum limit of a nonperturbative lattice formulation in terms of causal dynamical triangulations
- how we can analyze the scaling behaviour of (bare) couplings and try to relate them to a RG treatment (c.f. talks by Martin Reuter and Daniel Litim)

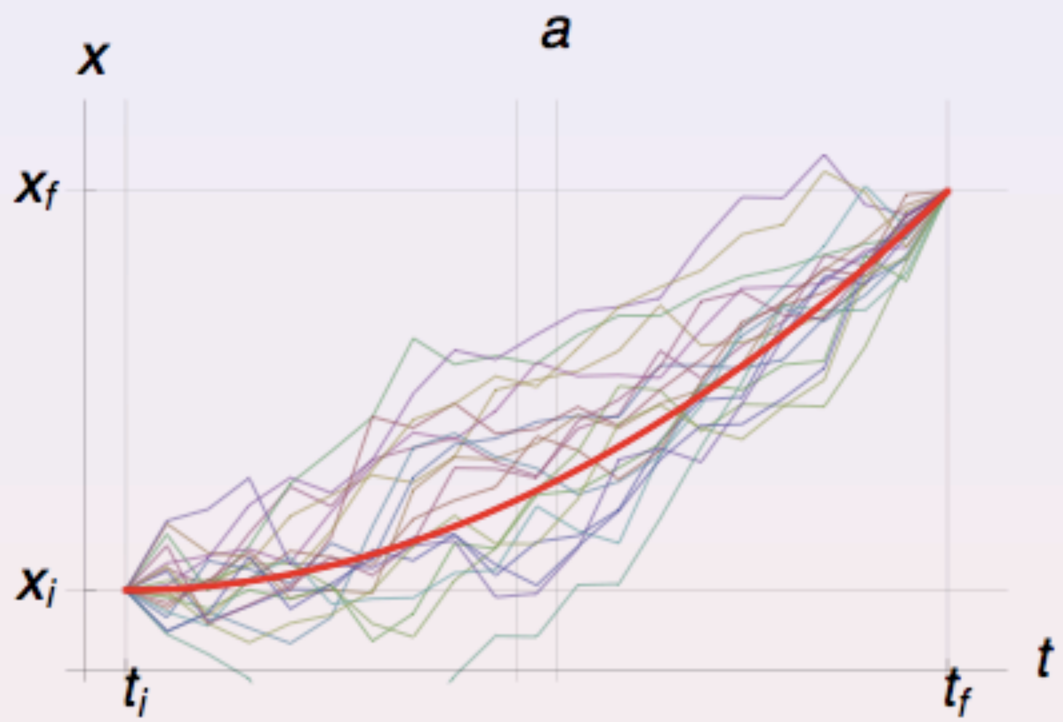
today's talk:

- recap some of the essentials
- nature of the dynamically generated quantum universe
- the issue of "observables"

$$S^{EH} = \frac{1}{G_N} \int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g] - 2\Lambda)$$

Basic tool: the good old path integral

Textbook example: the nonrelativistic particle (h.o.) in one dimension



$$\begin{aligned} x(t) &= \langle x(t)\rangle + y(t) \\ \langle|y|\rangle &\propto \sqrt{\hbar/m\omega} \end{aligned}$$

In QG we want $\langle x(t)\rangle$

$$\langle|y|\rangle \propto \sqrt{\hbar G}$$

Quantum superposition principle: the transition amplitude from $x_i(t_i)$ to $x_f(t_f)$ is given as a weighted sum over amplitudes exp $iS[x(t)]$ of all possible trajectories, where $S[x(t)]$ is the classical action of the path.

(here, time is discretized in steps of length *a*, and the trajectories are piecewise linear)

The same superposition principle, applied to gravity

"Sum over histories"

a.k.a. gravitational path integral

cosmol. const.

$$Z(G_N, \Lambda) = \int \mathcal{D}g \; e^{iS^{E\text{-}H}_{G_N,\Lambda}[g]}$$

Newton const.

spacetime geom.s $g \in \mathcal{G}$

Each "path" is now a four-dimensional, curved spacetime geometry g, which can be thought of as a three-dimensional, spatial geometry developing in time. The weight associated with each g is given by the corresponding Einstein-Hilbert action $S^{EH}[g]$,

$$S^{\mathrm{EH}} = \frac{1}{G_{\mathrm{N}}} \int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g] - 2\Lambda)$$

How can we make $Z(G_N, \Lambda)$ into a meaningful, well-defined quantity?

Regularizing the path integral via CDT

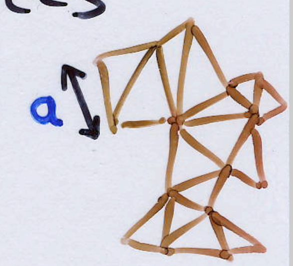
Sum over histories $Z(G_N, \Lambda)$:

$$\int \mathcal{D}g \; e^{\,i S^{EH}[g]} \;\underset{CDT}{:=}\; \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\, e^{\,i S^{Regge}[T]}$$

spacetime geom.s $g\in\mathcal{G}$

$C(T)$ ← $|Aut(T)|$

curved spacetime geometry g $\xmapsto[\text{CDT regular.}]{}$ gluing T of N simplices (piecewise flat manifold)

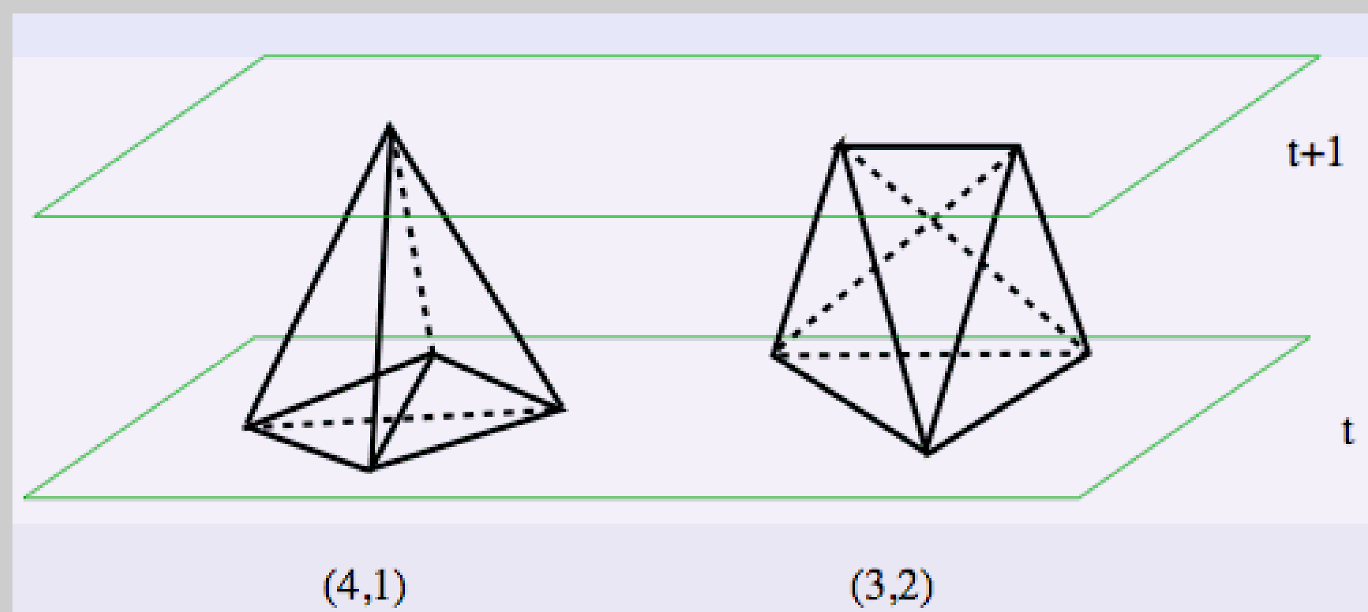


'democratic', regularized sum over piecewise flat spacetimes, *doesn't need coordinates* (Regge); continuum limit required to obtain universal results independent of the regularization

Elementary four-simplex, building block for a causal dynamical triangulation:

a ~ edge length; diffeomorphism-invariant UV regulator

Micro-causality is essential! This does not work in Euclidean signature - get only branched polymers (~mid-90s).



CDT's proper-time slicing

Wick rotation and analogy with statistical mechanics

- each regularized Lorentzian geometry T allows for a rotation to a unique regularized Euclidean geometry T_{eu}, such that the Feynman amplitude of a path is turned into a Boltzmann weight, as in statistical mechanics

$$e^{iS^{\text{Regge}}(T)} \longrightarrow e^{-S_{\text{eu}}^{\text{Regge}}(T_{\text{eu}})}$$

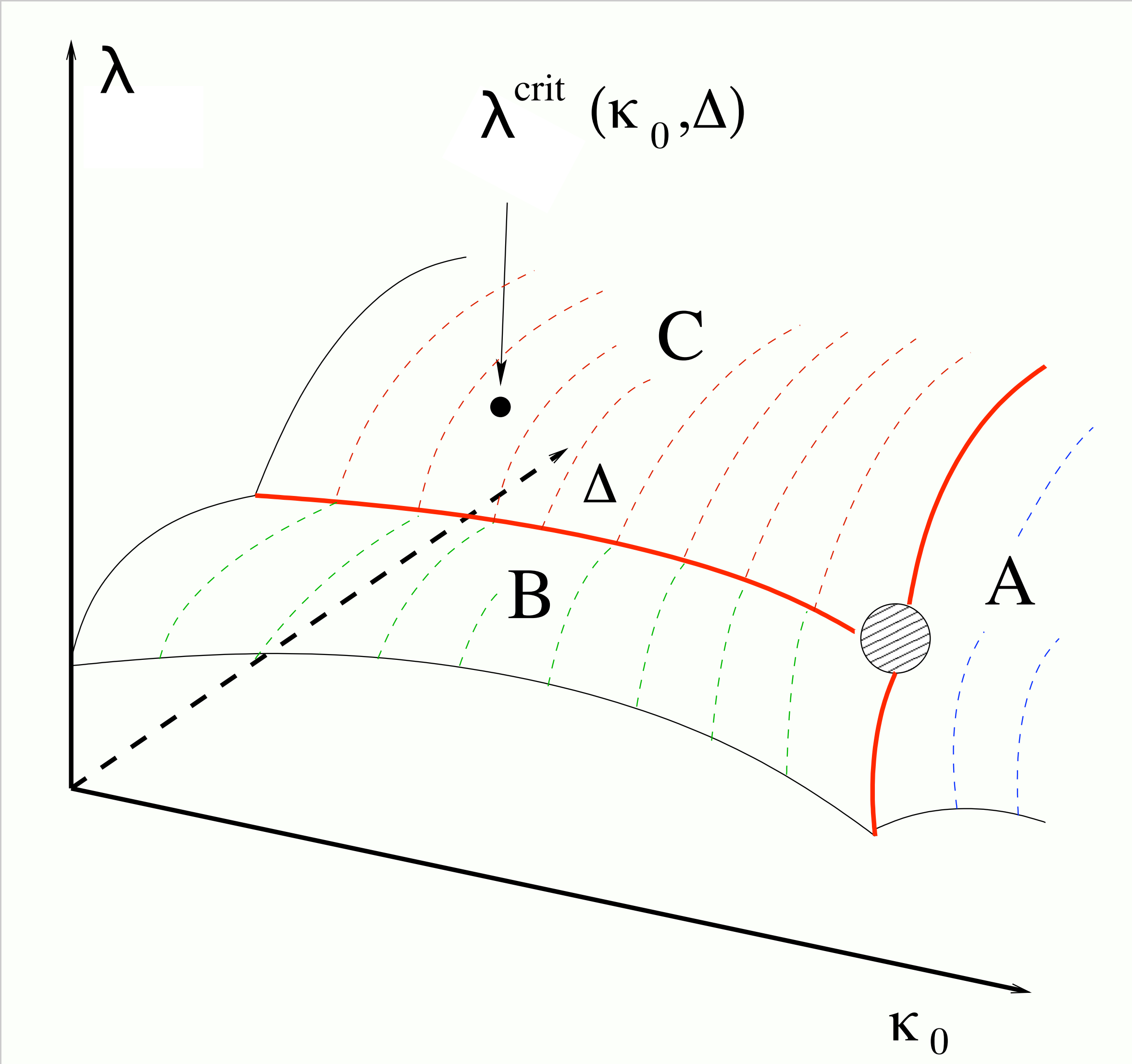
- this turns the quantum amplitude Z into a partition function Z_{eu} and allows us to use powerful numerical methods from statistical mechanics, like Monte Carlo simulations

- a ‘classical trajectory’ is an average over quantum trajectories in the statistical ensemble of trajectories (the Euclideanized ‘sum over histories’)

- taking the continuum limit of this regularized theory means studying the critical behaviour of the underlying statistical theory

- performing an ‘inverse Wick rotation’ on quantities computed in the continuum limit is in general nontrivial

The phase diagram of Causal Dynamical Triangulations



$$S_{eu}^{\mathrm{Regge}} = -\kappa_0 N_2 + N_4(c\kappa_0 + \lambda) + \\ +\Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

λ ~ cosmological constant

κ_0 ~ $1/G_N$ inverse Newton's constant

Δ ~ relative time/space scaling

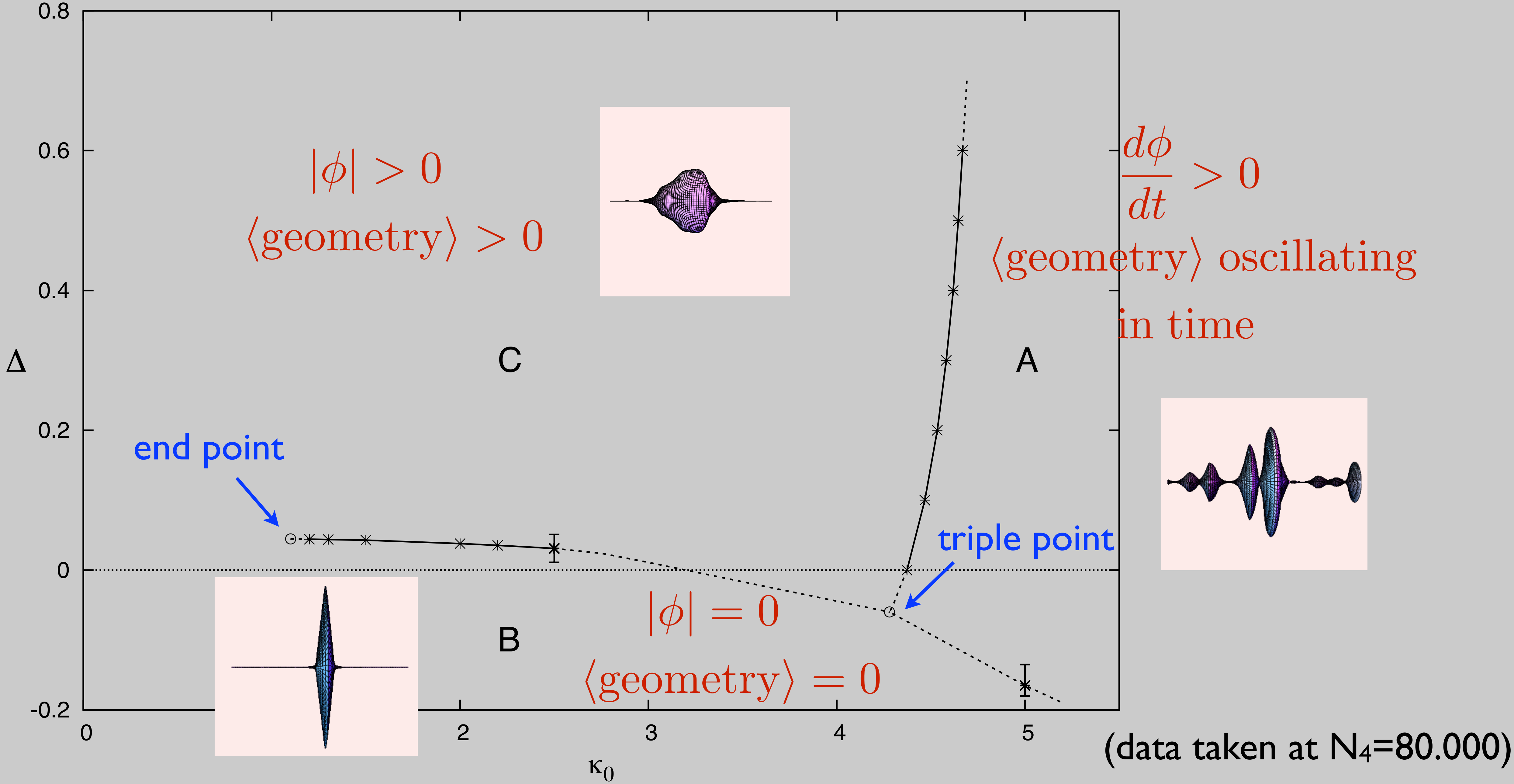
c ~ numerical constant, >0

N_i ~ # of triangular building blocks of dimension i

The partition function is defined for $\lambda > \lambda^{crit}(\kappa_0,\Delta)$;
approaching the critical surface = taking infinite-volume limit.
red lines ~ phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;
J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413;
definite phase transition analysis: work with S. Jordan, to appear)

The phase diagram of CDT in the κ_0-Δ plane



(data taken at N_4=80.000)

Similar to a Lifshitz phase diagram (cf. P. Hořava's anisotropic gravities), where Φ is an order parameter of a mean field Lifshitz theory, with free energy

$$F = a_2\phi^2 + a_4\phi^4 + \cdots + c_2(\partial_x\phi)^2 + d_2(\partial_t\phi)^2 + \dots$$

Conjecture: this phase structure is generic for Lorentzian higher-dim. geometries.

The dynamical emergence of spacetime as we know it

CDT is the so far only candidate theory of nonperturbative quantum gravity where a classical extended geometry is generated from nothing but Planck-scale quantum excitations.

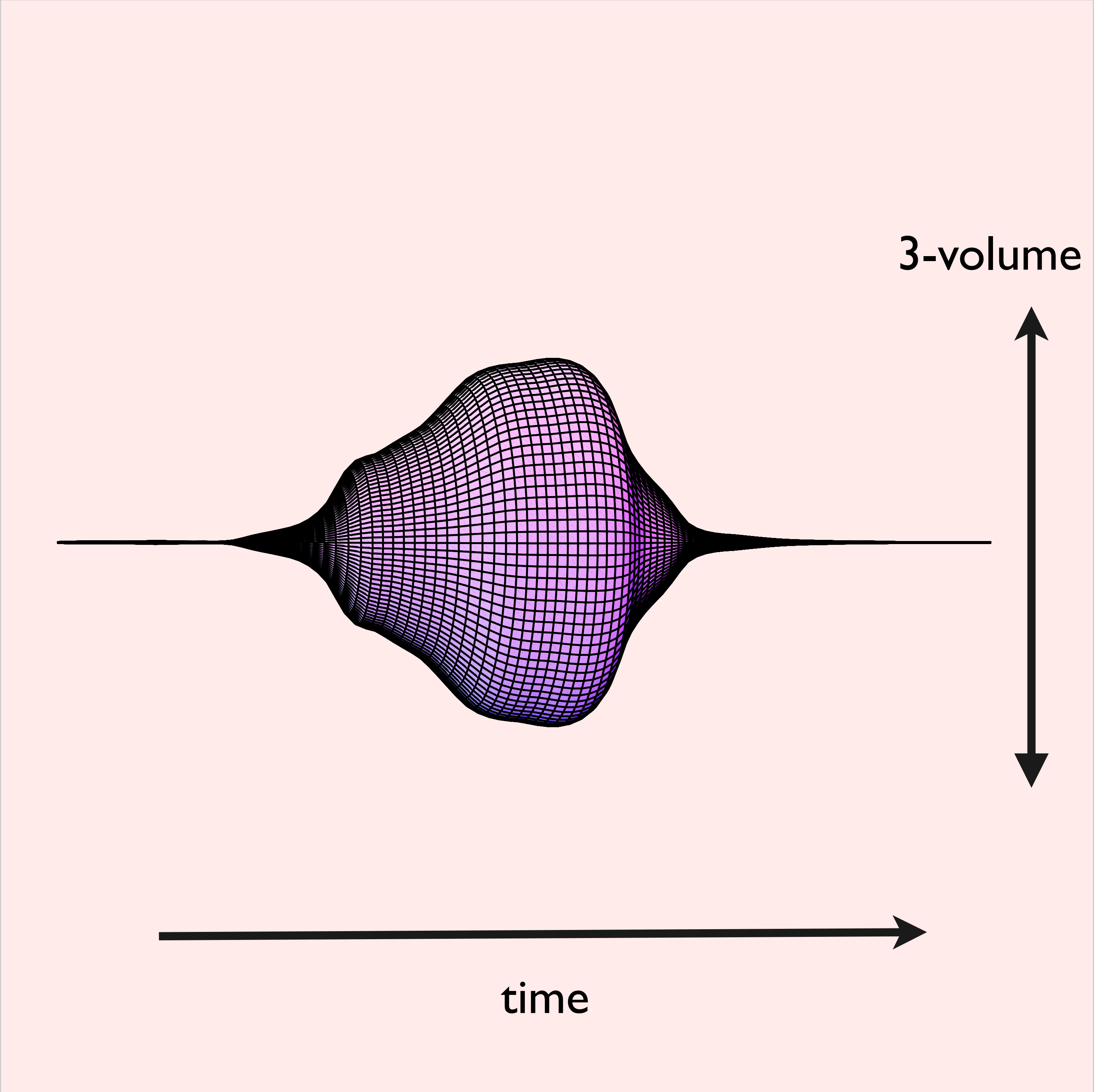
This happens by a nonperturbative, entropic* mechanism:

Magically, the many microscopic building blocks in the quantum superposition arrange themselves into an extended quantum spacetime whose macroscopic shape is that of a well known cosmology.

When, from all the gravitational degrees of freedom present, we monitor only the spatial three-volume $\langle V_3(t) \rangle$ of the universe as a function of proper time t, we find a distinct "volume profile".

*entropy = number of microscopic geometric realizations of a given value of the action

Dynamically generated four-dimensional quantum universe, obtained from a path integral over causal spacetimes



This is a Monte Carlo “snapshot” - still need to average to obtain the *expectation value* of the volume profile.

Our “self-organized quantum spacetime” has the shape of a de Sitter universe!

(A solution to the classical Einstein equations in the presence of “dark energy” - a.k.a. a cosmological constant Λ.)

Classical de Sitter space has

$$V_3(\tau) = 2\pi^2 \left(c \ \cosh \frac{\tau}{c}\right)^3$$ constant ,

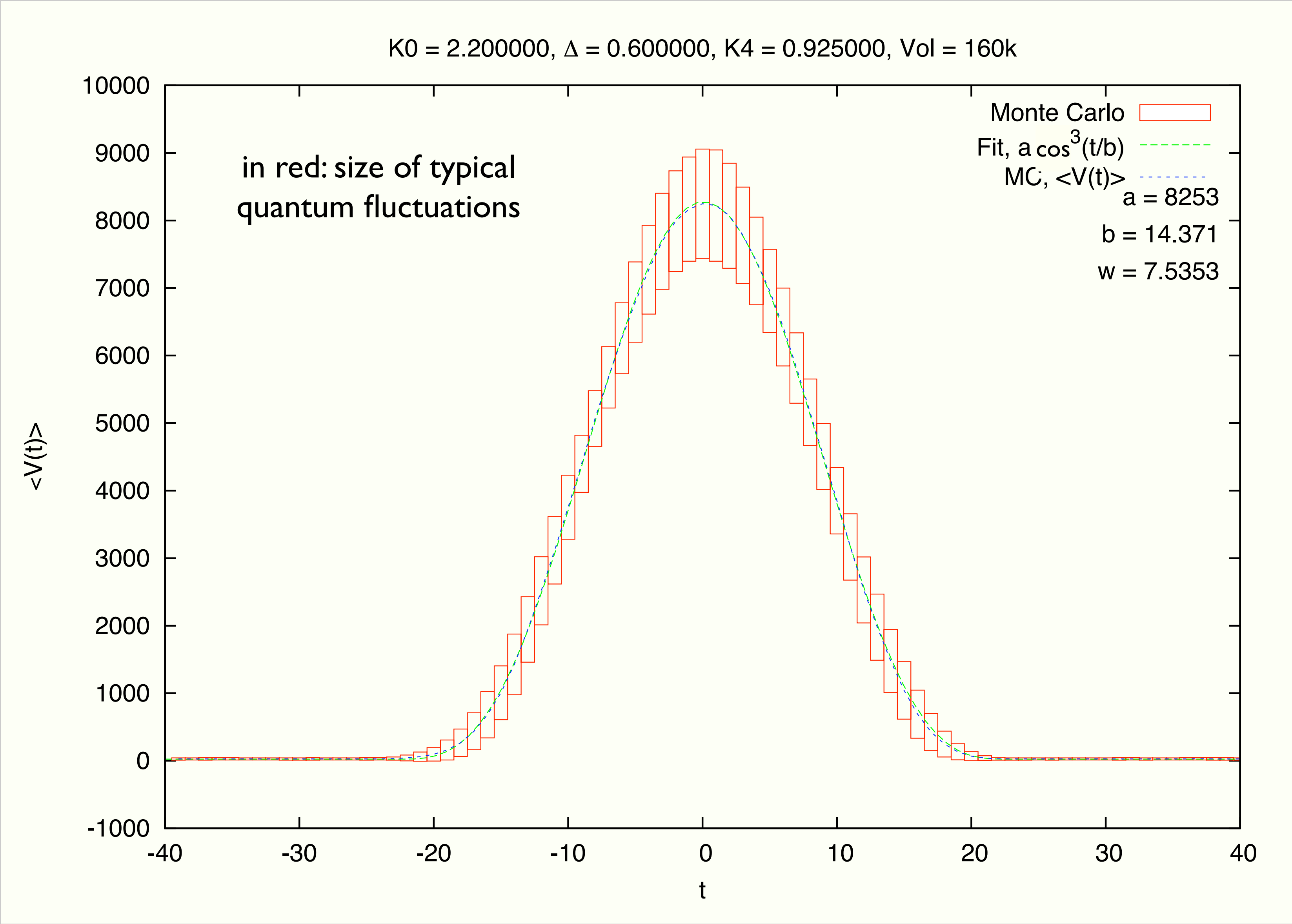
giving rise to an exponentially expanding universe, $V_3 \sim e^{c'\tau}$, for $\tau > 0$.

- We measure this for Euclidean time $t = i\tau$

→ a very nontrivial test of the classical limit; strong flavour of condensed matter phenomena

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (w/ J. Gizbert-Studnicki, T. Trzesniewski)

What is the concrete evidence for de Sitter space?



The volume profile <V3(t)>, as function of Euclidean proper time t=iτ, perfectly matches that of a Euclidean *de Sitter space*, with scale factor $a(t)^2$ given by

$$ds^2 = dt^2 + a(t)^2 d\Omega^2_{(3)} = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega^2_{(3)} \longleftarrow \text{volume el. } S^3$$

N.B.: we are *not* doing quantum cosmology

Remarkably, having started from the Wick-rotated path integral,

$$\int \mathcal{D}[g]\, \mathrm{e}^{-S_{\mathrm{eu}}}, \quad S_{\mathrm{eu}} = \int dt\, \left(-\frac{\dot{V}_3^2}{V_3} - V_3^{1/3} + \dots\right)$$

by integrating out everything *but* the global conformal mode $V_3(t)$, we obtain an effective dynamics for $V_3(t)$, given by

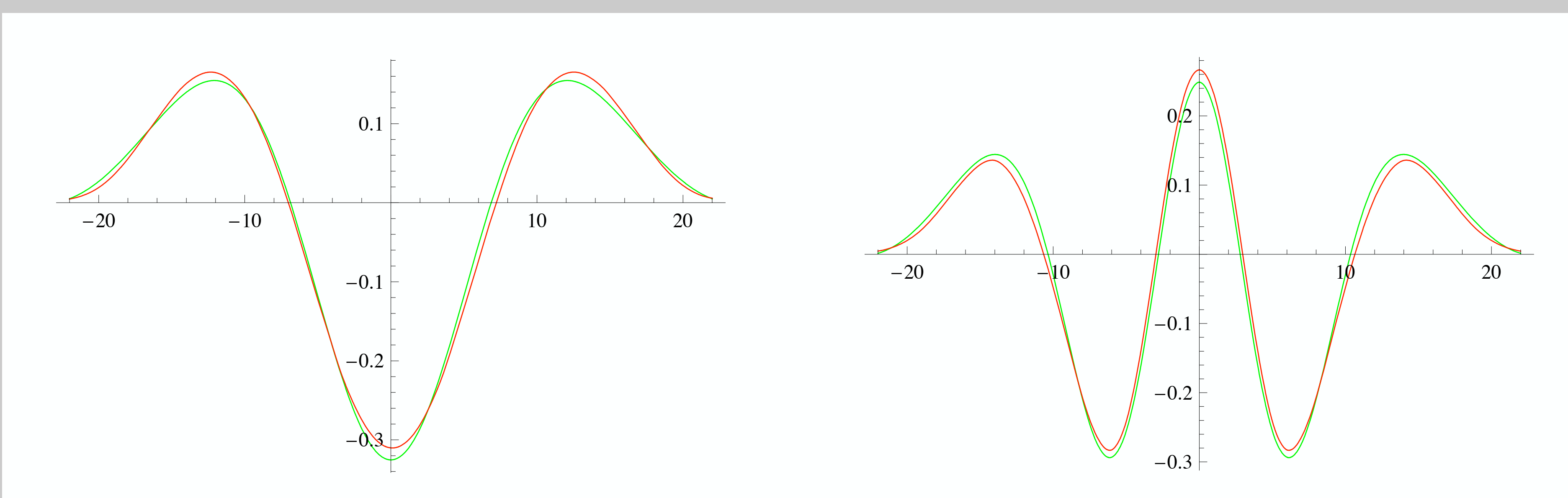
$$\mathrm{e}^{-S_{\mathrm{eu}}^{\mathit{eff}}}, \quad S_{\mathrm{eu}}^{\mathit{eff}} = \int dt\, \left(+\frac{\dot{V}_3^2}{V_3} + V_3^{1/3} + \dots\right)$$

This is entirely due to nonperturbative, entropic effects (the PI measure).

In addition, expanding the minisuperspace action around the de Sitter solution,

$$S_{\mathrm{eu}}(V_3) = S(V_3^{\mathrm{dS}}) + \kappa \int dt\; \delta V_3(t) \hat{H} \delta V_3(t)$$

the eigenmodes of $\hat{H}$ match well with those extracted from the simulations:



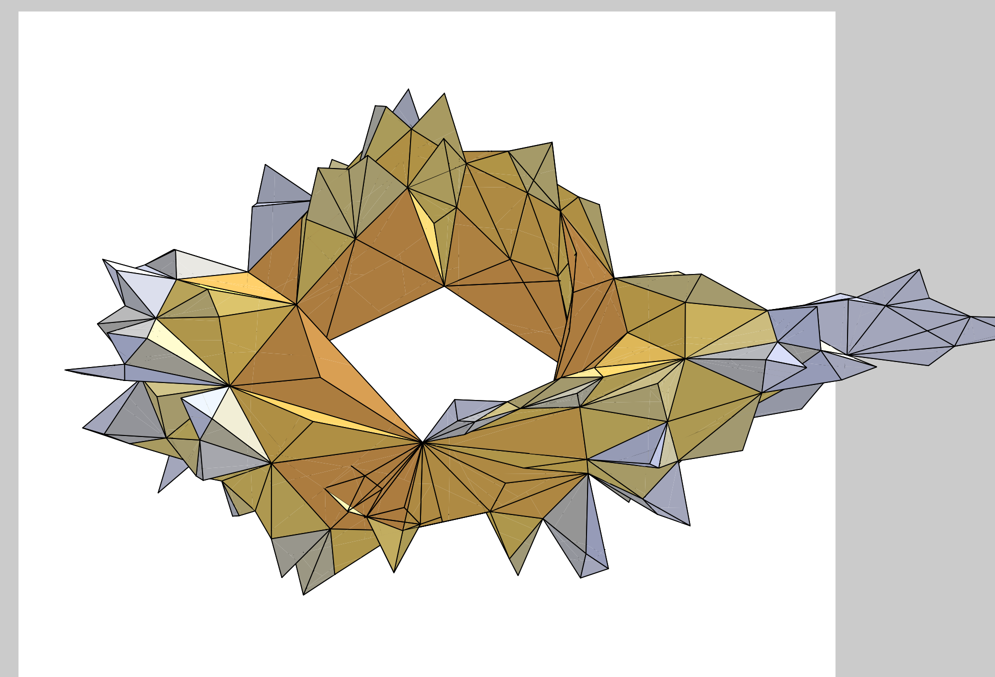
(N.B.: no further fitting necessary)

Going beyond the global conformal (“Friedmann”) mode

Interesting toy model with global metric d.o.f.: studying the effective action for a CDT path integral for 2+1 gravity on spatial tori

$$ds^2 = \mathrm{e}^{2\phi(x)} \bar{g}_{\mu\nu} dx^\mu dx^\nu \qquad \bar{g}_{\mu\nu} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}$$

Try to match to an extended effective action with kinetic terms



$$(1/2 - \lambda) \frac{\dot{V}_2^2}{V_2} + \alpha[V_2, \cdot] \frac{\dot{\tau}_1^2 + \dot{\tau}_2^2}{\tau_2^2}$$

where λ appears in the Wheeler-de Witt metric

$$\mathcal{G}^{\mu\nu\rho\sigma}_{(\lambda)} = \frac{\sqrt{g}}{4} \left(\frac{1}{2} (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) - \lambda g^{\mu\nu} g^{\rho\sigma} \right)$$

(T. Budd, RL, to appear)

Getting a handle on Planckian physics

(or, another nonperturbative surprise!)

A diffusion process is sensitive to the dimension of the medium where the "spreading" takes place. We have implemented such a process on the quantum superposition of spacetimes. By measuring a suitable "observable"*, we have extracted the spectral dimension D_s of the quantum spacetime.

Quite remarkably, we find that it depends on the length scale probed: D_s changes smoothly from 4 on large scales to ~2 on short scales.

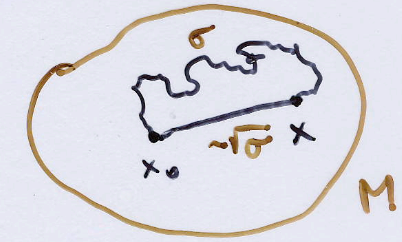
(J. Ambjorn, J. Jurkiewicz, RL, PRL 95 (2004) 171301)

* average return probability

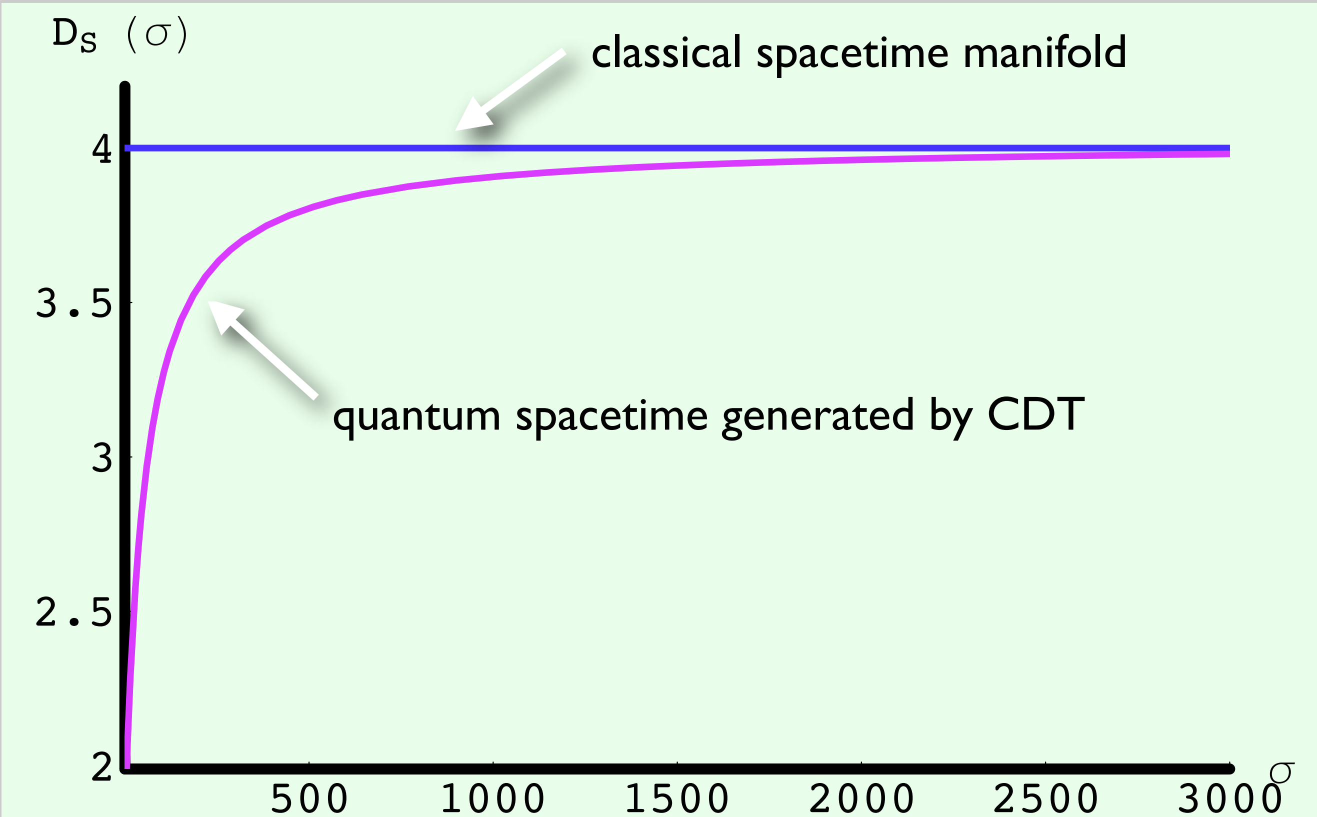
$$\mathcal{R}_V(\sigma) := \frac{1}{V(M)} \int_M d^dx \; P(x,x;\sigma) \sim \frac{1}{\sigma^{D_s/2}}$$

diffusion time

sol.n to heat eqn.



$D_S(\sigma)$ probes properties of the geometry on linear length scales ~ $\sigma^{1/2}$:



on short scales, our "ground state of geometry" is definitely *not* a classical manifold.

Instead, we find evidence for the presence of a random fractal structure.

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PLB 690 (2010) 420)

Intriguingly, the same short-scale *"dynamical dimensional reduction"* has since been found in a couple of disparate, (also quantum field-theoretic) approaches:

- nonperturbative renormalization group flow analysis (M. Reuter, O. Lauscher, JHEP 0510:050, 2005)
- nonrelativistic "Lifshitz quantum gravity" (P. Hořava, PRL 102 (2009) 161301)

3d: relating the curve $D_S(\sigma)$ of CDT (D. Benedetti, J. Henson, PRD 80 (2009) 124036) to dispersion relations of suitable differential operators on 3d flat space (T. Sotiriou, M. Visser, S. Weinfurtner, arXiv:1105.5646)

The quest for observables

- $D_S(\sigma)$ is the "dimension felt by a scalar test particle" - not really a true observable near the Planck scale in the sense of phenomenological implications

- still useful and "covariantly defined" (meaningful in the sum over geometries and after averaging over the starting point of the diffusion process) and can be computed; characteristically nonlocal

- can play an important role in discriminating between different candidate theories of quantum gravity, akin to the computation of "black hole entropy" S=A/4, but arguably one that probes the nonperturbative structure, not just semiclassical properties

- various computations of $D_S(\sigma)$ on short scales for nonclassical geometries: noncommutative geometry/κ-Minkowski space (D. Benedetti, PRL 102 (2009) 111303), three-dimensional CDT (D. Benedetti, J. Henson, PRD 80 (2009) 124036), from area operator in loop quantum gravity (L. Modesto, CQG 26 (2009) 242002), possible relation with strong-coupling limit of WdW equation (S. Carlip, arXiv: 1009.1136), modelling from dispersion relations on flat spaces (T. Sotiriou, M. Visser, S. Weinfurtner, arXiv: 1105.6098), modelling by multifractal spacetimes (G. Calcagni, arXiv: 1106.0295)

Causal Dynamical Triangulations - Summary & Outlook

CDT is a path integral formulation of gravity, which incorporates the dynamical and causal nature of geometry. It depends on a minimal number of assumptions and ingredients and has few free parameters. Its associated toolbox provides us with an "experimental lab" - a nonperturbative calculational handle on (near-)Planckian physics (c.f. lattice QCD).

- We have begun to make quantitative statements/predictions.
- We can in principle also test nonperturbative predictions from other fundamental theories containing gravity.
- Many nonperturbative lessons learned so far: relevance of metric signature in the path integral; tendency of geometric superpositions to degenerate; dynamical nature of "dimension"; emergence of classicality from quantum dynamics; crucial role of "entropy"; cure of the conformal divergence; role of "time" and "causality" as fundamental, and not emergent quantities
- Hopefully we are seeing glimpses of an essentially unique quantum theory of gravity; is your approach seeing the same? - Watch this space!

Where to learn more

- CDT light: "The self-organizing quantum universe", by J. Ambjørn, J. Jurkiewicz, RL (Scientific American, July 2008)
- A nontechnical review in Contemp. Phys. 47 (2006) [arxiv: hep-th/0509010]
- recent reviews/lecture notes: arXiv 0906.3947, 1004.0352, 1007.2560, Physics Report to appear
- links to both review and popular science material can be found on my homepage http://www.phys.uu.nl/~loll

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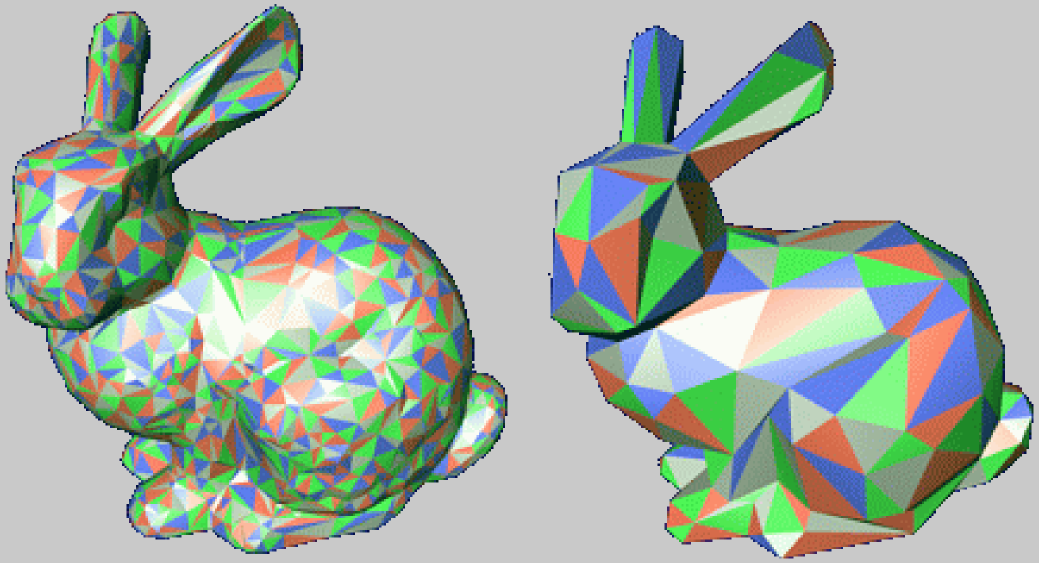
The End

$$S^{\mathrm{EH}} = \frac{1}{G_{\mathrm{N}}} \int d^4x \sqrt{-\det g}\left(R[g, \partial g, \partial^2 g] - 2\Lambda\right)$$

Regularizing gravity by "dynamical triangulations"

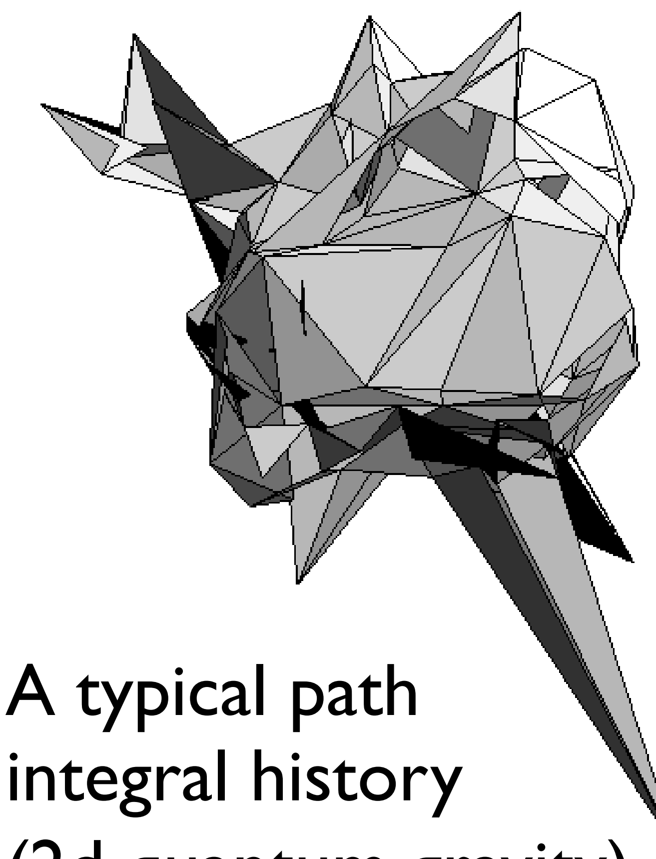


classical problem: approximate a curved surface through triangulation

triangulation = regularization

N.B.: no coordinates!

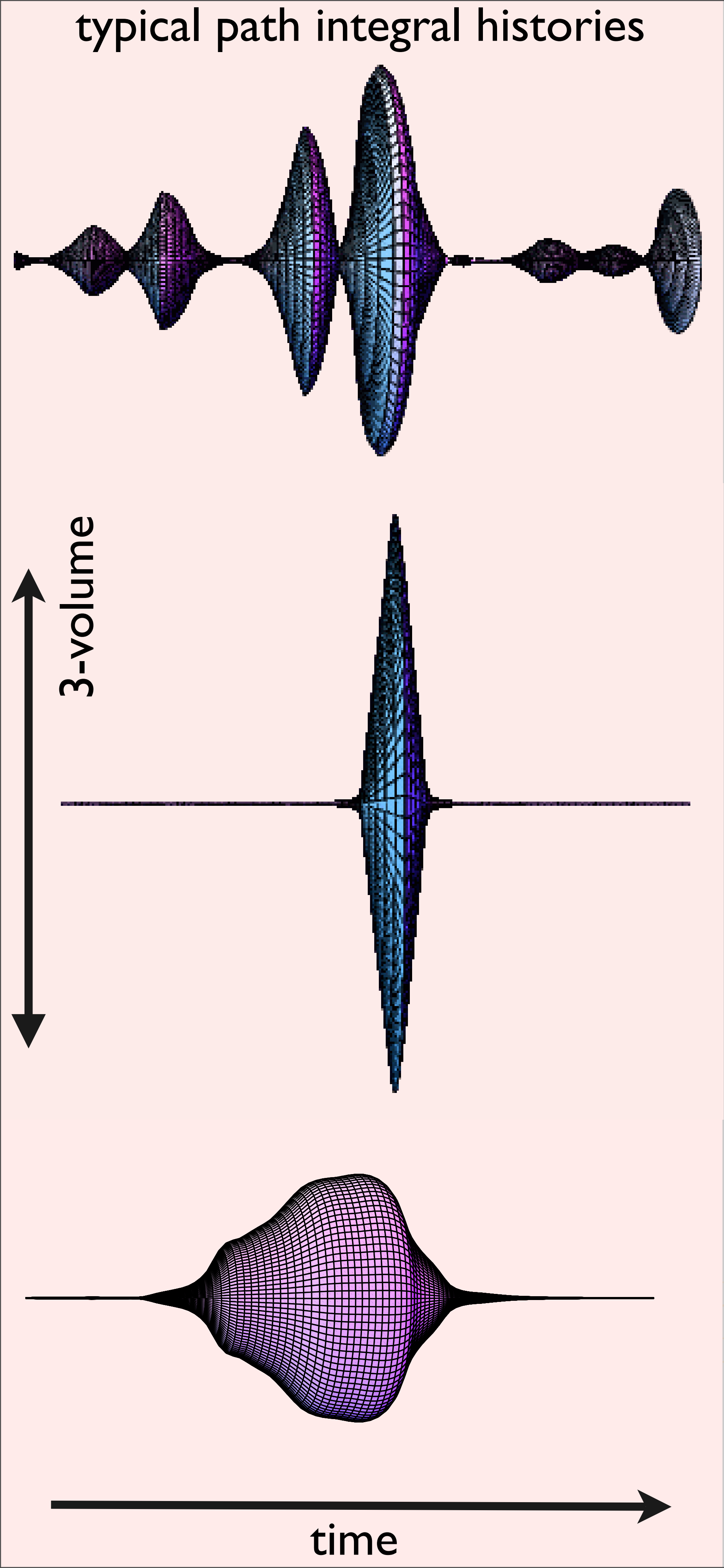
(T. Regge's 1961 idea of "GR without coordinates")



A typical path integral history (2d quantum gravity)

quantum theory: approximate the space of all curved geometries by a space of triangulations and subsequently integrate over this space(*)

(*) in d=4 by Monte Carlo simulations (for CDT models in lower dimensions also have exact stat. mech. solutions methods, see e.g. D. Benedetti, F. Zamponi, R.L., PRD 76 (2007) 104022; in d=2, the problem becomes purely combinatorial)



- Phase A: (sufficiently large $\kappa_0=1/G_N$) inhomogeneous in time, a Lorentzian version of conformal factor dominance, individual "universes" remain small

- Phase B: (small κ_0, small Δ) phase of "no geometry" - collapse along the time direction, but also space is "crumpled", without linear extension (Hausdorff dimension $d_H \approx \infty$!)

- Phase C: (small κ_0, large Δ) *physical phase of extended geometry!* - canonical scaling in the large, $\langle T \rangle \propto N_4^{1/4}$, $\langle V_3 \rangle \propto N_4^{3/4}$, $d_H \approx 4$

Nonperturbative semiclassicality

The semiclassical limit of CDT quantum gravity which gives rise to the de Sitter universe is truly nonperturbative: it is located in a region of coupling constant space where the entropy of the geometric configurations is *as* important as the contribution from the exponential of the action.

This is similar to what happens at a Kosterlitz-Thouless transition in the XY model of 2D spins on a two-dimensional lattice.

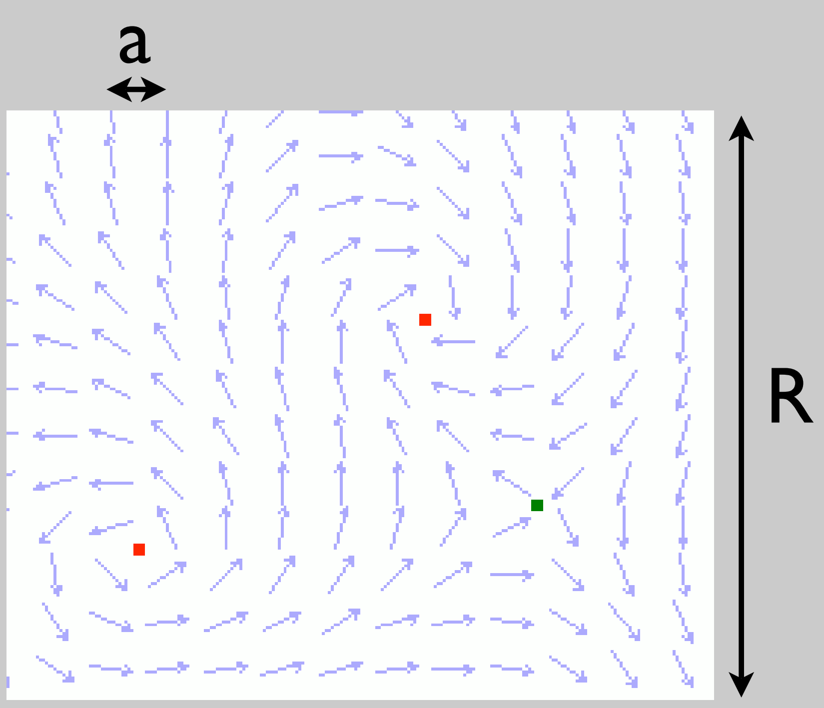
$$Z = \mathrm{e}^{-F/k_B T} = \sum_{\text{spin configs}} \mathrm{e}^{-E[spin]/k_B T} \simeq \left(\frac{R}{a}\right)^2 \mathrm{e}^{-[\kappa \ln(R/a)]/k_B T}$$

a single vortex has E = κ ln(R/a)

$$F = E - ST = (\kappa - 2k_B T)\ln(R/a)$$

F=0 is far away from the naive weak coupling limit.

$$S^{\mathrm{EH}} = \frac{1}{G_N}\int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g]$$



(J. Ambjørn, A. Görlich, J. Jurkiewicz, J. Gizbert-Studnicki, T. Trzesniewski, RL, NPB 849 (2011) 144)

CDT - fresh from the press

- analyzing the phase structure of QG from CDT, establishing order of critical transition lines/points (J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413, and ongoing work)

- establishing further aspects of the correct semiclassical limit of the nonperturbative CDT formulation (J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, J. Gizbert-Studnicki, T. Trzesniewski, to appear in Nucl. Phys. B),

- studying matter coupling (influence of matter on geometry, extract Newton's law, the early universe coupled to a scalar field) - we have recently been able to quantify the expected effect of a single pointlike mass on the volume profile of CDT's de Sitter background (I. Khavkine, P. Reska, RL, Class. Quant. Grav. 27 (2010) 185025)

- investigating short-scale quantum structure of spacetime, and coming up with an effective description of Planckian dynamics - we have recently analyzed the fractality of the quantum geometry by using geodesic shell decompositions (J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PLB 690 (2010) 420)