GROUP FIELD THEORY:
A BRIEF REVIEW OF RECENT DEVELOPMENTS

Daniele Oriti

Max Planck Institute for Gravitational Physics (Albert Einstein Institute)

Quantum Theory and Gravitation Conference
ETH - Zurich, Switzerland
16/06/2011

Group Field Theory formalism

general introductions:
L. Freidel, [arXiv: hep-th/0505016]
D. Oriti, [arXiv: gr-qc/0512103]
D. Oriti, [arXiv: gr-qc/0607032].
D. Oriti, [arXiv: 0912.2441 [hep-th]]
V. Rivasseau, [arXiv:1103.1900 [gr-qc]]
D. Oriti, in *Foundations of space and time*, G. Ellis, J. Marugan, A. Weltman (eds.), Cambridge University Press (2011)

work by:
Baratin, Ben Geloun, Bonzom, Boulatov, Carrozza, De Pietri, Fairbairn, Freidel, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Ooguri, Oriti, Perez, Raasakka, Reisenberger, Riello, Rivasseau, Rovelli, Ryan, Sindoni, Smerlak, Tanasa, Vitale, ...

..... growing area of research ....

Plan of the talk

- GFT: roots
- GFT: basic idea and main open issues/directions
- GFT models: 3d gravity - 4d constructions
- a brief survey of recent and current developments
- conclusions

GFT: roots

Loop Quantum Gravity meets Discrete (Simplicial) Quantum Gravity ......
(and non-commutative geometry).........

FIRST ROOT: MATRIX MODELS

(quantum) 2d spacetime as a (statistical) superposition of discrete surfaces

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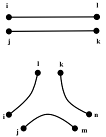
- microscopic dynamics:

$$S(M,\lambda) = \frac{1}{2} M^i{}_j M^j{}_i - \frac{\lambda}{\sqrt{N}} M^i{}_j M^j{}_k M^k{}_i$$

$$Z = \int \mathcal{D}M \, e^{-S(M)} = \sum_\Gamma \left(\frac{\lambda}{\sqrt{N}}\right)^{\frac{1}{2}} Z_\Gamma = \sum_\Gamma \lambda^{V_\Gamma} N^{\chi_\Gamma}$$

$(K^{-1})_{jkli}$

V_{jmknli}



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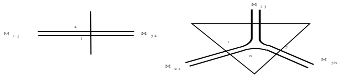
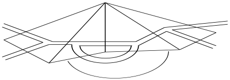


$\Gamma \simeq$ 2d simplicial complex Δ (triangulation)
$\simeq$ 2d discrete spacetime



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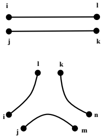

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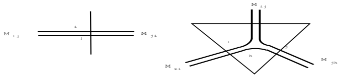

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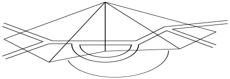

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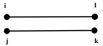

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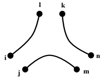

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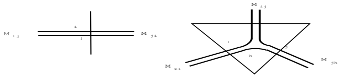

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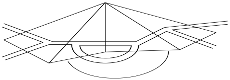

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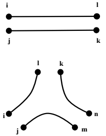

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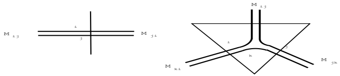

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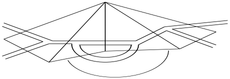

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First root: Matrix Models

- discretization of (Riemannian) 2d GR: replace surface S with equilateral triangles of area a:
  $S_{GR}^{2d} = \int_S d^2x \sqrt{g}\,(-R(g) + \Lambda) = -4\pi\chi + \Lambda A_S \quad \rightarrow \quad S_\triangle = -\frac{4\pi}{G}\chi + \frac{\Lambda a}{G} t$
- from matrix model (with $\lambda = e^{-\frac{\Lambda a}{G}}$ and $N = e^{+\frac{4\pi}{G}}$):

$$Z = \sum_\Gamma \lambda^{V_\Gamma} N^\chi = \sum_\triangle e^{+\frac{4\pi}{G}\chi(\triangle) - \frac{a\Lambda}{G} t_\triangle} \simeq \int \mathcal{D}g_\triangle\, e^{-S_\triangle(g)}$$

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MATRIX MODELS AND CONTINUUM 2D GR

- control over sum over triangulations/topologies?
  - large-N limit - sum governed by topological parameters

$$Z = \sum_{\Delta} \lambda^{t_\Delta} N^{2-2h} = \sum_h N^{2-2h} Z_h(\lambda) = N^2 Z_0(\lambda) + Z_1(\lambda) + N^{-2} Z_2(\lambda) + \ldots..$$

  - $N \rightarrow \infty$ (semi-classical approx) $\rightarrow$ only planar diagrams contribute
- does it match results from continuum 2d gravity path integral?
  - re-sum Feynman expansion in large-N limit
  - expectation value of area of surface:
    $\langle A \rangle = a \langle t_\Delta \rangle = \langle V_\Gamma \rangle = a \frac{\partial}{\partial \lambda} \ln Z_0(\lambda) \simeq \frac{a}{(\lambda - \lambda_c)^{\gamma - 1}}$, for $V_\Gamma >> 1, \gamma = 1/2$
  - continuum limit: area of triangles $a \rightarrow 0$ and number $t_\Delta = V_\Gamma \rightarrow \infty, \lambda \rightarrow \lambda_c$, with finite continuum macroscopic area (phase transition of discrete system)
  - results match continuum 2d gravity path integral (Liouville gravity): loop correlations and SD equations $\simeq$ Wheeler-De Witt eqns (GR as effective field theory)
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Going up in dimension: Tensor Models

- generalize in (combinatorial) dimension from 1d objects (edges) to 2d objects (triangles) - from 2d simplicial complexes as FD to 3d ones

- $M^i{}_j \longrightarrow T_{ijk} \qquad N \times N \times N$ tensor

$$S(T) = \frac{1}{2} trT^2 - \lambda\, trT^4 = \frac{1}{2} \sum_{i,j,k} T_{ijk} T_{kji} - \lambda \sum_{ijklmn} T_{ijk} T_{klm} T_{mjn} T_{nli}$$

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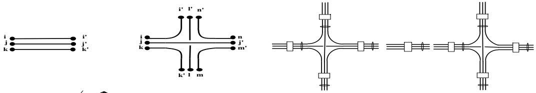
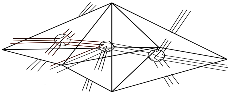
- Feynman diagrams are dual to 3d simplicial complexes





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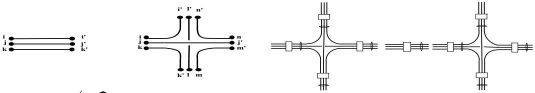

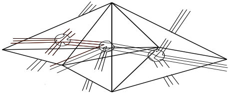

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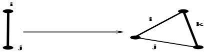

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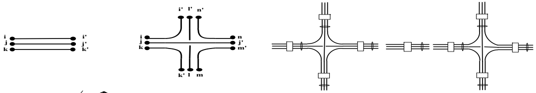

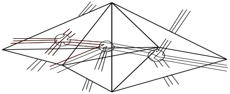

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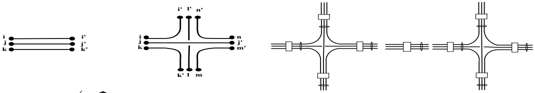

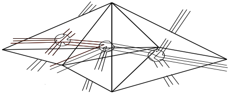

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  - no topological expansion of amplitudes - no control over topology of diagrams
  - no way to separate manifolds from pseudo-manifolds
  - no direct/nice relation with 3d simplicial (classical and quantum) gravity - not enough structure/data in the amplitudes, and in boundary states
- in $d > 2$, topology of simplicial complexes and gravity are -much- less trivial.........
- two possible ways forward:
  - define 'constructively'restricted sum over triangulations $\Rightarrow$ (causal) dynamical triangulations (see talks by Ambjorn and Loll)
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  - need to add (pre-geometric) data and d.o.f. $\Rightarrow$ Group Field Theory (see this talk)

ASIDE: QUANTUM REGGE CALCULUS

Adding data on a simplicial complex, to define quantum gravity path integral......

- discrete spacetime obtained by gluing d-simplices
- metric discretized to set of edge lengths $\{L_e\}$, in units of lattice scale a
- theory defined by:

$$Z(\Lambda, a, G, \Delta) = \prod_e \int d\mu(L_e)\, e^{-S_{Regge}(\Delta, a, \{L_e\}, G, \Lambda)}$$

$\Delta =$ triangulation (fixed topology), $S_{Regge} =$ Regge action, with measure $d\mu(L_e)$

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which kind of d.o.f should be added?

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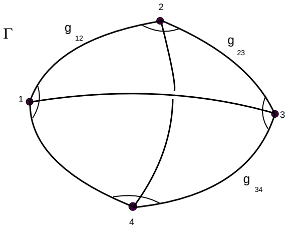

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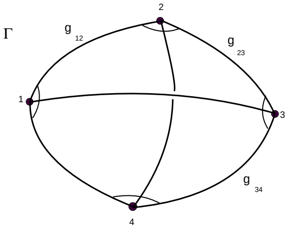

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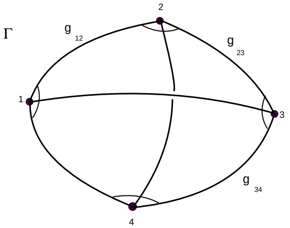

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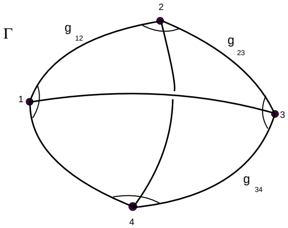

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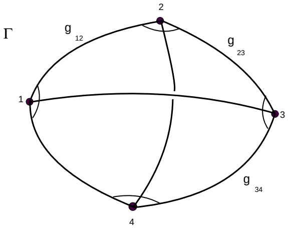

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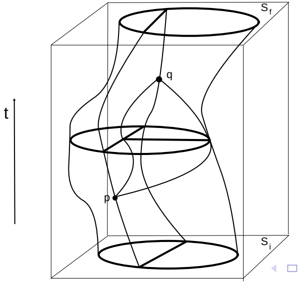

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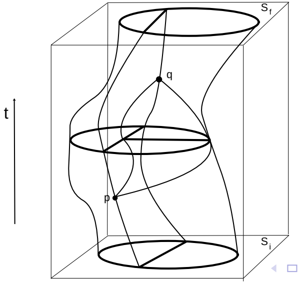

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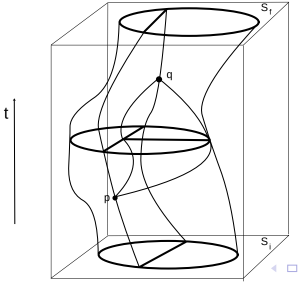

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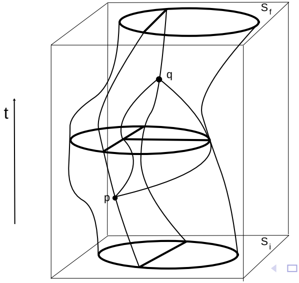

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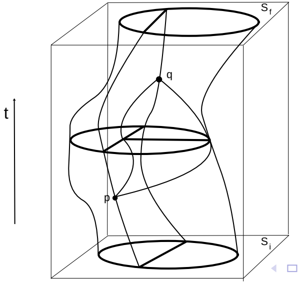

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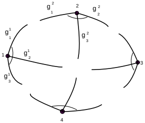
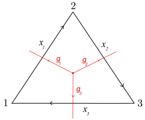
quantum of space: graph vertex $\leftrightarrow$ elementary cell





quantum field theory for vertices/cells $\Rightarrow$ GFT $\varphi(g_1, g_2, g_3) \leftrightarrow \varphi(x_1, x_2, x_3)$

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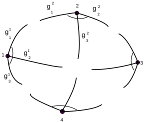

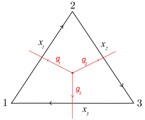

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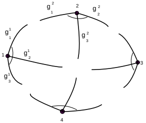

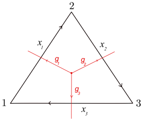

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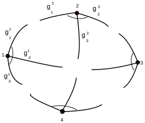

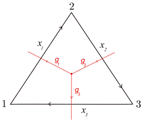

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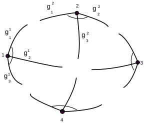

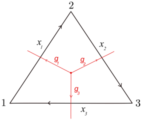

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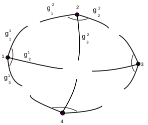

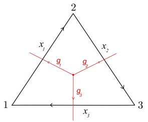

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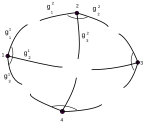

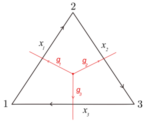

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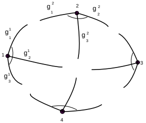
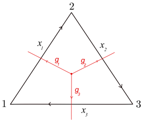




quantum field theory for vertices/cells $\Rightarrow$ GFT

$$\varphi(g_1, g_2, g_3) \leftrightarrow \varphi(x_1, x_2, x_3)$$

where to look for quantum dynamics of spacetime (e.g. LQG)?

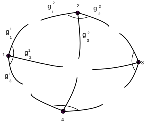
- microscopic dynamics can be quite different from continuum classical dynamics
- dynamics of single interaction process/history of fundamental excitations $\longrightarrow$ GFT Feynman amplitudes
  (any spin foam model (given complex) is a GFT Feynman amplitude)
- full (discrete) quantum dynamics $\longrightarrow$ GFT n-point functions and associated eqns (Ward ids, SD eqns)
- full continuum quantum dynamics $\longrightarrow$ same eqns but in continuum limit: critical points, effective actions, etc

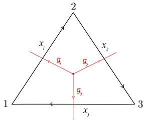
SECOND ROOT: LOOP QUANTUM GRAVITY

Whence the GFT idea (from LQG perspective)?

want quantum theory of dynamics of (very) many d.o.f. $\Rightarrow$ natural QFT framework

quantum of space: graph vertex $\leftrightarrow$ elementary cell





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GFT: BASIC IDEA

- group field theories are combinatorially non-local field theories on (Lie) groups or algebras which
  - bring the fundamental d.o.f. of quantum space, as identified by canonical LQG
  - in the general framework of matrix/tensor models for simplicial quantum gravity
- GFT field ϕ = 2nd quantized spinnet vertex or simplex $\rightarrow$ $\rightarrow$ quantum field theory of spin networks $\simeq$ of simplicial geometry
- tentative definition of complete dynamics of quantum space, from microscopic, pre-geometric, quantum to macroscopic, geometric, (semi-)classical
- can be studied using (almost) standard QFT methods
- classical dynamics $S(\phi)$
- quantum dynamics: perturbation theory around no-space vacuum $\phi = 0$:

$$Z = \int [d\phi]\, e^{-S(\phi)} = \sum_{\Gamma} A(\Gamma)$$

  Γ = possible interaction/evolution process of spin networks/simplices = cellular complex of arbitrary topology and complexity
- $A(\Gamma)$ = spin foam model or, equivalently, discrete gravity path integral

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  - any spin foam construction has a direct GFT counterpart
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  - main strategy (up to now) 4d gravity as BF theory + constraints
  - matter coupling, gauge theory, etc
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(COLORED) GFT FOR 3D EUCLIDEAN GRAVITY

(Boulatov, hep-th/9202074) (Gurau, arXiv:0907.2582 [hep-th])

- 4 fields φ_ℓ for $\ell = 1, .., 4$ function on $\mathrm{SO}(3)^{\otimes 3}$, subject to gauge invariance:

$$\forall h \in \mathrm{SO}(3), \qquad \varphi_\ell(hg_1, hg_2, hg_3) = \varphi_\ell(g_1, g_2, g_3)$$

- action $S[\varphi_\ell] = S_{kin}[\varphi_\ell] + S_{int}[\varphi_\ell]$:

$$S_{kin}[\varphi_\ell] = \int [\mathrm{d}g_i]^3 \sum_{\ell=1}^{4} \varphi_\ell(g_1, g_2, g_3)\overline{\varphi_\ell}(g_1, g_2.g_3),$$

$$\begin{aligned} S_{int}[\varphi_\ell] \quad &= \quad \lambda \int [\mathrm{d}g_i]^6 \varphi_1(g_1, g_2, g_3)\varphi_2(g_3, g_4, g_5)\varphi_3(g_5, g_2, g_6)\varphi_4(g_6, g_4, g_1) \\ &\quad + \lambda \int [\mathrm{d}g_i]^6 \overline{\varphi_4}(g_1, g_4, g_6)\overline{\varphi_3}(g_6, g_2, g_5)\overline{\varphi_2}(g_5, g_4, g_3)\overline{\varphi_1}(g_3, g_2, g_1) \end{aligned}$$

- spin network representation obtained by Peter-Weyl expansion ($j \in \mathbb{N}$)

$$\varphi_\ell(g_1, g_2, g_3) = \sum C^{j_1, j_2, j_3}_{m_1, m_2, m_3} \phi^{j_1, j_2, j_3}_{\ell, n_1, n_2, n_3} D^{j_1}_{m_1 n_1}(g_1) D^{j_2}_{m_2 n_2}(g_2) D^{j_3}_{m_3 n_3}(g_3)$$

- field $\leftrightarrow$ spin network vertex

COLORED GFT FOR 3D EUCLIDEAN GRAVITY

- non-commutative triad (flux) representation with $x \in \mathfrak{su}(2) \sim \mathbb{R}^3$

  (A. Baratin, DO, arXiv:1002.4723 [hep-th]), (A. Baratin, B. Dittrich, DO, J. Tambornino, arXiv:1004.3450 [hep-th])

- use group Fourier transform of the fields (E. Livine, L. Freidel, S. Majid, K. Noui, E. Joung, J. Mourad)

$$\widehat{\varphi}_\ell(x_1, x_2, x_3) := \int [dg_i]^3 \, \varphi_\ell(g_1, g_2, g_3) \, e_{g_1}(x_1) e_{g_2}(x_2) e_{g_3}(x_3),$$

  with plane-waves $e_g := e^{i\mathrm{Tr}(xg)} : \mathfrak{su}(2) \sim \mathbb{R}^3 \to U(1)$ ($g = e^{\theta \vec{n} \cdot \vec{\tau}}$ and $x = \vec{x} \cdot \vec{\tau}$)

- non-commutative product dual to convolution product on the group:

$$(e_g \star e_{g'})(x) := e_{gg'}(x),$$

- gauge invariance condition is 'closure constraint' for x_i

$$\widehat{\mathcal{P} \triangleright \varphi_\ell} = \widehat{C} \star \widehat{\varphi}_\ell \quad \widehat{C}(x_1, x_2, x_3) := \delta_0(x_1 + x_2 + x_3) \quad \mathcal{P} \triangleright \varphi_\ell = \int [dh] \, \varphi_\ell(hg_1, hg_2, hg_3)$$

$$\delta_x(y) := \int [dh] \, e_{h^{-1}}(x) e_h(y) \qquad \int [d^3y] \, (\delta_x \star f)(y) = f(x)$$

- $x_i =$ closed edges vectors of a triangle in $\mathbb{R}^3 \Rightarrow$ field $\leftrightarrow$ geometric simplex

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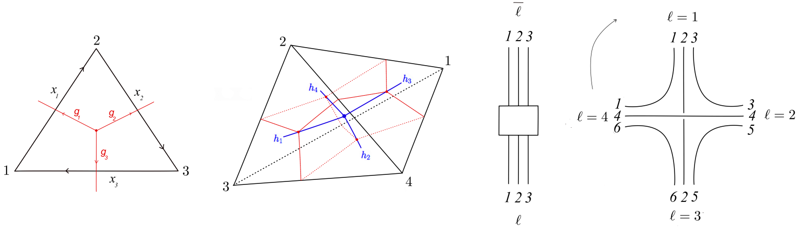

- Feynman diagrams Γ are dual to 3d simplicial complexes
- amplitudes $\mathcal{A}_\Gamma$ written in group, representation or algebra variables

$$\mathcal{A}_\Gamma = \int \prod_l dh_l \prod_f \delta\left(H_f(h_l)\right) = \int \prod_l dh_l \prod_f \delta\left(\overrightarrow{\prod}_{l\in\partial f} h_l\right) =$$

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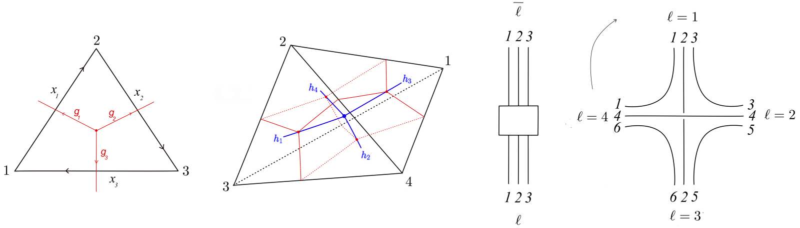
- last line is discretized path integral for 3d gravity $S(e,\omega) = \int Tr(e \wedge F(\omega))$
- exact duality: simplicial gravity path integral $\leftrightarrow$ spin foam model

COLORED GFT FOR 3D EUCLIDEAN GRAVITY



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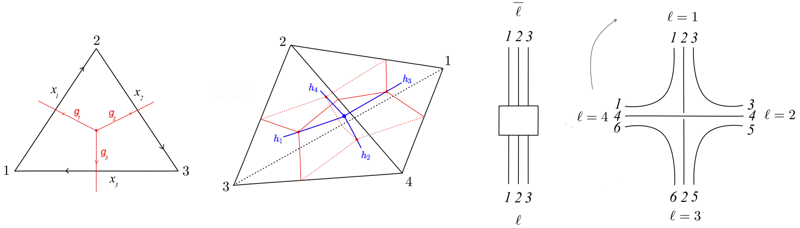

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CONSTRUCTION OF 4D GRAVITY MODELS

- 4d gravity is constrained BF theory: $\mathfrak{so}(4)$-Plebanski action

$$S(\omega, B, \phi) = \int_{\mathcal{M}} \left[B^{IJ} \wedge F_{IJ}(\omega) - \frac{1}{2}\phi_{IJKL} B^{KL} \wedge B^{IJ} \right]$$

- strategy: start from GFT for 4d BF theory and apply on them suitable (discrete) constraints - (Rovelli's talk)

$$S[\phi] = \frac{1}{2}\int dg_i \, [\phi(g_1, g_2, g_3, g_4)]^2 + \frac{\lambda}{5!}\int dg_j \, [\phi(g_1, g_2, g_3, g_4)\phi(g_4, g_5, g_6, g_7) \phi(g_7, g_3, g_8, g_9)\phi(g_9, g_6, g_2, g_{10})\phi(g_{10}, g_8, g_5, g_1)]$$

- impose constraints at level of quantum states $\longrightarrow$ restrictions on SO(4) representations and embedding of SU(2) into SO(4)
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  - EPRL/FK$_\gamma$ model Ben Geloun, Gurau, Rivasseau, arXiv:1008.0354 [hep-th] (see Rovelli's talk)
  - EPRL, $\gamma \longrightarrow \infty \Rightarrow$ Barrett-Crane model De Pietri et al. , hep-th/9907154; Perez, Rovelli, gr-qc/0006107
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V. Bonzom, E. Livine, arXiv:0812.3456 [gr-qc]; A. Baratin, DO, arXiv:1002.4723 [hep-th]; A. Baratin, DO, to appear

using noncommutative *B* representation of GFT dynamics

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- define interesting models for quantum gravity, i.e. $S(\phi)$ and thus $A(\Gamma)$
  - any spin foam construction has a direct GFT counterpart
  - same issues can be tackled from simplicial gravity path integral side
  - main strategy (up to now) 4d gravity as BF theory + constraints
  - matter coupling, gauge theory, etc
- understand encoding of quantum geometry and its GFT dynamics
  - geometric d.o.f. and symmetries in $S(\phi)$ and in $A(\Gamma)$
  - statistics (what is the statistics of spin networks?)
  - n-point functions, Ward identities and Schwinger-Dyson equations (here is WdW equation)
- control over perturbative expansion and perturbative renormalization
  - topology of diagrams, dependence of amplitudes on topology
  - GFT scaling and power counting
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- non-perturbative aspects and continuum/thermodynamic limit
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  - quantum dynamics in continuum limit: can take into account all d.o.f. of the theory?
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- effective (classical and quantum) dynamics (and new physics?)
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DIFFEOMORPHISMS IN DISCRETE (QUANTUM) GRAVITY

- QG models based on discrete structures → continuum diffeo symmetry generically broken
- need to identify discrete symmetry on 'pre-geometric' data
- extensive studies in Regge calculus B. Dittrich, arXiv:0810.3594 [gr-qc]; B. Bahr, B. Dittrich, arXiv:0905.1670 [gr-qc]:
  - 'discrete'diffeomorphisms → translations of vertices of triangulation in $\mathbb{R}^d$
  - invariance of Regge action exact in 3d without cosmological constant (flat space)
  - invariance only approximate in 4d → recovered in continuum limit
  - invariance of action related to Bianchi identities at vertices of triangulation
- diffeos in 3d Ponzano-Regge spin foam model L. Freidel, D. Louapre, gr-qc/0212001

GFT DIFFEOS - A. BARATIN, F. GIRELLI, DO, ARXIV:1101.0590 [HEP-TH]

- symmetries of GFT model for 3d Euclidean gravity = subset of $\mathcal{D}SO(3)^{\otimes 4}$ (deformation of Poincare group), one for each vertex of a tetrahedron
- translation (diffeo) symmetry:
  - transformations generated by four $\mathfrak{su}(2)$-translation parameters ε_v, one per vertex of tetrahedron
  - in metric representation, it shifts $x_i^{\ell\neq 3}$ by $\pm\varepsilon_3$ according to orientation:

$$x_i^\ell \mapsto x_i^\ell + \varepsilon_3 \quad \text{if } i \text{ outgoing} \qquad\qquad x_i^\ell \mapsto x_i^\ell - \varepsilon_3 \quad \text{if } i \text{ incoming}.$$

$$\begin{aligned}
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_1(x_1,x_2,x_3) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_1(x_1-\varepsilon_3, x_2, x_3+\varepsilon_3)\\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_2(x_3,x_4,x_5) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_2(x_3-\varepsilon_3, x_4+\varepsilon_3, x_5)\\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_4(x_6,x_4,x_1) &:= \bigstar_{\varepsilon_3}\, \widehat{\varphi}_4(x_6, x_4-\varepsilon_3, x_1+\varepsilon_3)\\
\mathcal{T}_{\varepsilon_3} \triangleright \widehat{\varphi}_3(x_5,x_2,x_6) &:= \widehat{\varphi}_3(x_5,x_2,x_6)
\end{aligned}$$

- geometric meaning: when translanting a vertex, one translates the edge vectors sharing this vertex
- these transformations leave GFT action (more: the integrands) invariant

INVARIANCE OF GFT INTERACTION VERTEX AND QUANTUM GEOMETRY

(A. Baratin, F. Girelli, DO, arXiv:1101.0590 [hep-th])

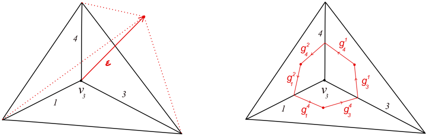
- non-commutative triad representation → invariance of geometry of tetrahedron under translation of vertices in $\mathbb{R}^3$
- group representation
  - vertex function is:

$$V(g_i^\ell, g_i^{\ell'}) = \int \prod_{\ell=1}^{4} \mathrm{d}h_\ell \prod_{i=1}^{6} \delta((g_i^\ell)^{-1} h_\ell h_{\ell'}^{-1} g_i^{\ell'})$$

  - its invariance under translations of the vertex v_3 means that, for all $\epsilon_3 \in \mathfrak{su}(2)$:

$$\mathrm{e}_{G_{v_3}}(\varepsilon_3) V(g_i^\ell, g_i^{\ell'}) = V(g_i^\ell, g_i^{\ell'}) \qquad G_{v_3} = (g_1^1)^{-1} g_3^1 (g_3^2)^{-1} g_4^2 (g_4^4)^{-1} g_1^4$$

  - G_{v_3} is the holonomy along a loop circling the vertex v_3 of the tetrahedron.
  - symmetry → boundary connection is *flat* → Hamiltonian and vector constraints → constraint on tetrahedral wave-function constructed from the GFT field



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- spin representation
  - the vertex function takes the form of SO(3) 6j-symbols:

$$\sum_{\{m_i^\ell\}} \prod_\ell i^\ell_{m_i^\ell} V^{j_i}_{m_i^\ell n_i^\ell} = \prod_i \delta_{n_i^\ell, -n_i^{\ell'}} \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

  - the flatness constraint (WdW equation) on boundary connection becomes algebraic (recursion) identity for 6j-symbols $\longrightarrow$

$$\left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\} =$$

$$= \sum_{k_i, j} d_{k_1} d_{k_3} d_{k_4} d_j \widehat{\chi}^j(\varepsilon) \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j & k_1 & k_3 \end{array} \right\} \left\{ \begin{array}{ccc} j_1 & j_5 & j_4 \\ j & k_4 & k_1 \end{array} \right\} \left\{ \begin{array}{ccc} j_3 & j_6 & j_4 \\ j & k_3 & k_4 \end{array} \right\} \left\{ \begin{array}{ccc} k_1 & k_3 & j_2 \\ k_4 & j_5 & j_6 \end{array} \right\}$$

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- nice duality between simplicial geometric, quantum geometric and algebraic implementations of diffeo invariance (WdW equation)

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SYMMETRY OF GFT AMPLITUDES, BIANCHI IDENTITY

- amplitude associated to vertex: function of x_e, $\forall e \supset v$, choose ordering
- act with non-commutative translation $x_e \to x_e + \varepsilon_v^e(\epsilon_v)$, $\varepsilon_v^e(\epsilon_v) = k_e^v \epsilon_v (k_e^v)^{-1}$ (k_e^v = parallel transport from fixed vertex in L_v to reference vertex of face f_e)
- function gets transformed into a $\star$-product of functions of ϵ_v:

$$\prod_{e \supset v} e^{i\mathrm{Tr} x_e H_e} \mapsto \overrightarrow{\bigstar}_{e \supset v} \prod_{e \supset v} e^{i\mathrm{Tr}(x_e + \varepsilon_v^e) H_e}(\epsilon_v)$$

- non-commutative translation acts by multiplication by plane wave:

$$e^{i\mathrm{Tr}\left[\epsilon_v\left(\overrightarrow{\prod}_{e \supset v} (k_v^e)^{-1} H_e\, k_v^e\right)\right]} = 1,$$

  which is trivial $\to$ Bianchi identity
- trivial braiding for GFT fields *does not* intertwine the GFT diffeomorphism symmetry $\to$ symmetry is broken at full quantum level $\to$ GFT n-point functions are *not* covariant
  $\Rightarrow$ introduction of non trivial *braiding* map among GFT fields $\to$ braided statistics (of spin nets!!!)

GFT: BRAIDING AND STATISTICS

Baratin-Girelli-DO, 1101.0590 [hep-th], S. Carrozza, DO, 1104.5158 [hep-th], A. Baratin, S. Carrozza, F. Girelli, DO, M. Raasakka, to appear

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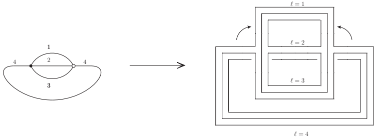
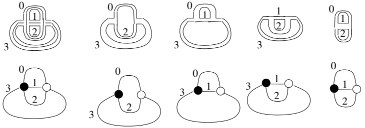
TOPOLOGICAL STRUCTURE OF GFT DIAGRAMS

- Feynman diagrams include manifolds as well as pseudo-manifolds with at most point-like singularities (at vertices of triangulation)
  (De Pietri, Petronio, gr-qc/0004045, Gurau, arXiv:1006.0714 [hep-th] , Smerlak, arXiv:1102.1844 [hep-th])
  - nature of singularities depend on topology of 3-cells dual to vertices of Δ (bubbles)
- one can use cellular (co-)homology (and its twisting by flat connection) to study the topology of diagrams and of bubbles (Bonzom, Smerlak, arXiv:1103.3961 [gr-qc])
- an alternative is available in *colored* GFT models (Gurau, arXiv:0907.2582 [hep-th]), Gurau, arXiv:1006.0714 [hep-th]) - non-standard (co-)homology defined in terms of *colors* - e.g. bubble = connected component made of 3 colors only





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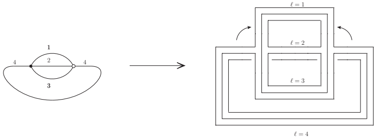

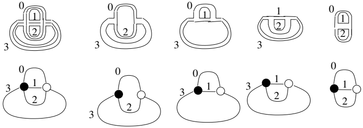

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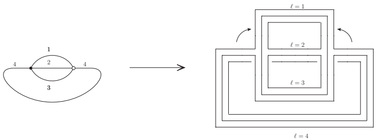

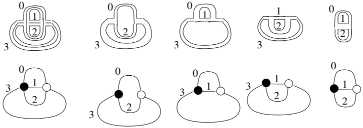

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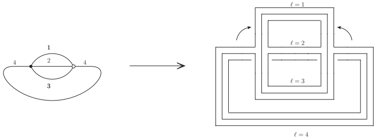

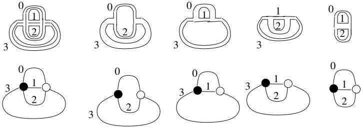

Divergences and scaling

- detailed analyses and most results for BF models or independent identically distributed (i.i.d.) models (equivalent to tensor models/dynamical triangulations)
- GFT amplitudes (SF models) (generically) diverge for large representations/fluxes - need to understand scaling with cut-off
- power counting theorems:
  - simple cases: ‘contractible’diagrams (Freidel, Gurau, DO, arXiv:0905.3772 [hep-th]) - abelianized GFTs (Ben Geloun, Krajewski, Magnen, Rivasseau, arXiv:1002.3592 [hep-th])
  - general power counting theorems
    - using twisted co-homology, for diagrams Γ which are 2-skeletons of cellular decompositions Δ of M:
      $\Omega(\Gamma, G) = I(M, G) + \omega(\Delta_M, G)$,
      where $I(\Gamma, G)$ depends only on the topology of M (Euler character, fundamental group), $\omega(\Delta_M, G)$ depends on particular cellular decomposition
      (Bonzom, Smerlak, arXiv:1004.5196 [gr-qc], arXiv:1008.1476 [math-ph], arXiv:1103.3961 [gr-qc])
    - similar, purely combinatorial results using color structure (Gurau, arXiv:1011.2726 [gr-qc], arXiv:1102.5759 [gr-qc]; Gurau, Rivasseau, arXiv:1101.4182 [gr-qc])

DIVERGENCES AND SCALING

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  - general optimal bounds: $Z_\Lambda(\Gamma) \le K^n \Lambda^{3(D-1)(D-2)n/4+3(D-1)}$, with n vertices , for colored models in D dimensions (Ben Geloun, Magnen, Rivasseau, arXiv:0911.1719 [hep-th])
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  - for i.i.d. and BF models in any dimension, need color structure
  - in the large cut-off limit $\Lambda \to \infty$ only diagrams corresponding to manifolds of trivial topology dominate the perturbative GFT expansion
  - not *all* diagrams of trivial topology appear at leading order - the same topology can appear at arbitrary order
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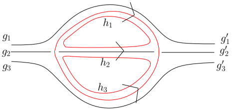
PERTURBATIVE GFT RENORMALIZATION

- radiative corrections to the GFT 2-point function of the BF GFT models

Ben Geloun, Bonzom, arXiv:1101.4294 [hep-th]



- two leading divergences:
  - a mass renormalization
  - a divergence proportional to the second derivatives of the propagator
- this needs to be balanced by a new counter-term in the GFT Boulatov action:

$$m^2 \int [dg] \phi^2(g_1, g_2, g_3) \quad \rightarrow \quad \int [dg] \phi(g_1, g_2, g_3) \left[\sum_{i=1}^{3} \Delta_i + m^2 \right] \phi(g_1, g_2, g_3)$$

- BF GFT model could be fixed point of more general GFT dynamics - attractive or repulsive? role of symmetries?

- need to tackle intensively all 4d gravity models!!!
- perturbative GFT renormalization vs renormalization of discrete gravity?

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NON-PERTURBATIVE RESULTS

- re-sum the perturbation expansion?
- if it is, it means we can give a non-perturbative definition of the theory, taking into account all its infinite degrees of freedom
- for i.i.d. models in any dimension (equivalent to dynamical triangulations), the dominant configurations at large-Λ (spheres) can be resummed exactly!
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  - when $g \rightarrow g_c$ ($g = |\lambda|^2$), the free energy has critical behavior (phase transition) $F = (g_c - g)^{2-\gamma}$ with $\gamma = \frac{1}{2}$
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GFT: MAIN QUESTIONS AND DIRECTIONS

- define interesting models for quantum gravity, i.e. $S(\phi)$ and thus $A(\Gamma)$
  - any spin foam construction has a direct GFT counterpart
  - same issues can be tackled from simplicial gravity path integral side
  - main strategy (up to now) 4d gravity as BF theory + constraints
  - matter coupling, gauge theory, etc
- understand encoding of quantum geometry and its GFT dynamics
  - geometric d.o.f. and symmetries in $S(\phi)$ and in $A(\Gamma)$
  - statistics (what is the statistics of spin networks?)
  - n-point functions, Ward identities and Schwinger-Dyson equations (where is the LQG dynamics?)
- control over perturbative expansion and perturbative renormalization
  - topology of diagrams, dependence of amplitudes on topology
  - GFT scaling and power counting
  - (perturbative) GFT renormalization - is your candidate QG mode renormalizable?
- non-perturbative aspects and continuum/thermodynamic limit
  - (Borel) summability at least in some sector of full theory?
  - quantum dynamics in continuum limit: can take into account all d.o.f. of the theory?
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- effective (classical and quantum) dynamics (and new physics?)
  - extract effective dynamics for geometry and matter in appropriate phase
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COLLECTIVE STATES, SIMPLIFIED MODELS

- best case scenario: evaluate GFT n-point functions and solve Schwinger-Dyson equations in continuum limit, extract QG interpretation
- need approximations, new tools, physically relevant simplifications
- one important tool: semi-classical coherent states as boundary GFT states
  - standard construction (Thiemann and collaborators):
- for effective continuum physics: *collective coherent states* - (DO, Pereira, Sindoni, to appear)
  - states semi-classical w.r.t. observables depending on many fundamental d.o.f.
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- other avenue: simplified models (see Ashtekar's talk)
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EFFECTIVE GFT DYNAMICS

General idea:

- $S_{GFT}(\phi)$ fundamental dynamics - adapted for perturbation theory around $\phi = 0$
- need to *change vacuum* : $\phi \rightarrow \phi_0(g_1, .., g_D) + \psi(g_1, .., g_D)$
- ϕ_0 = new (non-perturbative) vacuum - satisfies (approximately) dynamical eqns coming from GFT action:

$$\left[\phi + \lambda \frac{\delta}{\delta\phi}\mathcal{V}(\phi)\right]_{\phi=\phi_0} \approx 0$$

- effective dynamics for $\psi(g_i)$:

$$S_{eff}(\psi) = S(\phi_0 + \psi) - S(\phi_0) = \frac{1}{2}\psi\,\mathcal{H}\,\psi + \mu\,\mathcal{U}(\psi)$$

$$\mathcal{H} = \mathcal{H}(\lambda, \phi_0) \qquad \mu = \mu(\lambda, \phi_0)$$

- possible interpretations:
  - ϕ_0 non-trivial background quantum geometry (eqns satisfied encode quantum GR)
  - ψ = quantum gravity wave function around ϕ_0
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  - derive effective $\mathcal{H}$ which acts on spin network states as non-graph changing operator
  - action compatible with 3d WdW operator
  - computed spectrum of $\mathcal{H}$ - modes for eigenvalues > 0 excited by interaction
- emergent non-commutative matter fields

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- effective Hamiltonian constraint (3d) (Livine, DO, Ryan, arXiv:1104.5509 [gr-qc]):
  - ϕ_0 = exact solution of GFT classical eqns
  - derive effective $\mathcal{H}$ which acts on spin network states as non-graph changing operator
  - action compatible with 3d WdW operator
  - computed spectrum of $\mathcal{H}$ - modes for eigenvalues > 0 excited by interaction
- emergent non-commutative matter fields
  (Fairbairn, Livine, gr-qc/0702125), (Girelli, Livine, DO, arXiv:0903.3475 [gr-qc]), (DO, arXiv:0903.3970 [hep-th]), (Di Mare, DO, arXiv:1001.2702 [gr-qc])
  - both in 3d euclidean (clean) and in 4d lorentzian (less clean), for BF GFT models
  - ϕ_0 = classical solution of GFT eqns
  - $\psi = \psi(g)$ emergent matter field with momentum space = group manifold, Lie algebra = configuration space
  - non-commutative matter field theory, curved momenta (relative locality)
  - possible interest for phenomenology

Conclusions

- GFTs define a tentative but complete quantum dynamics for spin networks/simplices
  - bring the fundamental d.o.f. of quantum space, as identified by canonical LQG
  - in the framework of matrix/tensor models for simplicial quantum gravity
- tentative definition of complete dynamics of geometry (and topology) of quantum space, from microscopic, pre-geometric, quantum to macroscopic, geometric, (semi-)classical

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Thank you for your attention!