Einstein - Cartan Gravity
and
Asymptotic Safety

Martin Reuter

1. Introduction to Asymptotic Safety

2. Recent results on the $(e_\mu^a, \omega_\mu^{ab})$ - universality class

The key question:

Is it possible to construct a consistent and predictive quantum field theory of gravity as the (non-perturbative) "continuum" limit of some appropriate system involving an ultraviolet cutoff ?

The Asymptotic Safety idea:

- Take the infinite-cutoff limit of an UV-regularized quantum theory of gravity at a non-trivial RG fixed point with a finite dimensional UV-critical hypersurface, assuming it exists.

- The resulting continuum theory is predictive and well behaved at arbitrarily short distances.

(S. Weinberg, 1979, 2009)

Concrete implementation: the gravitational average action

(M.R., 1996)

- Fix a set of fields Ψ supposed to carry the degrees of freedom: $\Psi = (g_{\mu\nu})$, $\Psi = (e^a_\mu, \omega^{ab}_\mu)$, ...
- Fix a group $\mathcal{G}$ of (gauge) symmetries acting on Ψ.
- Define "theory space" to consist of all invariant fctl's:

$$\mathcal{T} = \Big\{ A[\Psi] \;\Big|\; \text{action functional } A \text{ is inv. under } \mathcal{G} \Big\}$$

- Fix a coarse graining scheme on $\mathcal{T}$ (background covariant continuum analogue of Kadanoff's block spin idea)
- Compute the resulting "renorm. group flow" $(\mathcal{T}, \beta)$.

  β: vector field on $\mathcal{T}$ obtained by applying infinitesimal coarse graining steps to all $A \in \mathcal{T}$

$$A \xrightarrow[\text{RG step}]{\text{infinites.}} A + \underbrace{\beta(A)}_{\in T_A \mathcal{T}}$$

- Compute "RG trajectories" $\Gamma_\bullet : \mathbb{R} \to \mathcal{T},\ k \mapsto \Gamma_k$ as integral curves of β:

$$\frac{d}{d\ln k}\Gamma_k = \beta(\Gamma_k)$$

"FRGE"
"flow eq."

concretely: $\beta(\Gamma_k) = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$; $\Gamma_0 = \Gamma$, $\Gamma_\infty \sim S$

- Determine fixed points of the flow: $\beta(A_*) = 0$

- Linearize flow about A_*: $\Gamma_k = A_* + \delta\Gamma_k$

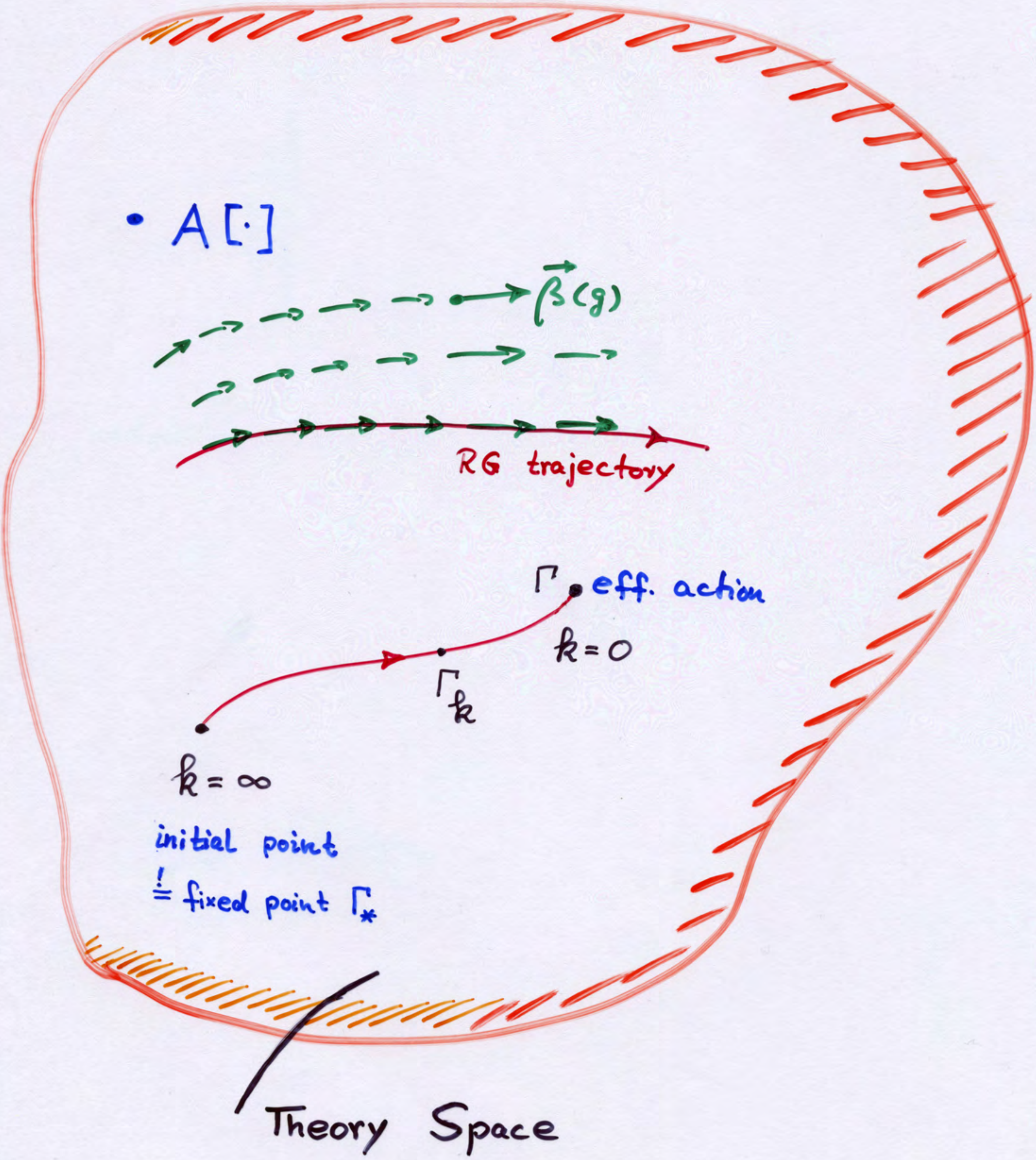
$$\frac{d}{d\ln k}\delta\Gamma_k = \left[\begin{matrix}\text{stability}\\ \text{matrix}\end{matrix}\right]_{A_*}\delta\Gamma_k$$

$\Longrightarrow$
- eigenvectors = "scaling fields"
- eigenvalues = "critical exponents" θ_i

$$(\delta\Gamma_k)_i \sim k^{-\theta_i}$$

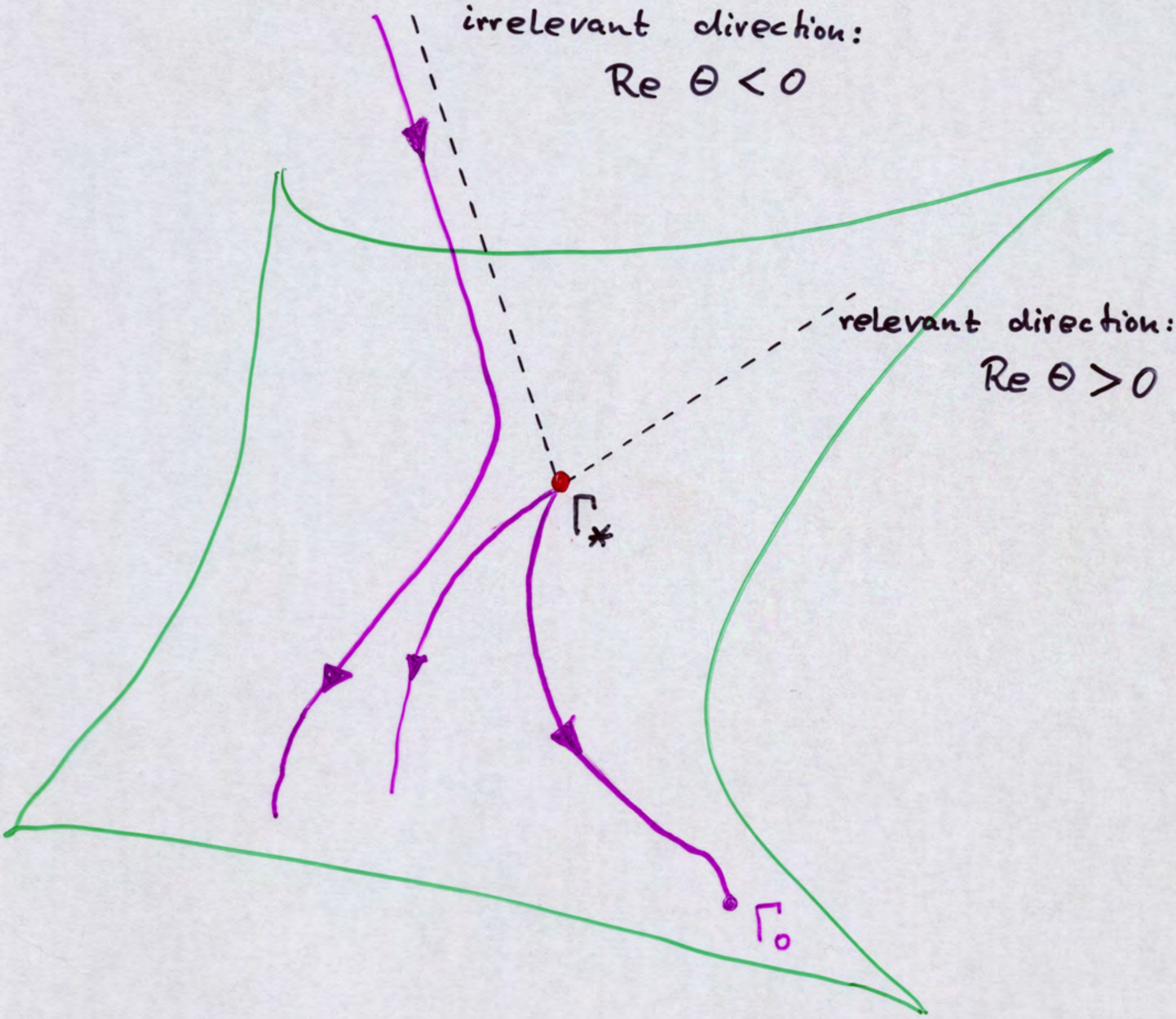
- Try to find <u>complete</u> RG trajectories for which the limits $k \to \infty$ (UV) and $k \to 0$ (IR) exist. Every one of them defines a quantum theory.

- The Asymptotic Safety construction: perform UV limit at a fixed point: $\Gamma_k \longrightarrow A_*$ for $k \to \infty$.



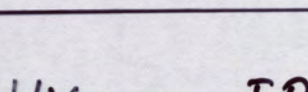
• A[·]
$\vec{\beta}(g)$
RG trajectory
Γ eff. action
k=0
Γ_k
k=∞
initial point
$\overset{!}{=}$ fixed point Γ_*
Theory Space

The UV-critical hypersurface $\mathcal{S}_{UV}$:



$\Delta_{UV} \equiv \dim \mathcal{S}_{UV}$ = # relevant directions

= # free parameters in the a.s. quantum field theory



Θ : critical exponent (neg. eigenvalue of lin. flow)

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k} \int d^dx \sqrt{g} \left\{ R - 2\Lambda_k \right\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k , dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k , dimensionless: $\lambda(k) = \Lambda_k / k^2$

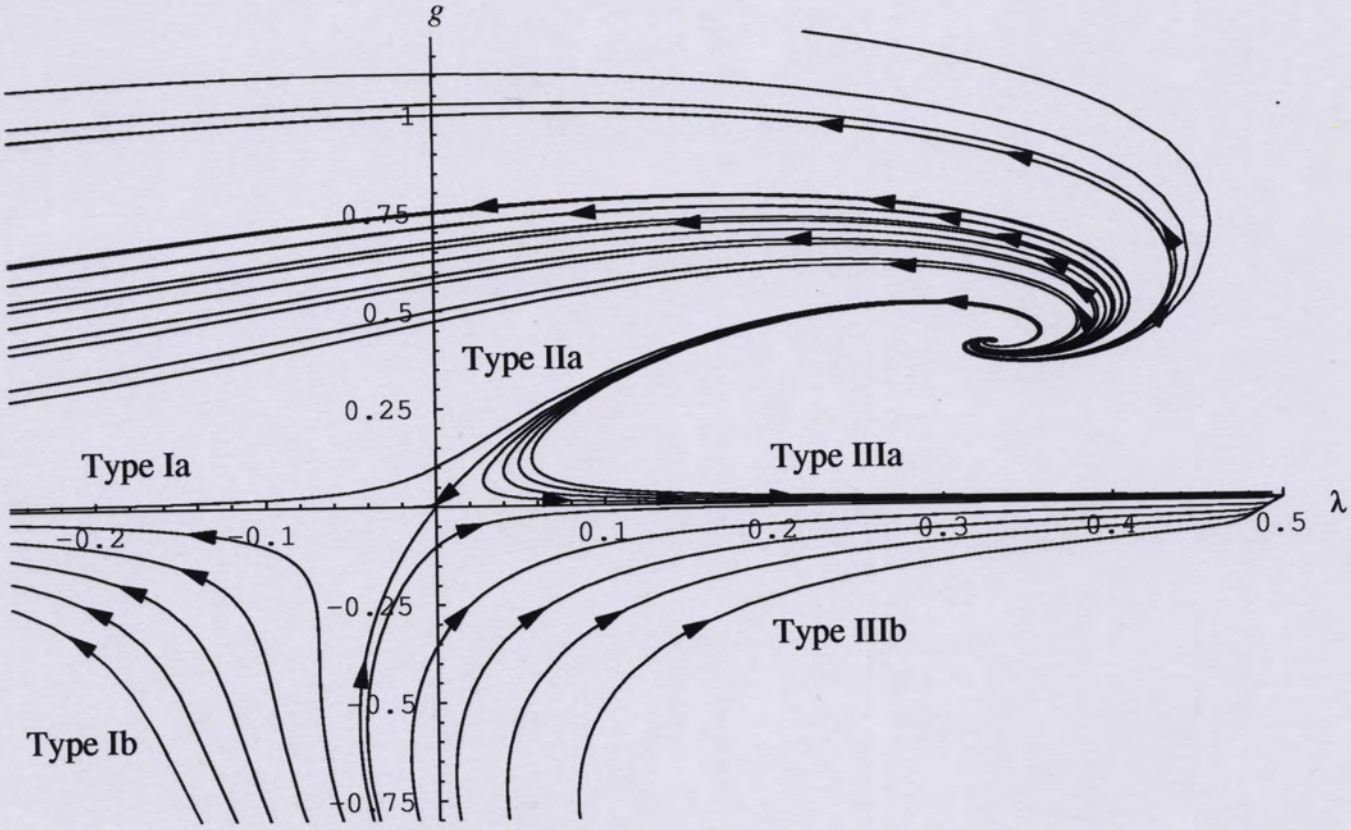
insert ansatz into flow equation, expand

$$\mathrm{Tr}[\cdots] = (\cdots) \int \sqrt{g} + (\cdots) \int \sqrt{g} R + \cdots$$

$$\rightsquigarrow \quad \boxed{\begin{aligned} k\,\partial_k\, g(k) &= \beta_g(g,\lambda) \\ k\,\partial_k\, \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

Einstein - Hilbert Truncation :

RG Flow on the $g-\lambda$ plane



M.R., F. Saueressig, hep-th / 0110054

Einstein - Cartan Gravity

("e - ω theory")

The fundamental problem:

Give a meaning to ("define", "renormalize", "take the continuum limit" of, ...) a quantum gravity functional integral of the form

$$\int \mathcal{D}\hat{e}_\mu^{\,a}\, \mathcal{D}\hat{\omega}_\mu^{\,ab}\, e^{-S[\hat{e},\hat{\omega}]}$$

vielbein   spin connection   bare action, inv. under $\mathrm{Diff}(\mathcal{M}) \otimes O(4)_{loc}$

- In particular:

  S = Holst action $\equiv$ Hilbert–Palatini action of Einstein–Cartan gravity + Immirzi term (γ)

- Resulting quantum theory inequivalent to metric gravity: $\omega_\mu^{\,ab}$ carries **torsion** !?

- Related to: • Ashtekar variables,
  • Loop Quantum Gravity (LQG)
  (γ enters eigenvalues of area, volume, ... operators and black hole entropy)
  • Spin Foams, Group Field Theory, ...

Our approach:

J.-E. Daum, M.R.
2010

- Reformulate functional integral as gauge-fixed $\mathrm{Diff}(\mathcal{M}) \ltimes O(4)_{\mathrm{loc}}$ gauge theory using the background field method.

- Explore the Wilsonian renormalization group flow in this "universality class".

- Try to construct an "asymptotically safe" ultraviolet limit.

Seems to work for metric gravity!

The dynamical variables:

- Vielbein (co-frame) : $e^a = e^a{}_\mu \, dx^\mu$
- $O(4)$ - connection : $\omega^a{}_b = \omega^a{}_{b\mu} \, dx^\mu$

  - covariant derivative: $\nabla_\mu \equiv \partial_\mu + \frac{1}{2}\omega^{ab}{}_\mu M_{ab}$
  - curvature: $F^{ab}{}_{\mu\nu} \equiv \partial_\mu \omega^{ab}{}_\nu + \omega^a{}_{c\mu}\omega^{cb}{}_\nu - (\mu \leftrightarrow \nu)$
  - torsion: $T^a{}_{\mu\nu} \equiv \partial_\mu e^a{}_\nu + \omega^a{}_{c\mu} e^c{}_\nu - (\mu \leftrightarrow \nu)$

Under local $O(4)$ / Lorentz transformations:

$$\delta_L(\lambda)\, e^a{}_\mu = \lambda^a{}_b(x)\; e^b{}_\mu$$

$$\delta_L(\lambda)\, \omega^{ab}{}_\mu = -\partial_\mu \lambda^{ab} + \lambda^a{}_c \omega^{cb}{}_\mu + \lambda^b{}_c \omega^{ac}{}_\mu$$

Under (ordinary) diffeomorphisms:

$$\delta_D(v)\, e^a{}_\mu = \mathcal{L}_v\, e^a{}_\mu \equiv v^\nu \partial_\nu e^a{}_\mu + (\partial_\mu v^\nu) e^a{}_\nu$$

$$\delta_D(v)\, \omega^{ab}{}_\mu = \mathcal{L}_v\, \omega^{ab}{}_\mu \equiv v^\nu \partial_\nu \omega^{ab}{}_\mu + (\partial_\mu v^\nu) \omega^{ab}{}_\nu$$

Under $O(4)$-covariant diffeo's $\tilde{\delta}_D(v) \equiv \delta_D(v) + \delta_L(v^\mu \omega_\mu^{ab})$:

$$\tilde{\delta}_D(v)\, e^a{}_\mu = v^\nu \nabla_\nu e^a{}_\mu + (\nabla_\mu v^\nu) e^a{}_\nu$$

$$\tilde{\delta}_D(v)\, \omega^{ab}{}_\mu = -F^{ab}{}_{\mu\nu} v^\nu$$

Implementation on fctls. : **Ward operators**

$$A[e+\delta_{\underset{D}{L}} e, \omega+\delta_{\underset{D}{L}} \omega] - A[e,\omega] =: - W_{\underset{D}{L}} A[e,\omega]$$

$$W_L(\lambda) = - \int d^4x \left\{ \delta_L(\lambda) e^a{}_\mu(x) \frac{\delta}{\delta e^a{}_\mu(x)} + \cdots \right\}$$

Algebra of infinites. $\mathrm{Diff}(\mathcal{M}) \otimes O(4)_{Loc}$ transf's :

$$[W_D(v_1), W_D(v_2)] = W_D([v_1, v_2])$$

$$[W_L(\lambda_1), W_L(\lambda_2)] = W_L([\lambda_1, \lambda_2])$$

$$[W_D(v), W_L(\lambda)] = W_L(\underbrace{\mathcal{L}_v \lambda}_{\stackrel{!}{=} v^\mu \partial_\mu \lambda^{\cdot\cdot} + \cdots})$$

not $O(4)_{Loc}$ - covariant !

Equivalent $O(4)_{Loc}$ - covariant algebra :

$$[\tilde{W}_D(v_1), \tilde{W}_D(v_2)] = \tilde{W}_D([v_1,v_2]) - W_L(v_1^\mu v_2^\nu F_{\mu\nu}{}^{\cdot\cdot})$$

$$[W_L(\lambda_1), W_L(\lambda_2)] = W_L([\lambda_1, \lambda_2])$$

$$[\tilde{W}_D(v), W_L(\lambda)] = 0$$

The Background Split

- Fix arbitrary background configuration $\{\bar{e}^a{}_\mu, \bar{\omega}^{ab}{}_{\mu\nu}\}$
- Decompose integration variables:

$$\hat{e}^a{}_\mu =: \bar{e}^a{}_\mu + \varepsilon^a{}_\mu$$
$$\hat{\omega}^{ab}{}_\mu =: \bar{\omega}^{ab}{}_\mu + \underbrace{\tau^{ab}{}_\mu}_{\text{non-linear fluctuations}}$$

$\Longrightarrow$

$$\int \mathcal{D}\hat{e}\, \mathcal{D}\hat{\omega}\, e^{-S[\hat{e},\hat{\omega}]} = \int \mathcal{D}\varepsilon\, \mathcal{D}\tau\, e^{-S[\bar{e}+\varepsilon, \bar{\omega}+\tau]}$$

- Gauge-fix à la Faddeev-Popov
- use $(\tilde{\delta}_D, \delta_L)$-parametrization of gauge transf's: necessary to obtain a δ^B-invariant ghost action!

Two ways of splitting the gauge transf's:

True gauge transformations δ^G:

- leave the background unchanged
- will be broken by gauge fixing

Background gauge transformations δ^B:

- change the background
- will always remain intact

True gauge tr.	Backgrd. gauge tr.
$\delta^{G}_{D}(v)\,\bar{e} = 0$	$\delta^{B}_{D}(v)\,\bar{e} = \mathcal{L}_v\,\bar{e}$
$\delta^{G}_{D}(v)\,\bar{\omega} = 0$	$\delta^{B}_{D}(v)\,\bar{\omega} = \mathcal{L}_v\,\bar{\omega}$
$\delta^{G}_{D}(v)\,\varepsilon = \mathcal{L}_v(\bar{e}+\varepsilon)$	$\delta^{B}_{D}(v)\,\varepsilon = \mathcal{L}_v\,\varepsilon$
$\delta^{G}_{D}(v)\,\tau = \mathcal{L}_v(\bar{\omega}+\tau)$	$\delta^{B}_{D}(v)\,\tau = \mathcal{L}_v\,\tau$
$\delta^{G}_{L}(\lambda)\,\bar{e} = 0$	$\delta^{B}_{L}(\lambda)\,\bar{e}^{a}{}_{\mu} = \lambda^{a}{}_{b}\,\bar{e}^{b}{}_{\mu}$
$\delta^{G}_{L}(\lambda)\,\bar{\omega} = 0$	$\delta^{B}_{L}(\lambda)\,\bar{\omega}^{ab}{}_{\mu} = -\bar{\nabla}_{\mu}\lambda^{ab}$
$\delta^{G}_{L}(\lambda)\,\varepsilon^{a}{}_{\mu} = \lambda^{a}{}_{b}\,(\bar{e}+\varepsilon)^{b}{}_{\mu}$	$\delta^{B}_{L}(\lambda)\,\varepsilon^{a}{}_{\mu} = \lambda^{a}{}_{b}\,\varepsilon^{b}{}_{\mu}$
$\delta^{G}_{L}(\lambda)\,\tau^{ab}{}_{\mu} = -\nabla_{\mu}\lambda^{ab}$	$\delta^{B}_{L}(\lambda)\,\tau^{ab}{}_{\mu} = [\lambda,\tau_{\mu}]^{ab}$

O(4) covariantization after the split

$$\widetilde{\widetilde{\delta}}_{D}^{G}(v) := \delta_{D}^{G}(v) + \delta_{L}^{G}(v^{\mu}\,\overline{\omega}_{\mu}{}^{\cdot\cdot})$$

$$\widetilde{\widetilde{\delta}}_{D}^{B}(v) := \delta_{D}^{B}(v) + \delta_{L}^{B}(v^{\mu}\,\overline{\omega}_{\mu}{}^{\cdot\cdot})$$

Lie algebra of true gauge transformation:

$$[\widetilde{\widetilde{W}}_{D}^{G}(v_1), \widetilde{\widetilde{W}}_{D}^{G}(v_2)] = \widetilde{\widetilde{W}}_{D}^{G}([v_1,v_2]) + W_{L}^{G}(v_1^{\mu} v_2^{\nu}\, \overline{F}_{\mu\nu}{}^{\cdot\cdot})$$

$$[W_{L}^{G}(\lambda_1), W_{L}^{G}(\lambda_2)] = W_{L}^{G}([\lambda_1,\lambda_2])$$

$$[\widetilde{\widetilde{W}}_{D}^{G}(v), W_{L}^{G}(\lambda)] = \widetilde{\widetilde{W}}_{D}^{G}(v^{\mu}\,\overline{\nabla}_{\mu}\lambda)$$

$\rightsquigarrow$ determines Faddeev-Popov operator

Lie algebra of background gauge transformations:

$$[\widetilde{\widetilde{W}}_{D}^{B}(v_1), \widetilde{\widetilde{W}}_{D}^{B}(v_2)] = \widetilde{\widetilde{W}}_{D}^{B}([v_1,v_2]) - W_{L}^{B}(v_1^{\mu} v_2^{\nu}\, \overline{F}_{\mu\nu}{}^{\cdot\cdot})$$

$$[W_{L}^{B}(\lambda_1), W_{L}^{B}(\lambda_2)] = W_{L}^{B}([\lambda_1,\lambda_2])$$

$$[\widetilde{\widetilde{W}}_{D}^{B}(v), W_{L}^{B}(\lambda)] = 0$$

$\rightsquigarrow$ determines theory space $\mathcal{T}$

Generating fctl. with IR cutoff

$$\exp W_k[s^\mu_a, t^\mu_{ab}, \cdots ; \bar{e}, \bar{\omega}]$$

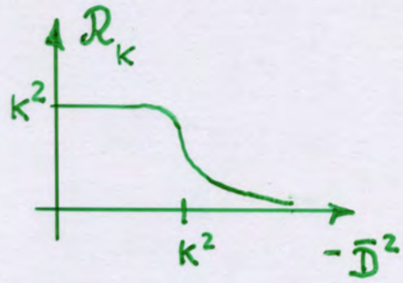
$$= \int \mathcal{D}\varepsilon^a_\mu \, \mathcal{D}\tau^{ab}_\mu \; e^{-S[\bar{e}+\varepsilon, \bar{\omega}+\tau]} \cdot e^{-\Delta_k S} \cdot$$

$$\cdot \, e^{-S_{gf}} \underbrace{\int \mathcal{D}\xi \, \mathcal{D}\bar{\xi}}_{\substack{\text{Diff.} \\ \text{ghosts}}} \; \underbrace{\mathcal{D}\Upsilon \, \mathcal{D}\bar{\Upsilon}}_{\substack{O(4) \\ \text{ghosts}}} \; e^{-S_{gh}} \cdot$$

$$\cdot \exp \int d^4x \, \bar{e} \left\{ s^\mu_a \, \varepsilon^a_\mu + t^\mu_{ab} \, \tau^{ab}_\mu + \text{ghost terms} \right\}$$

Cutoff action:

$$\Delta_k S = \frac{1}{2} \int \bar{e} \; (\varepsilon, \tau) \, \mathcal{R}_k(-\bar{D}^2) \begin{pmatrix} \varepsilon \\ \tau \end{pmatrix} \quad + \text{ghost terms}$$



Ghost action:

$$S_{gh} = \int \bar{e} \; (\bar{\xi}_\mu, \bar{\Upsilon}_{ab}) \begin{bmatrix} \text{Faddeev-} \\ \text{Popov} \\ \text{operator} \end{bmatrix} \begin{pmatrix} \xi^\mu \\ \Upsilon^{ab} \end{pmatrix}$$

Gauge fixing action:

Diff $(\mathcal{M})$ g.f. condition: $\mathcal{F}_\mu[\bar{e},\bar{\omega}]$ (4)

$O(4)_{Loc}$ g.f. condition: $\mathcal{G}^{ab}[\bar{e},\bar{\omega}]$ (6)

$$S_{gf} = \frac{1}{2\alpha_D}\int d^4x\, \bar{e}\, \bar{g}^{\mu\nu}\mathcal{F}_\mu\mathcal{F}_\nu + \frac{1}{2\alpha_L}\int d^4x\, \bar{e}\;\; \mathcal{G}^{ab}\mathcal{G}_{ab}$$

chosen such that $\tilde{\tilde{W}}_D^{G} S_{gf} \neq 0\,,\; W_L^{G} S_{gf} \neq 0$

but $\tilde{\tilde{W}}_D^{B} S_{gf} = 0\,,\; W_L^{B} S_{gf} = 0$

Example:

$$\mathcal{F}_\mu = \bar{e}_a{}^\nu\left[\bar{D}_\nu\, \varepsilon^a{}_\mu + \beta\, \bar{D}_\mu\, \varepsilon^a{}_\nu\right]$$

$$\mathcal{G}^{ab} = \tfrac{1}{2}\,\bar{g}^{\mu\nu}\left[\varepsilon^a{}_\mu\,\bar{e}^b{}_\nu - \varepsilon^b{}_\mu\,\bar{e}^a{}_\nu\right]$$

$$\rightsquigarrow \quad \bar{e}_a^\mu\,\bar{e}_b^\nu\,\mathcal{G}^{ab} = \varepsilon^{[\mu\nu]}$$

Derived objects:

$(\bar{e}_a{}^\mu) := (\bar{e}^a{}_\mu)^{-1}$; $\quad \bar{g}_{\mu\nu} := \bar{e}^a{}_\mu\,\bar{e}^b{}_\nu\,\delta^{ab}$

$\bar{D} := \partial + \bar{\omega} + \bar{\Gamma} = \bar{\nabla} + \bar{\Gamma}$

with $\bar{\Gamma} \equiv \bar{\Gamma}(\bar{e},\bar{\omega})$ such that $\bar{D}_\mu\,\bar{e}^a{}_\nu \overset{!}{=} 0$

The eff. average action:

$$\Gamma_k[\langle\varepsilon\rangle, \langle\tau\rangle, \langle \text{ghosts}\rangle \,;\, \bar{e}, \bar{\omega}]$$

$$\equiv \Gamma_k[e, \omega, \bar{e}, \bar{\omega}, \langle\text{ghosts}\rangle]$$

$$:= \text{Legendre transf. of } W_k \quad - \Delta_k S[\langle\varepsilon\rangle, \langle\tau\rangle, \cdots]$$

Full expct. val. fields:

$$e^a{}_\mu := \bar{e}^a{}_\mu + \langle \varepsilon^a{}_\mu\rangle$$

$$\omega^{ab}{}_\mu := \bar{\omega}^{ab}{}_\mu + \langle \tau^{ab}{}_\mu\rangle$$

By construction, Γ_k is invariant under background gauge transformations! It satisfies the FRGE:

$$k\partial_k \Gamma_k[e, \omega, \bar{e}, \bar{\omega}, \xi, \bar{\xi}, \Upsilon, \bar{\Upsilon}]$$

$$= \frac{1}{2}\,\mathrm{STr}\Big[\big(\Gamma_k^{(2)} + \mathcal{R}_k\big)^{-1} k\partial_k \mathcal{R}_k\Big]$$

Hessian w.r.t. $(e, \omega, \text{ghosts})$

Generates coarse graining flow on the theory space:

$$\mathcal{T} = \Big\{ A[e^a_\mu, \omega^{ab}_\mu, \bar{e}^a_\mu, \bar{\omega}^{ab}_\mu, \xi^\mu, \bar{\xi}_\mu, \Upsilon^{ab}, \bar{\Upsilon}_{ab}] \;\Big|\; \tilde{\tilde{W}}{}^{\mathbf{B}}_{D}(v)\, A = 0,\; W^{\mathbf{B}}_{L}(\lambda)\, A = 0 \quad \forall v, \lambda \Big\}$$

A Holst-type Truncation

$$\Gamma_k[e, \omega, \bar{e}, \bar{\omega}, \text{ghosts}]$$

$$= \frac{-1}{16\pi G_k} \int d^4x \, e \left[e_a{}^\mu e_b{}^\nu \left(F^{ab}{}_{\mu\nu} - \frac{1}{\gamma_k} * F^{ab}{}_{\mu\nu} \right) - 2\Lambda_k \right]$$

$$+ \left(S_{gf} + S_{gh} \right)[e, \omega, \bar{e}, \bar{\omega}, \text{ghosts}]$$

$$= \frac{-1}{16\pi G_k} \Big(\frac{1}{2} \int \varepsilon_{abcd} F^{ab} \wedge e^c \wedge e^d$$

$$- \frac{\Lambda_k}{12} \int \varepsilon_{abcd} \, e^a \wedge e^b \wedge e^c \wedge e^d$$

Hilbert-Palatini action of Einstein-Cartan gravity

$$- \frac{1}{\gamma_k} \int F^{ab} \wedge e_a \wedge e_b \Big)$$

Immirzi term

$$+ \cdots$$

Dualization: $* F^{ab}{}_{\mu\nu} \equiv \frac{1}{2} \varepsilon^{ab}{}_{cd} F^{cd}{}_{\mu\nu}$

Coordinates on $\mathcal{T}_{\text{trunc}}$: (g, λ, γ)

$$g(k) := k^2 \, G_k$$

$$\lambda(k) := \Lambda_k / k^2$$

The Results

- From "proper time" simplification of FRGE
- β-fcts. computed for all $(\alpha_D, \alpha_L, \beta ; \mu)$

$$\partial_t g = \beta_g \equiv [2 + \eta_N]\, g \quad , \quad \eta_N = 16\pi\, g\, f_+(\lambda, \gamma)$$

$$\partial_t \gamma = \beta_\gamma \equiv 16\pi\, g\, \gamma \left[\gamma f_-(\lambda, \gamma) - f_+(\lambda, \gamma) \right]$$

$$\partial_t \lambda = \beta_\lambda \equiv -2\lambda + 8\pi\, g \left[2\lambda f_+(\lambda, \gamma) + f_3(\lambda, \gamma) \right]$$

$$f_\pm(\lambda, -\gamma) = \pm f_\pm(\lambda, \gamma)$$

The g-λ-System: (Immirzi term discarded)

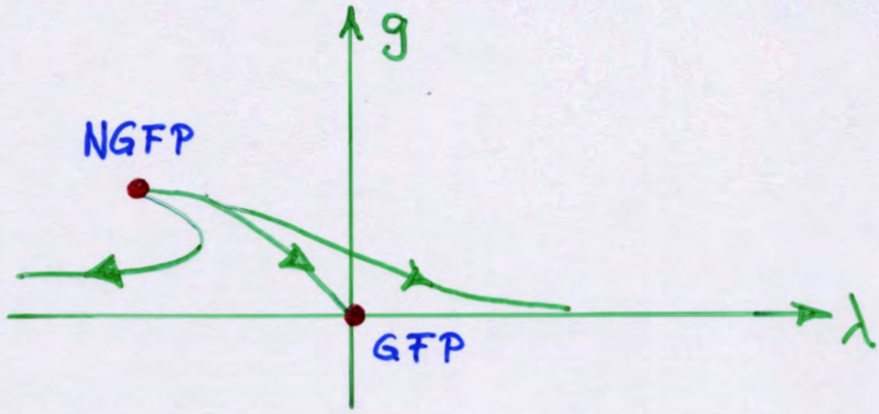
Flow is similar to Einstein-Hilbert truncation of metric gravity!

$\exists$ GFP: $g_* = 0\,,\ \lambda_* = 0$

$\exists$ NGFP: $g_* > 0\,,\ \lambda_* < 0$

Critical exponents Θ_1, Θ_2 real and positive.

Results robust w.r.t. variation of $\alpha \equiv (\alpha_D, \alpha_L, \beta; \rho)$.



The g-λ-γ-System:

The 2D NGFP splits into two 3D NGFPs:

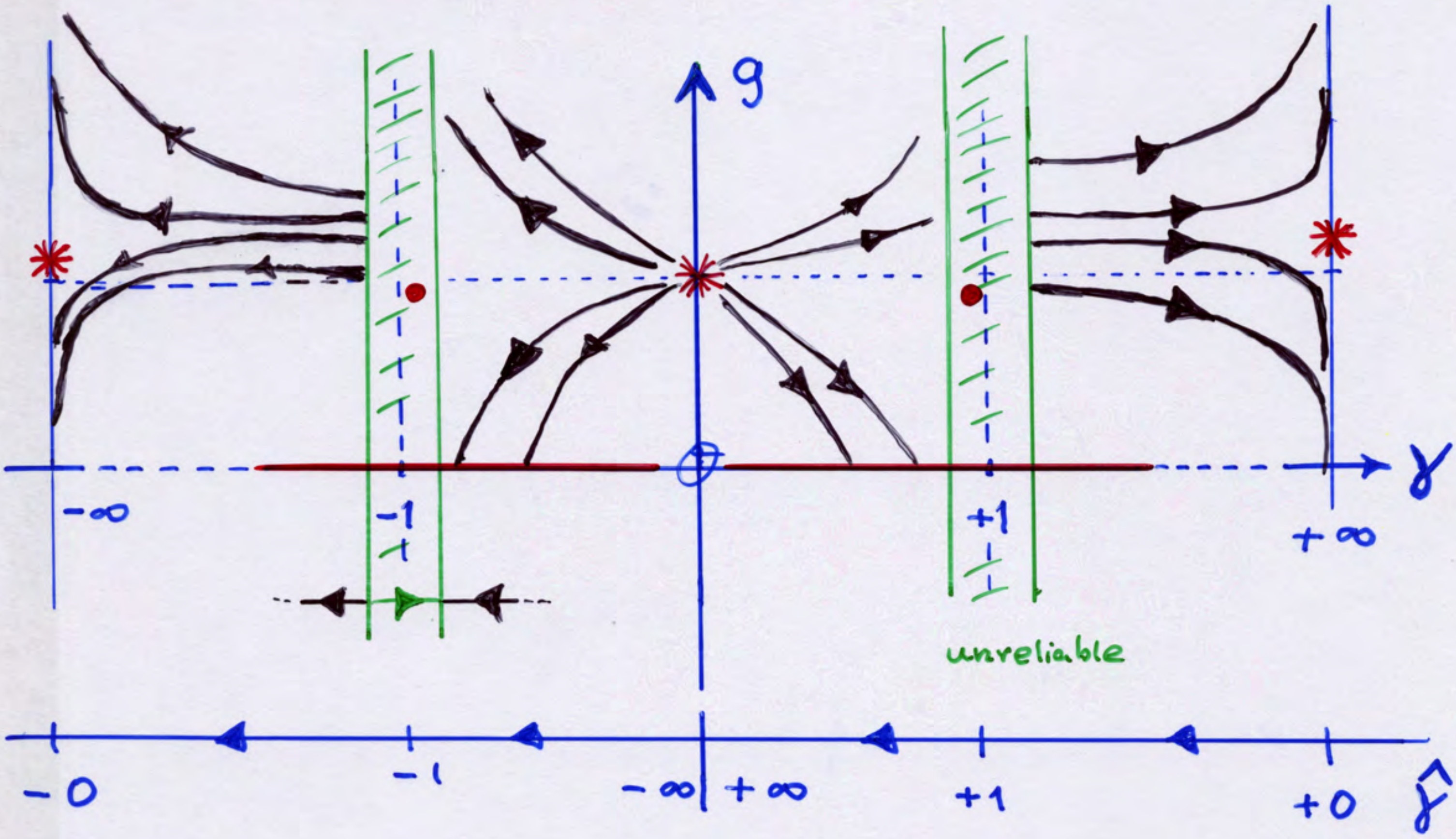
$$(\underbrace{g_*, \lambda_*}_{\approx \text{ 2D values}}, \gamma_*)$$

with $\gamma_* = 0$ and "$\gamma_* =$ infinity", respectively.

For $\alpha_D = \alpha_L = 1$: $g_* = 2.75\,,\ \lambda_* = -3.28$

Both NGFPs seem suitable for asym. safety!

The g-γ-System: (Λe^4-term discarded)



(1) Symmetry w.r.t. $\gamma \to -\gamma$

(2) $g = 0$ is fixed line

(3) $\exists$ NGFP $\left(\gamma_* = 0,\ g_*^{(0)} \gtrless 0\right)$; sign α dependent

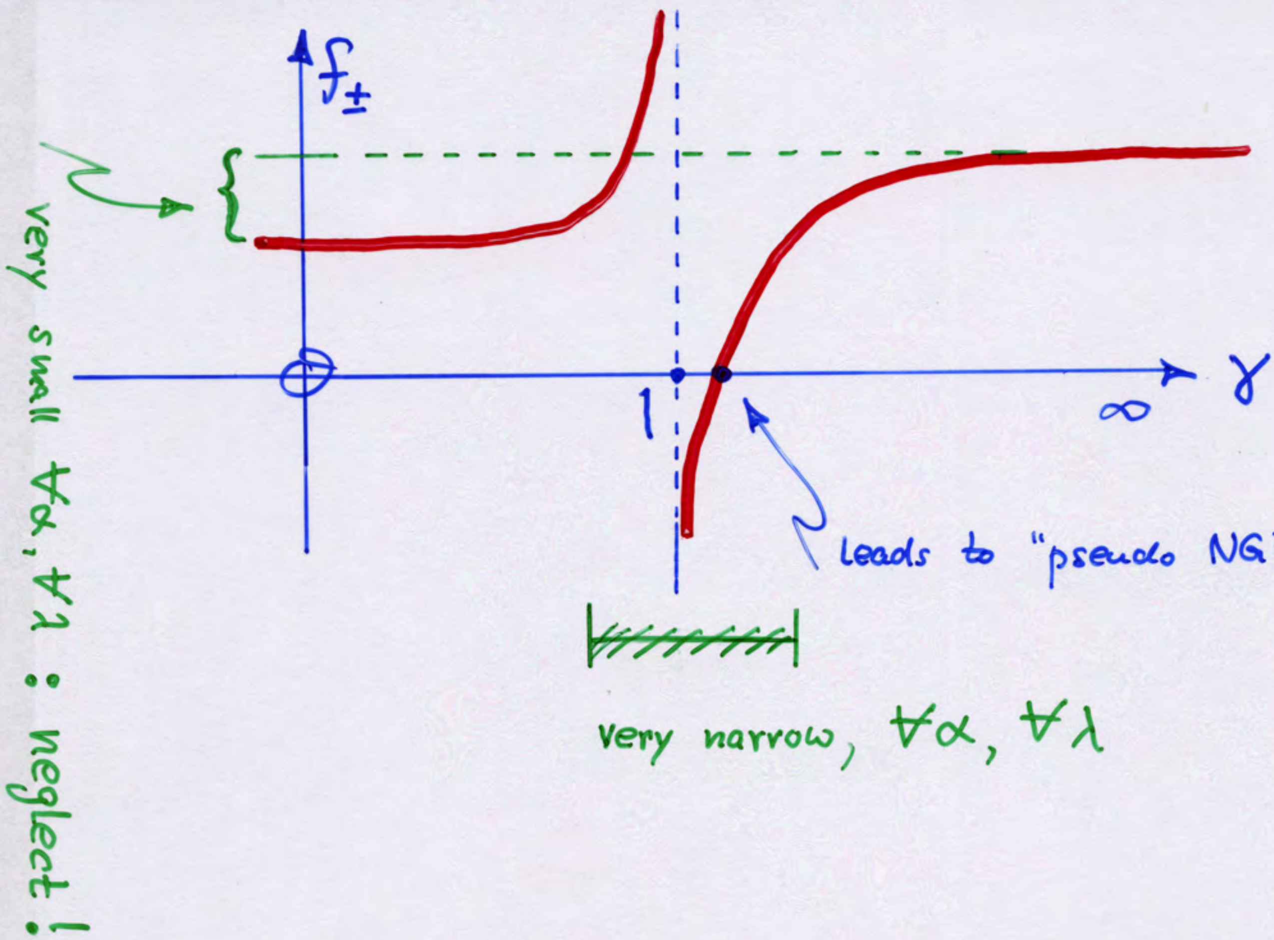
(4) $\exists$ NGFP $\left(\hat{\gamma}_* = 0,\ g_*^{(\infty)} \approx g_*^{(0)}\right)$; $\hat{\gamma} \equiv \gamma^{-1}$

(5) $\exists$ two NGFPs $\left(\pm \gamma_*^{(1)}, g_*^{(1)}\right)$ with

$\gamma_*^{(1)} \approx 1$ and $g_*^{(1)} \approx g_*^{(0)}$; probably unreliable

(6) Attractivity properties in γ-direction are α dependent.

$f_\pm(\lambda, \gamma)$ has a pole at $\gamma = \pm 1$ and a nearby zero:



For γ not too close to ± 1, the functions f_+ and γf_- are almost independent of γ:

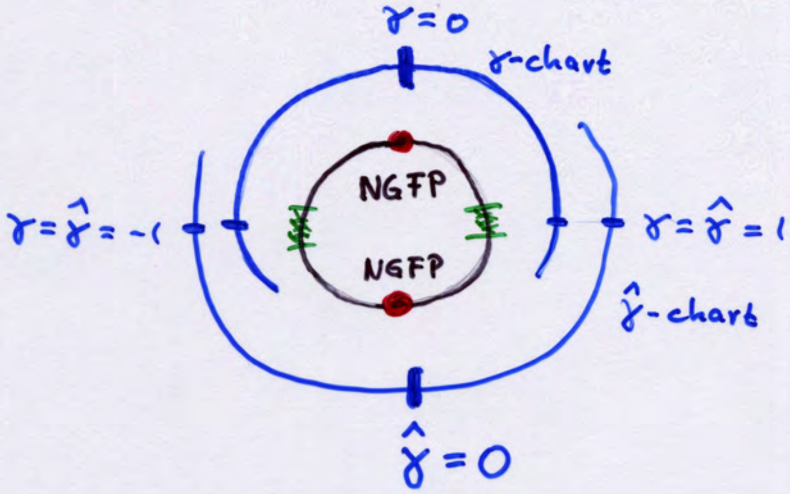
$$\left.\begin{aligned} f_+(\lambda, \gamma) &\approx \frac{1}{16\pi}\, b_+(\lambda) \\ f_-(\lambda, \gamma) &\approx \frac{1}{16\pi}\, \frac{1}{\gamma}\, b_-(\lambda) \end{aligned}\right\} (+)$$

Conjecture: In a more general truncation the pole and the zero might "annihilate" and $(+)$ is valid <u>for all γ</u> then.

γ-axis compactifies to a circle:

Cover 3D theory space by two coordinate charts with local coordinates (g, λ, γ) and $(g, \lambda, \hat{\gamma})$, respectively.

- Transition function on the overlaps: $\hat{\gamma} = \frac{1}{\gamma}$
- β-fcts. transform as vector components: $\beta_{\hat{\gamma}} = \left(\frac{\partial \hat{\gamma}}{\partial \gamma}\right) \beta_\gamma$

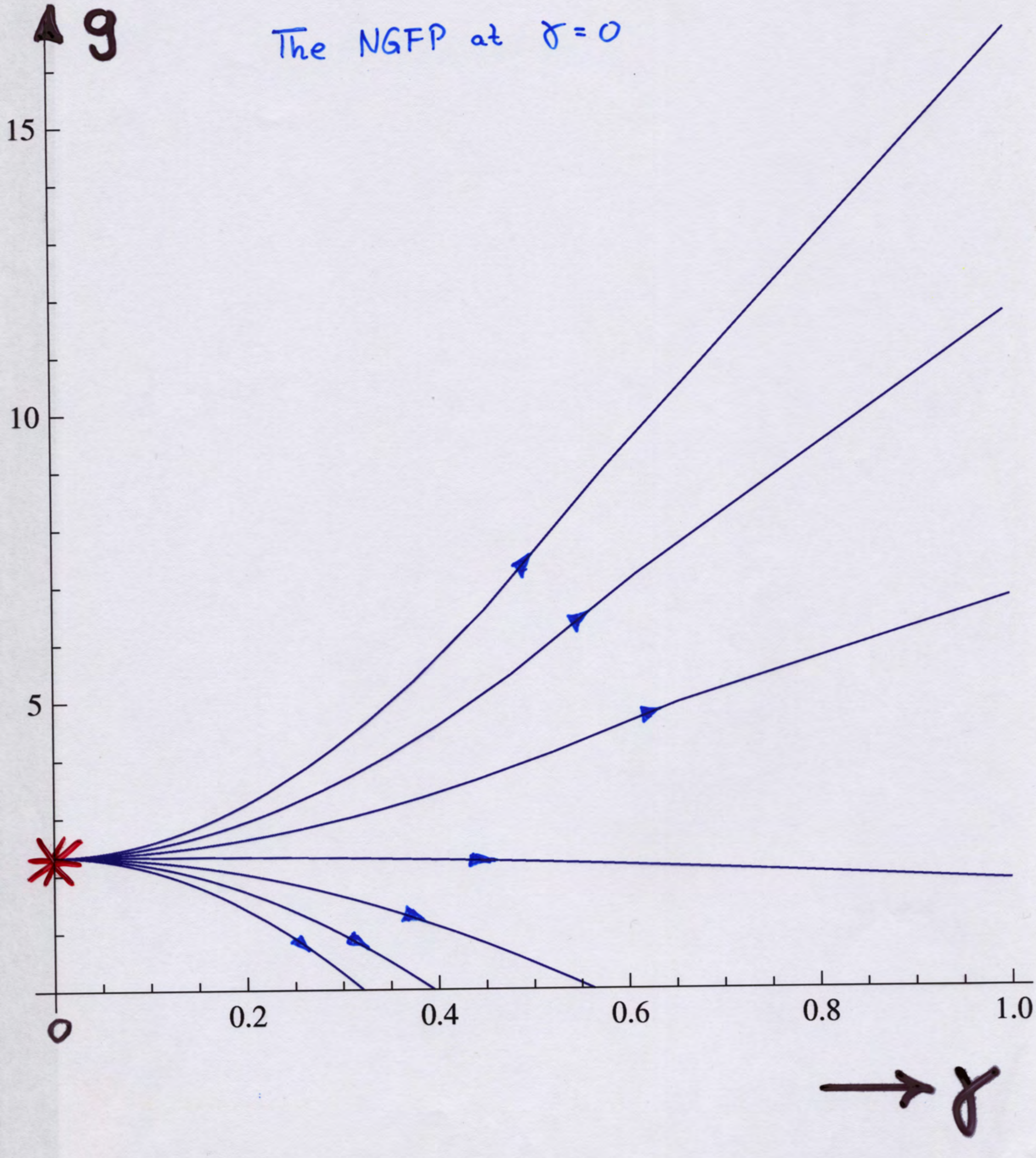


- Within the approximation (+):

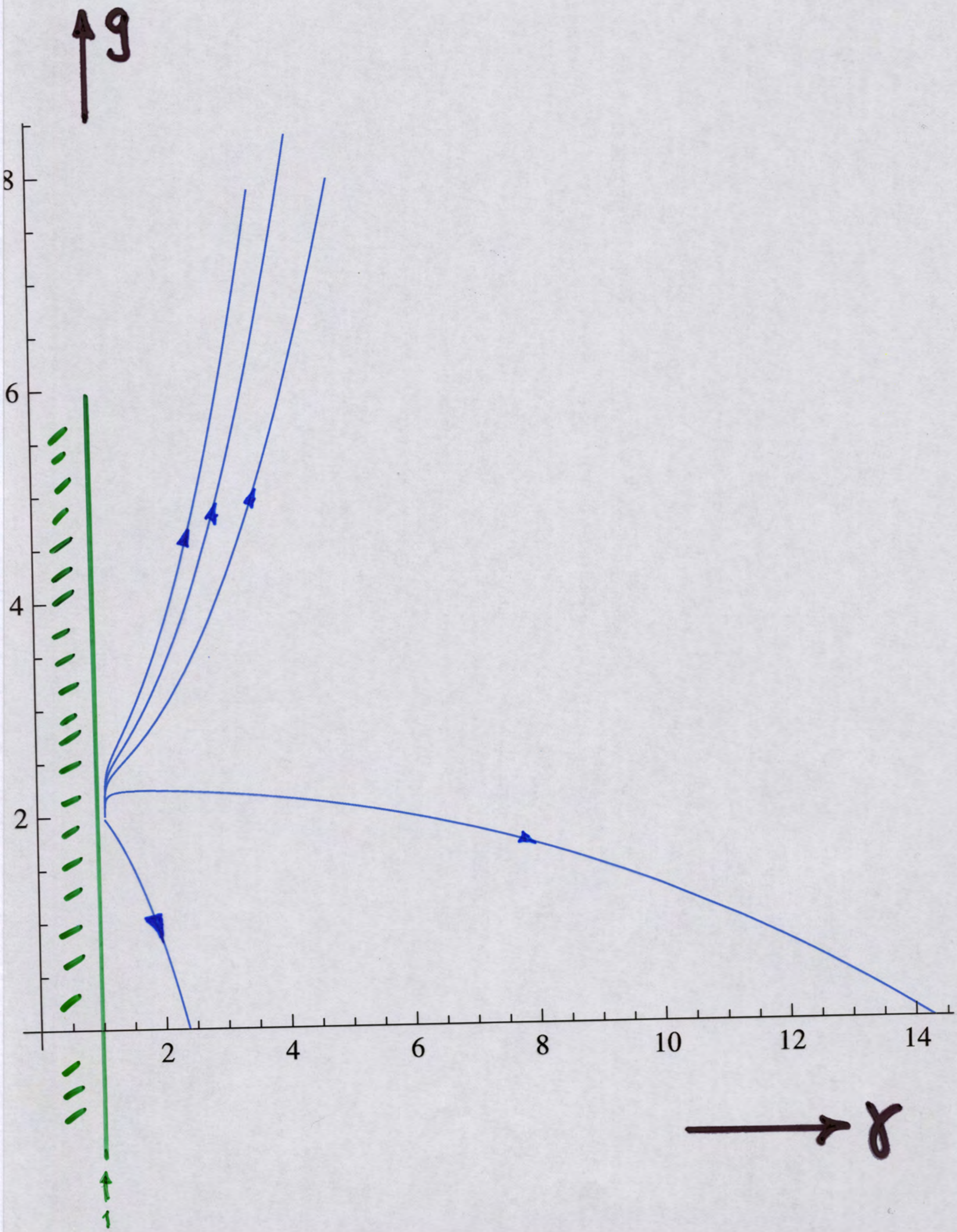
$$\beta_\gamma(\gamma) = -(b_+ - b_-)\, g\, \gamma$$

$$\beta_{\hat{\gamma}}(\hat{\gamma}) = +(b_+ - b_-)\, g\, \hat{\gamma}$$

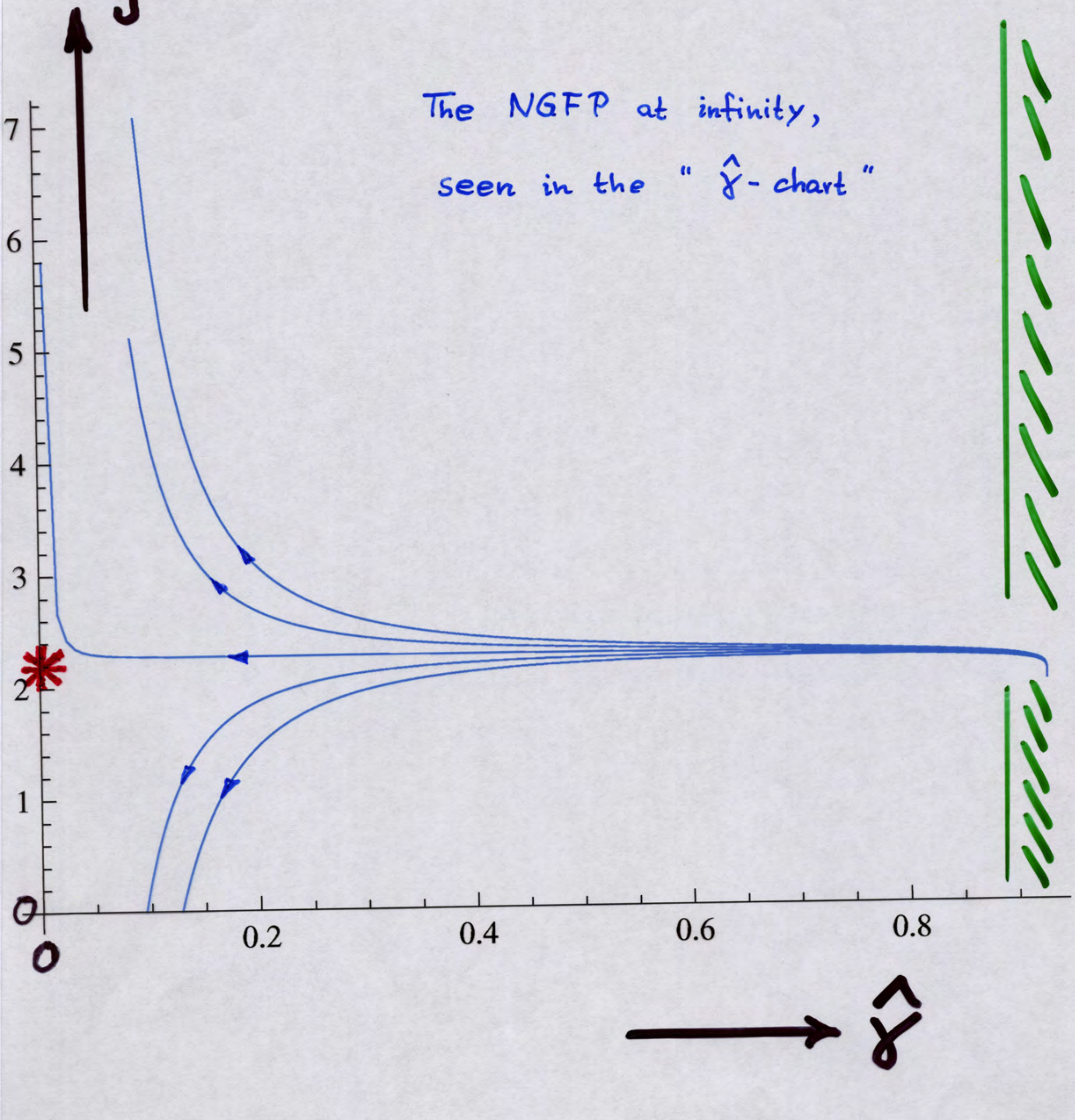
RG flow displays "duality" between small and large values of the Immirzi parameter!

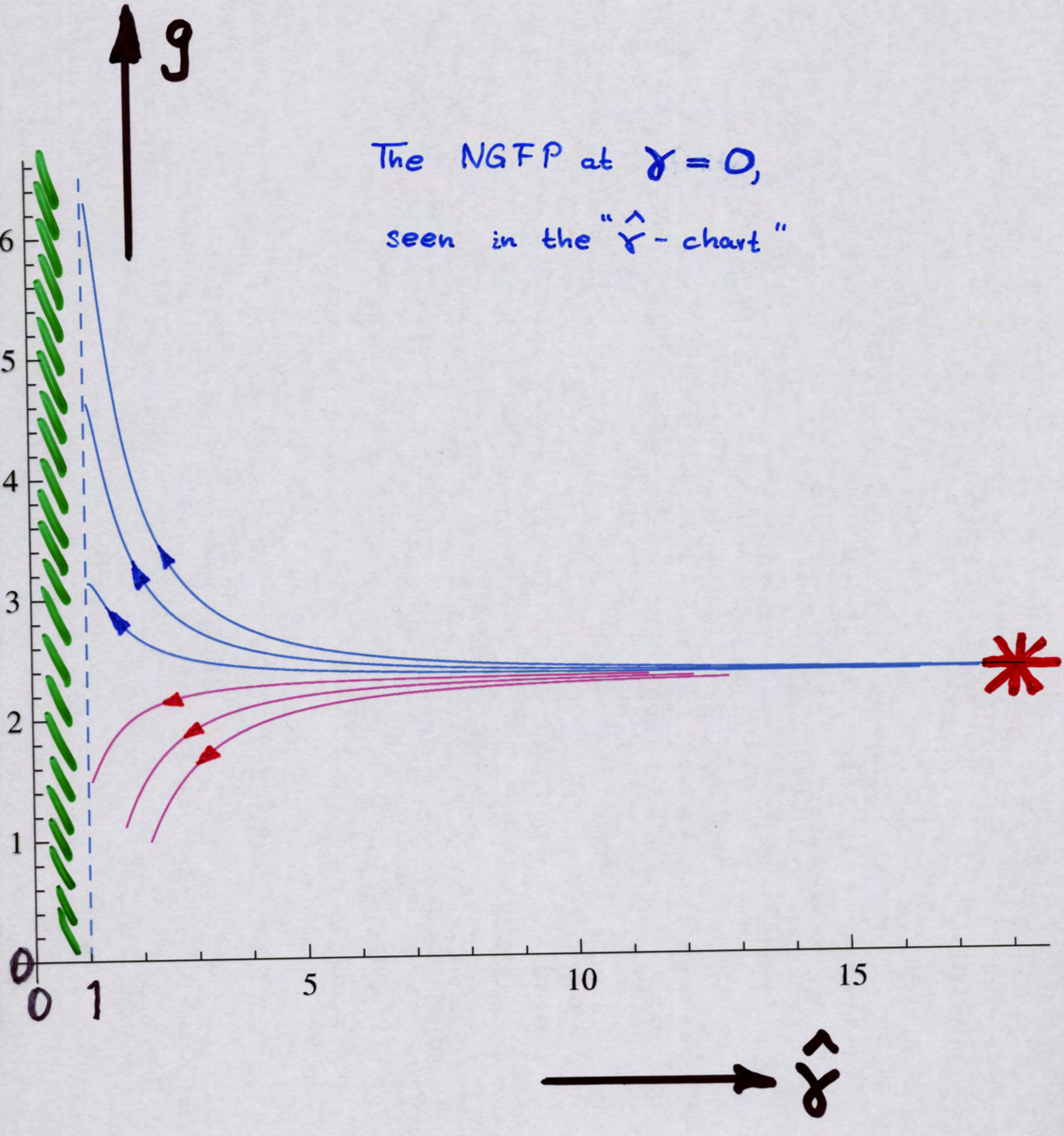


a



b


The NGFP at infinity,
seen in the " $\hat{\gamma}$ - chart "
g
$\hat{\gamma}$
0
1
2
3
4
5
6
7
0.2
0.4
0.6
0.8



The NGFP at γ = 0,
seen in the "γ̂ - chart"
g
γ̂
0
1
2
3
4
5
6
0
1
5
10
15

Conclusion

- Einstein - Cartan gravity successfully passed all tests for Asymptotic Safety performed so far.

- The Immirzi parameter shows a nontrivial, but in the present truncation gauge-fixing-dependent RG running.

- Its fixed point values are $\gamma = 0$ and "$\gamma = \infty$", respectively. Besides $\gamma = 0, \pm 1, \pm\infty$ no other values of γ seem distinguished....

Cf. also D. Benedetti, S. Speziale (2011) : pert. theory