ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Scale-invariant alternatives to general relativity

Mikhail Shaposhnikov

Zurich, 21 June 2011

Based on:

- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 162
- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 187
- Diego Blas, M.S., Daniel Zenhäusern, arXiv:1104.1392 [hep-th] (2011)

GR

Einstein gravity is a theory which is invariant under all diffeomorphisms, Diff: $x^\mu \rightarrow f^\mu(x^\nu)$.

Pros - consistence with all tests of GR. One of the main predictions - existence of massless graviton.

Problems of GR

- Large dimensionfull coupling constant $G_N^{-1} = M_P^2$, leading to hierarchy problem $m_H \ll M_P$.
- Extra arbitrary fundamental parameter - cosmological constant - which is known to be very small.
- Quantum gravity?

Our proposal

$$Diff \rightarrow TDiff \times Dilatations$$

TDiff - volume conserving coordinate transformations,

$$det\left[\frac{\partial f^{\mu}}{\partial x^{\nu}}\right] = 1 \, .$$

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \;\rightarrow\; \sigma^{n}\Psi(\sigma x) \; ,$$

$n = 0$ for the metric, $n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

Similarity with GR: consistency with all tests

Differences with GR:

- Dynamical origin of all mass scales
- Hierarchy problem gets a different meaning - an alternative (to SUSY, techicolor, little Higgs or large extra dimensions) solution of it may be possible.
- Cosmological constant problem acquires another formulation.
- Natural chaotic cosmological inflation
- Low energy sector contains a massless dilaton
- There is Dark Energy even without cosmological constant
- Quantum gravity?

Outline

- Field theory: classical scale invariance and its spontaneous breakdown
- Unimodular gravity
- Scale invariance, unimodular gravity, cosmological constant, inflation and dark energy
- Quantum scale invariance
- Dilaton as a part of the metric in TDiff gravity
- Conclusions

Scale invariance

Trivial statement: multiply all mass parameters in the theory

$$M_W, \;\; \Lambda_{QCD}, \;\; M_H, M_{Pl}, ...$$

by one and the same number : $M \to \sigma M$. Physics is not changed!

Indeed, this change, supplemented by a dilatation of space-time coordinates $x^\mu \to \sigma x^\mu$ and an appropriate redefinition of the fields does not change the complete quantum effective action of the theory.

This transformation, however, is not a symmetry of the theory (symmetry = transformation of dynamical variables which does not change the action)

Dilatations:

$$\phi(x) \ \rightarrow \ \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \ \rightarrow \ g_{\mu\nu}(\sigma x)$.

Dilatation symmetry forbids all dimensionfull couplings: Higgs mass, Newton gravity constant, cosmological constant, etc

Dilatations:

$$\phi(x) \rightarrow \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x)$.

Dilatation symmetry forbids all dimensionfull couplings: Higgs mass, Newton gravity constant, cosmological constant, etc

Ruled out by observations?

Dilatations:

$$\phi(x) \rightarrow \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x)$.

Dilatation symmetry forbids all dimensionfull couplings: Higgs mass, Newton gravity constant, cosmological constant, etc

Ruled out by observations?

No, if it is spontaneously broken!

Dilatation symmetry

Dilatations:

$$\phi(x) \rightarrow \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x)$.

Dilatation symmetry forbids all dimensionfull couplings: Higgs mass, Newton gravity constant, cosmological constant, etc

Ruled out by observations?

No, if it is spontaneously broken!

First step: consider classical physics only (no parameters like Λ_{QCD}), just tree explicit mass parameters such as M_H, M_W, M_{Pl}.

Classical scale invariant theory

Unique regular scale-invariant Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM[M\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger \varphi = 2h^2$):

$$V(\varphi, \chi) = \lambda \left(\varphi^\dagger \varphi - \frac{\alpha}{2\lambda} \chi^2 \right)^2 + \beta \chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi \right) \frac{R}{2} \, ,$$

Forget first about gravity. Consider scalar potential

$$V(\varphi,\chi)=\lambda\left(\varphi^\dagger\varphi-\frac{\alpha}{2\lambda}\chi^2\right)^2+\beta\chi^4,$$

Requirements: vacuum state exists if $\lambda\geq 0, \quad \beta\geq 0$

For $\lambda>0, \;\; \beta>0$ the vacuum state is unique: $\chi=0, \;\; \varphi=0$ and scale invariance is exact.

Field propagators: scalar $1/p^2$, fermion $\not{p}/p^2$. Greenberg, 1961:

free quantum field theory!!

If not - theory does not describe particles !!

Gravity included - argument for $\beta = \mathbf{0}$ gets weaker:

- for $\beta > \mathbf{0}$ there is a de Sitter solution
- for $\beta < \mathbf{0}$ there is an AdS solution

However,

- dS solution is not stable in the presence of massless scalar - no dS invariant ground sate exists
- AdS solution has different pathologies

For $\lambda > 0, \ \ \beta = 0$ the scale invariance can be spontaneously broken.

The vacuum manifold:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \ \ M_{Pl} \sim \chi_0$$

Phenomenological requirement:

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \lll 1$$

Good news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

Good news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

Bad news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

Good news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

Bad news: cosmological constant may be zero due to scale invariance and requirement of presence of particles

Universe is in the state of accelerated expansion, $\Omega_{DE} \simeq 0.7$!

Unimodular gravity

Ordinary gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates.

Invariant under general coordinate transformations

Unimodular gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates with set$[g] = -1$. Invariant under general coordinate transformations which conserve the 4-volume.

van red Bi, van Dam, N

Origin of UG: Field theory describing spin 2 massless particles is either GR or UG

Number of physical degrees of freedom is the same.

Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\,[R+\Lambda]$$

UG. Λ does not appear in the action:

$$S = -\frac{1}{M_P^2}\int d^4x R$$

Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -\frac{1}{M_P^2} \int d^4x \sqrt{-g}\, [R + \Lambda]$$

UG. Λ does not appear in the action:

$$S = -\frac{1}{M_P^2} \int d^4x R$$

Cosmological constant problem is solved in UG??!!

Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\,[R+\Lambda]$$

UG. Λ does not appear in the action:

$$S = -\frac{1}{M_P^2}\int d^4x R$$

Cosmological constant problem is solved in UG??!!

Wilczek, Zee: NO!

UG is equivalent to

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\left[R + \Lambda(x)\left(1 - \frac{1}{\sqrt{-g}}\right)\right]$$

Equations of motion ($G_{\mu\nu}$ - Einstein tensor):

$$G_{\mu\nu} = -\Lambda(x)\, g_{\mu\nu} \;, \sqrt{-g} = 1$$

Bianchi identity: $\Lambda(x)_{;} = 0 \rightarrow \Lambda(x) = const.$

Solutions of UG are the same as solutions of GR with an arbitrary cosmological constant.

Conclusion: in UG cosmological constant reappears, but as an integral of motion, related to initial conditions

Scale invariance + unimodular gravity

Solutions of scale-invariant UG are the same as the solutions of scale-invariant GR with the action

$$S = -\int d^4x\sqrt{-g}\left[\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2} + \Lambda + ...\right] ,$$

Physical interpretation: Einstein frame

$$g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu} \ , \ \ (\xi_\chi \chi^2 + \xi_h h^2)\Omega^2 = M_P^2$$

Λ is not a cosmological constant, it is the strength of a peculiar potential!

Relevant part of the Lagrangian (scalars + gravity) in Einstein frame:

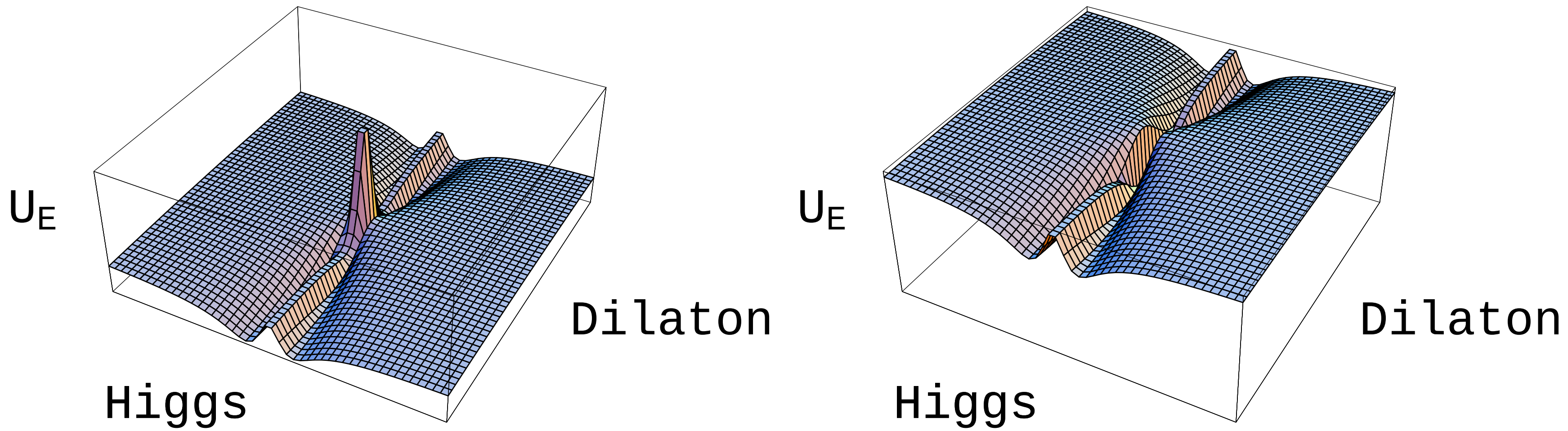
$$\mathcal{L}_E = \sqrt{-\tilde{g}} \left(-M_P^2 \frac{\tilde{R}}{2} + K - U_E(h,\chi) \right) ,$$

K - complicated non-linear kinetic term for the scalar fields,

$$K = \Omega^2 \left(\frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} (\partial_\mu h)^2 \right) - 3 M_P^2 (\partial_\mu \Omega)^2 .$$

The Einstein-frame potential $U_E(h,\chi)$:

$$U_E(h,\chi) = M_P^4 \left[\frac{\lambda \left(h^2 - \frac{\alpha}{\lambda} \chi^2 \right)^2}{4(\xi_\chi \chi^2 + \xi_h h^2)^2} + \frac{\Lambda}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \right] ,$$



Potential for the Higgs field and dilaton in the Einstein frame.

Left: $\Lambda > 0$, right $\Lambda < 0$.

50% chance ($\Lambda < 0$): inflation + late collapse

50% chance ($\Lambda > 0$): inflation + late acceleration

Inflation

Chaotic initial condition: fields χ and h are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \;\; \xi_\chi \ll 1$ (will be justified later)

Then - dynamics of the Higgs field is more essential, $\chi \simeq const$ and is frozen. Denote $\xi_\chi \chi^2 = M_P^2$.

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\tilde{h}}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \begin{cases} h \simeq \tilde{h} & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\tilde{h}}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

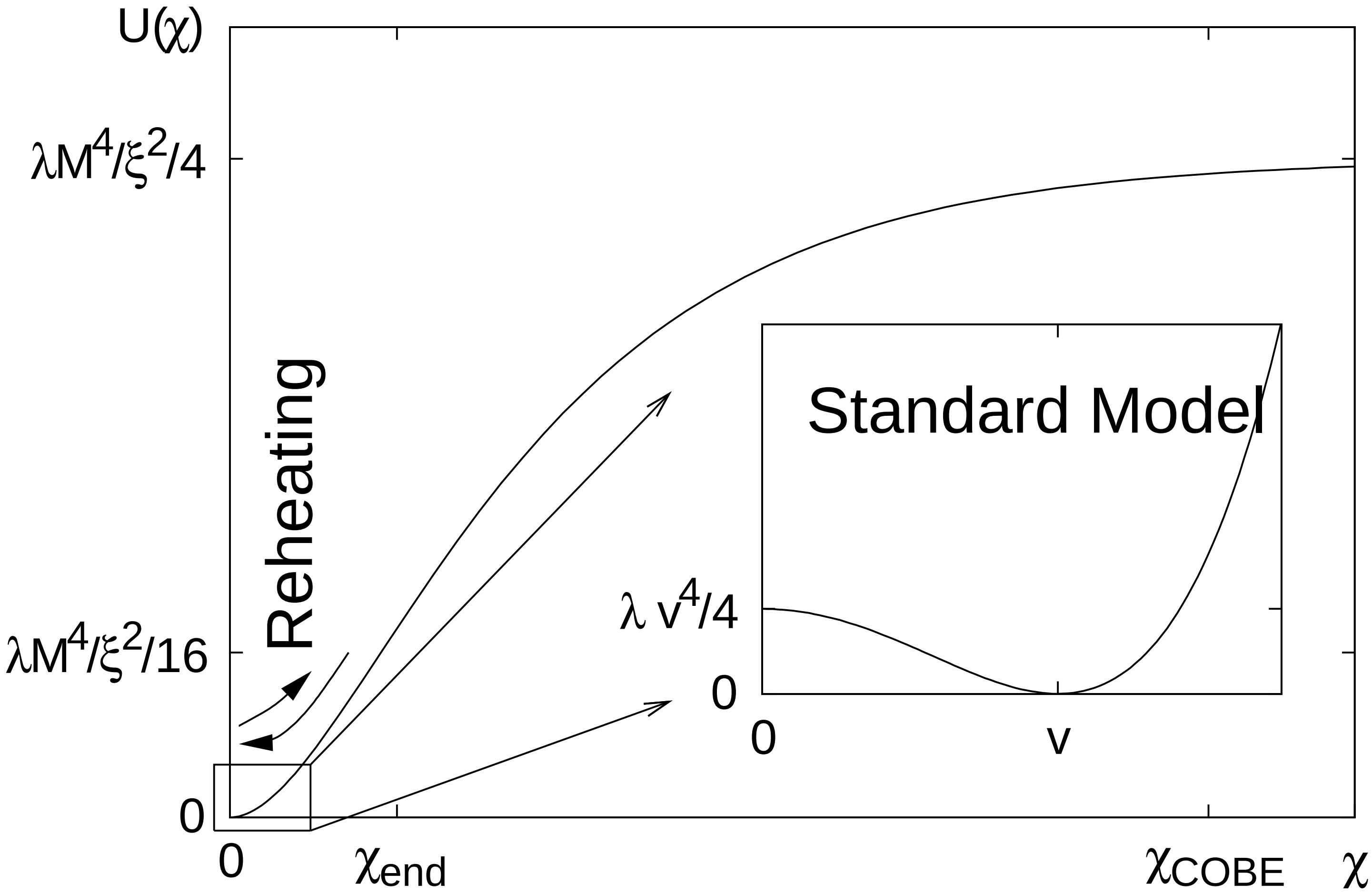
Resulting action (Einstein frame action)

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\tilde{h}\partial^\mu\tilde{h}}{2} - \frac{1}{\Omega(\tilde{h})^4}\frac{\lambda}{4}h(\tilde{h})^4\right\}$$

Potential:

$$U(\tilde{h}) = \begin{cases} \frac{\lambda}{4}\tilde{h}^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\tilde{h}}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Slow roll stage

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4}{3}\exp\left(-\frac{4\chi}{\sqrt{6}M_P}\right)$$

$$\eta = M_P^2\frac{d^2U/d\chi^2}{U} \simeq -\frac{4}{3}\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)$$

Slow roll ends at $\chi_{\rm end} \simeq M_P$

Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\rm end}^2}{M_P^2/\xi}$

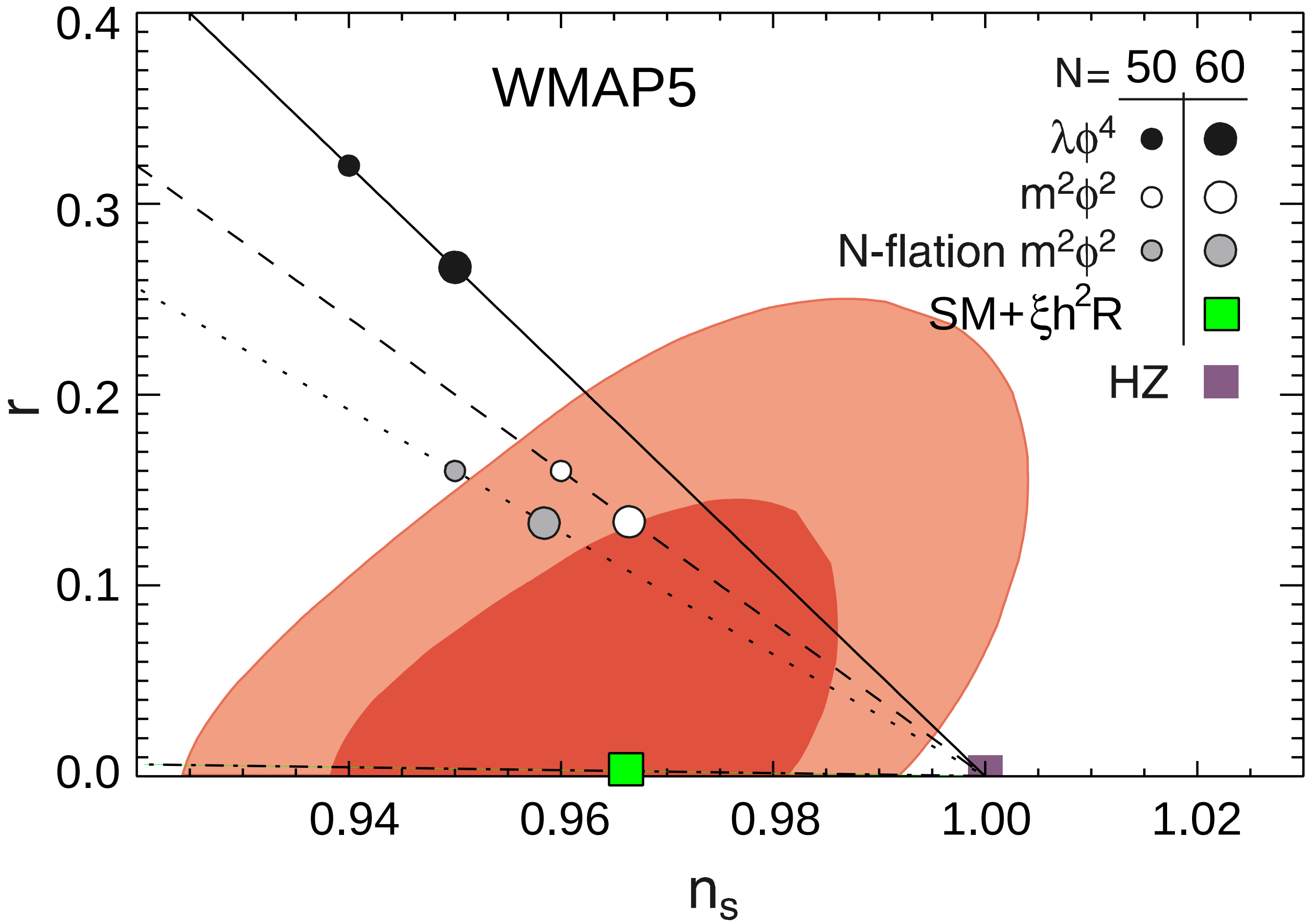
$$\chi_{60} \simeq 5M_P$$

COBE normalization $U/\epsilon = (0.027M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}}\frac{N_{\rm COBE}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

CMB parameters—spectrum and tensor modes



Dark energy

Late time evolution of dilaton ρ along the valley, related to χ as

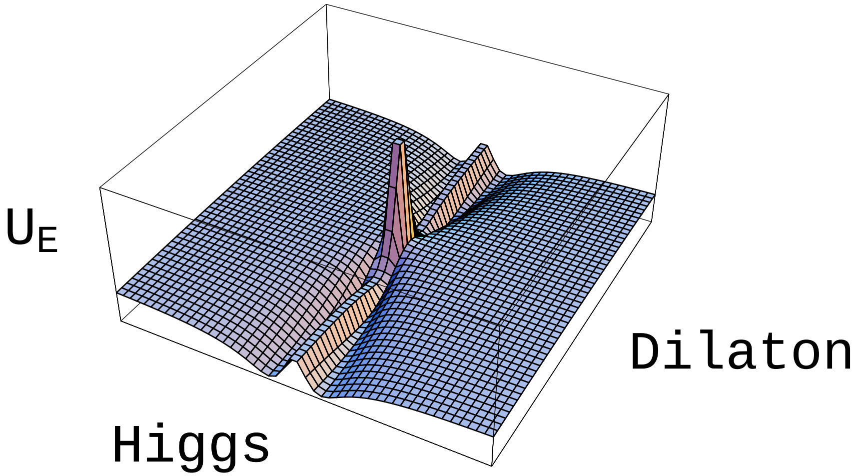
$$\chi = M_P \exp\left(\frac{\gamma\rho}{4M_P}\right), \quad \gamma = \frac{4}{\sqrt{6+\frac{1}{\xi_\chi}}} .$$

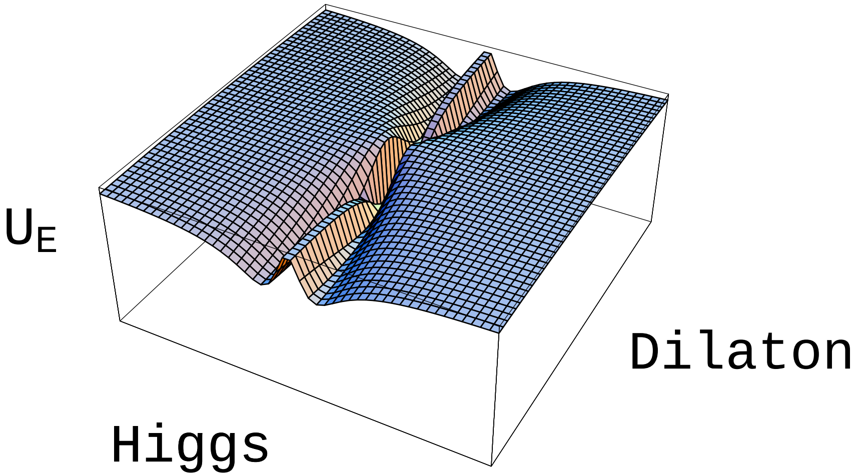
Potential: Wetterich; Ratra, Peebles

$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) .$$

From observed equation of state: $0 < \xi_\chi < 0.09$

Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.





Potential for the Higgs field and dilaton in the Einstein frame.

Left: $\mathbf{\Lambda > 0}$, right $\mathbf{\Lambda < 0}$.

$\mathbf{50\%}$ chance ($\mathbf{\Lambda < 0}$): inflation + late collapse

$\mathbf{50\%}$ chance ($\mathbf{\Lambda > 0}$): inflation + late acceleration

Intermediate summary

- Spontaneously broken scale invariance :
  - All mass scales originate from one and the same source - vev of the massless dilaton
  - Zero cosmological constant – $\beta = 0$ – existence of particles
  - Scale invariance naturally leads to flat potentials and thus to cosmological inflation
- TDiff or Unimodular gravity:
  - New parameter – strength of a particular potential for the dilaton
  - Dynamical Dark Energy

Quantum scale invariance

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} ,$$

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} \,,$$

Sidney Coleman: “For scale invariance,..., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way.”

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} \ ,$$

Sidney Coleman: “For scale invariance,..., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way.”

Known exceptions - not realistic theories like N=4 SYM

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} ,$$

Sidney Coleman: “For scale invariance,..., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way.”

Known exceptions - not realistic theories like N=4 SYM

Everything above does not make any sense???!!!

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

μ is a dimensionfull parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

Reason: mismatch in mass dimensions of bare (λ) and renormalized couplings (λ_R)

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

Reason: mismatch in mass dimensions of bare (λ) and renormalized couplings (λ_R)

Idea: Replace $\mu^{2\epsilon}$ by combinations of fields χ and h, which have the correct mass dimension:

$$\mu^{2\epsilon} \to \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$.

Zenhäusern, M.S

Englert, Truffin, Gastmans, 1976

Example of computation

Natural choice:

$$\mu^{2\epsilon} \to \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \;, (\xi_\chi \chi^2 + \xi_h h^2) \equiv \omega^2$$

Potential:

$$U = \frac{\lambda_R}{4} \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[h^2 - \zeta_R^2 \chi^2\right]^2 \;,$$

Counter-terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[A h^2 \chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B\chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + C h^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right],$$

To be fixed from conditions of absence of divergences and presence of spontaneous breaking of scale-invariance

$$U_1 = \frac{m^4(h)}{64\pi^2}\left[\log\frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right)\right]$$

$$+ \frac{\lambda_R^2}{64\pi^2}\left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4\right] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right),$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$C_0 = \frac{3}{2}\left[2a - 1 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + \frac{4}{3}\log 2\lambda_R + O(\zeta_R^2)\right],$$

$$C_2 = -3\left[2a - 3 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + O(\zeta_R^2)\right],$$

$$C_4 = \frac{3}{2}\left[2a - 5 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) - 4\log 2\lambda_R + O(\zeta_R^2)\right].$$

Origin of Λ_{QCD}

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \text{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \ .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

Quantum effective action is scale invariant in all orders of perturbation theory!!!

Quantum effective action is scale invariant in all orders of perturbation theory!!!

Problems

Quantum effective action is scale invariant in all orders of perturbation theory!!!

Problems

- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is “no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory

Quantum effective action is scale invariant in all orders of perturbation theory!!!

Problems

- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is “no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory
- Unitarity and high-energy behaviour: What is the high-energy behaviour $(E > M_{Pl})$ of the scattering amplitudes? Is the theory Unitary? Can it have a scale-invariant UV completion?

Consequences

- The dilaton is massless in all orders of perturbation theory

Consequences

- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)

Consequences

- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)
- Fifth force or Brans-Dicke constraints are not applicable to it

Consequences

- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)
- Fifth force or Brans-Dicke constraints are not applicable to it
- Higgs mass is stable against radiative corrections (in dimensional regularisation)

Consequences

- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)
- Fifth force or Brans-Dicke constraints are not applicable to it
- Higgs mass is stable against radiative corrections (in dimensional regularisation)
- Requirement of spontaneous breakdown of scale invariance - cosmological constant is tuned to zero in all orders of perturbation theory

Dilaton as a part of the metric

Previous discussion - ad hoc introduction of scalar field χ. It is massless, as is the graviton. Can it come from gravity?

Yes - it automatically appears in scale-invariant TDiff gravity as a part of the metric!

Consider arbitrary metric $g_{\mu\nu}$ (no constraints). Determinant g of $g_{\mu\nu}$ is TDiff invariant. Generic scale-invariant action for scalar field and gravity:

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\phi^2 f(-g)R - \frac{1}{2}\phi^2\mathcal{G}_{gg}(-g)(\partial g)^2$$

$$-\frac{1}{2}\mathcal{G}_{\phi\phi}(-g)(\partial\phi)^2 + \mathcal{G}_{g\phi}(-g)\phi\,\partial g\cdot\partial\phi - \phi^4 v(-g)\Big]\,.$$

Equivalence theorem

This TDiff theory is equivalent (at the classical level) to the following Diff scalar tensor theory:

$$\frac{\mathcal{L}_e}{\sqrt{-g}} = -\frac{1}{2}\phi^2 f(\sigma)R - \frac{1}{2}\phi^2\mathcal{G}_{gg}(\sigma)(\partial\sigma)^2 - \frac{1}{2}\mathcal{G}_{\phi\phi}(\sigma)(\partial\phi)^2$$

$$-\mathcal{G}_{g\phi}(\sigma)\phi\,\partial\sigma\cdot\partial\phi - \phi^4 v(\sigma) - \frac{\Lambda_0}{\sqrt{\sigma}}\,.$$

Transformation to Einstein frame:

$$\frac{\mathcal{L}_e}{\sqrt{-\tilde{g}}} = -\frac{1}{2}M^2\tilde{R} - \frac{1}{2}M^2\mathcal{K}_{\sigma\sigma}(\sigma)(\partial\sigma)^2 - \frac{1}{2}M^2\mathcal{K}_{\phi\phi}(\sigma)(\partial\ln(\phi/M))^2$$

$$-M^2\mathcal{K}_{\sigma\phi}(\sigma)\,\partial\sigma\cdot\partial\ln(\phi/M) - M^4V(\sigma) - \frac{M^4\Lambda_0}{\phi^4 f(\sigma)^2\sqrt{\sigma}}\,,$$

As expected, ϕ is a Goldstone boson with derivative couplings only (except the term containing Λ_0).

So, TDiff scale invariant theory automatically contains a massless dilaton. All previous results can be reproduced in it.

Conclusions

- Scale-invariance and TDiff gravity lead to:

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation
  - Higgs mass is stable against radiative corrections (at least if dimensional regularization is used) - no SUSY, or technicolor, or little Higgs, or large extra dimensions are needed

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation
  - Higgs mass is stable against radiative corrections (at least if dimensional regularization is used) - no SUSY, or technicolor, or little Higgs, or large extra dimensions are needed
  - Cosmological constant may be zero due to quantum scale-invariance and the requirement of existence of particles

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation
  - Higgs mass is stable against radiative corrections (at least if dimensional regularization is used) - no SUSY, or technicolor, or little Higgs, or large extra dimensions are needed
  - Cosmological constant may be zero due to quantum scale-invariance and the requirement of existence of particles
  - Even if $\Lambda = 0$, Dark Energy is present

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation
  - Higgs mass is stable against radiative corrections (at least if dimensional regularization is used) - no SUSY, or technicolor, or little Higgs, or large extra dimensions are needed
  - Cosmological constant may be zero due to quantum scale-invariance and the requirement of existence of particles
  - Even if $\Lambda = 0$, Dark Energy is present
  - The massless sector of the theory contains dilaton, which has only derivative couplings to matter and can be a part of the metric.

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.
- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.
- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).
- Why eventual cosmological constant is zero (or why $\beta = 0$)?

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.
- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).
- Why eventual cosmological constant is zero (or why $\beta = 0$)?
- Renormalizability

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.
- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).
- Why eventual cosmological constant is zero (or why $\beta = 0$)?
- Renormalizability
- Unitarity

Problems to solve

- Non-perturbative regularisation? Lattice proposal: Tkachev, M.S.
- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).
- Why eventual cosmological constant is zero (or why $\beta = 0$)?
- Renormalizability
- Unitarity
- High energy limit