Matrix models, noncommutative gauge theory and emergent geometry

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Motivation

- "fundamental" d.o.f. for quantum gravity might be different from macroscopic ones
  "emergence", quantum structure of space-time
- need quantum theory of all fund. interactions
- fine-tuning problems (Higgs, cosm. const.); mechanism ?

candidate: Matrix Models

- related to string theory ("nonpert. def.")
- describes dynamical NC space-time (branes)
- accessible, novel tools

NC gauge theory $\leftrightarrow$ gravity

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Matrix Models

IKKT (IIB) model Ishibashi, Kawai, Kitazawa and Tsuchiya 1996

$$S[X] = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]g_{aa'}g_{bb'} \; + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N, \mathbb{C}), \qquad a = 0, ..., 9$$

$$N \to \infty$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY, etc.

as
1) nonpert. def. of IIB string theory (on $\mathbb{R}^{10}$) ($IKKT$)
2) $\mathcal{N} = 4$ SUSY Yang-Mills gauge thy. on $\mathbb{R}^4_\theta$

dynamical NC branes $\mathcal{M} \subset \mathbb{R}^{10}$

carry (NC) gauge theory coupled to gravity (H.S. 2007 ff)

(brane-world scenarios)

Space-time & gravity from matrix models:

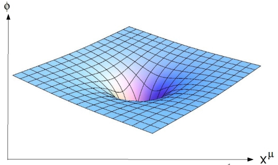
e.o.m.: $\delta S = 0 \Rightarrow [X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

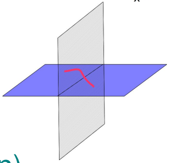
- $[X^a, X^b] = i\theta^{ab}\,\mathbf{1},$ "quantum plane" $\mathbb{R}^4_\theta$
- $[X^a, X^b] \sim i\theta^{ab}(x),$ generic quantum space

  $\rightarrow$ space-time as 3+1-dim. brane solution

  $$X^a \sim x^a: \quad \mathcal{M}^4 \hookrightarrow \mathbb{R}^{10}$$



- intersecting branes, stacks (as in string theory)



- compact extra dim $\mathcal{M}^4 \times T^2$, etc. (soon)

Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu}\,\mathbb{1}, \qquad \mu,\nu = 1,...,4$$

... Heisenberg algebra $\mathcal{A} = Mat(\infty, \mathbb{C})$ = space of functions on $\mathbb{R}^4_\theta$

$$\Delta\bar{X}^\mu \Delta\bar{X}^\nu \geq |\bar{\theta}^{\mu\nu}|$$

relation with classical $\mathbb{R}^4$:

$$\phi(x) = \int d^4k\,\tilde{\phi}(k)e^{ik_\mu x^\mu} \;\leftrightarrow\; \int d^4k\,\tilde{\phi}(k)e^{ik_\mu \bar{X}^\mu} =: \Phi(\bar{X}) \in Mat(\infty, \mathbb{C})$$

$\Phi(\bar{X}^\mu) \in Mat(\infty, \mathbb{C})$ ... general function on $\mathbb{R}^4_\theta$

$\bar{X}^\mu \in Mat(\infty, \mathbb{C})$ ... quantized coordinate functions on $\mathbb{R}^4_\theta$

$[\bar{X}^\mu, \Phi(\bar{X})] =: i\bar{\theta}^{\mu\nu}\partial_\nu\Phi(\bar{X}) \sim i\bar{\theta}^{\mu\nu}\partial_\nu\phi(x)$ $\rightarrow$ NC field theory

Quantized symplectic manifolds

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dim. manifold with Poisson structure

Its quantization is NC algebra $\mathcal{A}$ such that

$$\begin{aligned} \mathcal{I}: \ \mathcal{C}(\mathcal{M}) &\rightarrow \mathcal{A} \subset \mathcal{L}(\mathcal{H}) \\ f(x) &\mapsto \hat{f}(X) \end{aligned}$$

such that

$$\begin{aligned} \hat{f}\,\hat{g} &= \mathcal{I}(fg) + O(\theta) \\ [\hat{f}, \hat{g}] &= \mathcal{I}(i\{f, g\}) + O(\theta^2) \end{aligned}$$

(“nice“) $\Phi \in Mat(\infty, \mathbb{C}) \quad \leftrightarrow$ quantized function on $\mathcal{M}$

examples:

fuzzy sphere S^2_N
fuzzy torus T^{2n}_N
etc.

$\Big\}$ ... built-in UV cutoff, $\dim \mathcal{H} \sim \mathrm{Vol}(\mathcal{M})$

small deformation of $\mathbb{R}^4_\theta$:

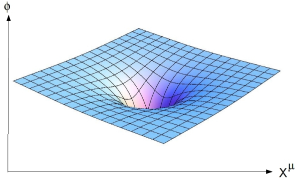
$$X^a = \bar{X}^a + A^a = \begin{pmatrix} \bar{X}^\mu + A^\mu(\bar{X}^\mu) \\ \phi^i(\bar{X}^\mu) \end{pmatrix}$$

- "old" interpretation: $U(1)$ NC gauge theory on $\mathbb{R}^4_\theta$
- more appropriate interpretation:

$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$ quantized embedding map

$[X^\mu, X^\nu] \sim i\{x^\mu, x^\nu\}$ ...Poisson bracket

... defines embedded NC space $\mathcal{M}$



consider scalar field φ ("test particle") on generic NC brane

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

$\varphi \in \mathcal{A}$, action

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad \text{(gauge inv.!)} \\ &\sim \int d^4x\,\sqrt{|\theta^{-1}_{\mu\nu}|}\,\theta^{\mu'\mu}\partial_{\mu'}x^a\partial_\mu\varphi\,\theta^{\nu'\nu}\partial_{\nu'}x^b\partial_\nu\varphi\,\eta_{ab} \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

(use $[f,\varphi] \sim i\theta^{\mu\nu}(x)\partial_\mu f\,\partial_\nu\varphi$)

where

$$\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,g_{\mu'\nu'}(x) && \text{effective metric (cf. open string m.)} \\ g_{\mu\nu}(x) &= \partial_\mu x^a\partial_\nu x^b\eta_{ab} && \text{induced metric on } \mathcal{M}^4_\theta \text{ (cf. closed string m.)} \end{aligned}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}$$

fluctuations of matrices X^a, Ψ around background
$\to$ gauge fields, scalar fields, fermions on $\mathcal{M}^4$ (NOT 10 dim!)

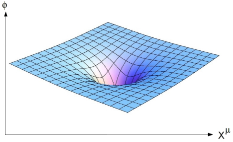
all fields couple to metric $G^{\mu\nu}(x)$
determined by $\theta^{\mu\nu}(x)$, embedding (scalar fields)
dynamical $\Rightarrow$ ("emergent") gravity

can show

$$\Box := [X^a, [X^b, .]] g_{ab} \sim e^\sigma \Box_G$$

matrix e.o.m $[X^a, [X^{a'}, X^b]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\begin{aligned} \Box_G x^a &= 0, \qquad \text{"minimal surface"} \\ \nabla^\mu (e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma} G_{\rho\nu} \theta^{\rho\mu} \partial_\mu \eta \\ & \qquad \eta \sim G^{\mu\nu} g_{\mu\nu} \end{aligned}$$



covariant formulation in semi-classical limit (H.S. 2007/08)

Gauge fields

M.M. is gauge invariant:

$$X^a \to U^{-1} X^a U$$

$\to$ fluctuations $X^\mu = \bar{X}^\mu + \theta^{\mu\nu} A_\nu$ transform as

$A_\mu \to U^{-1} A_\mu U + U^{-1} \partial_\mu U$ (nonabelian) gauge field!

note

$$[X^\mu, X^\nu] = i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}(\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}])$$
$$= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} F_{\mu'\nu'}$$

(use $[\bar{X}^\mu, \phi] \sim i\theta^{\mu\nu}\partial_\nu \phi$)

stack of n coinciding branes $X^a \otimes \mathbb{1}$

$\Rightarrow$ effective action

$$S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu} F_{\mu'\nu'} + \int \eta(x)\, tr\, F \wedge F$$

$\to$ $SU(n)$ YM gauge theory coupled to gravity

H.S. 2007 ff

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note:

- $SU(n)$ fluctuations ... nonabelian gauge fields
- $U(1)$ fluctuations $\rightarrow$ geometry, metric $G \sim \theta\theta g$

$$\left.\begin{array}{l} F_{\mu\nu}(x) \\ \phi^i(x) \end{array}\right\} \rightarrow \left\{\begin{array}{l} \theta^{\mu\nu}(x), \\ g_{\mu\nu}(x) = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi_i \end{array}\right.$$

- generic 4D metric on $\mathcal{M}^4 \subset \mathbb{R}^{10}$ (almost-Kähler)
- explains & takes advantage of UV/IR mixing

  (= new divergences in all NC models except IKKT)

Quantization

Quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]} \; = \; e^{-S_{eff}}$$ "path integral"

2 interpretations:

1. as NC gauge theory on $\mathbb{R}^4_\theta$: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$

   $\to$ new divergences (UV/IR mixing) in $U(1)$ sector

   except $\mathcal{N} = 4$ SUSY (≡IKKT)

2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\to$ induced gravity action (Sakharov 1967)

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 \, R[G] + ...)$$

- explanation UV/IR mixing, $U(1)$ entanglement (=gravity)
- IKKT model $\Rightarrow$ good quantum theory ! ($\mathcal{N} = 4$ SUSY)

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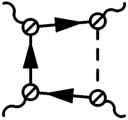
example: 1-loop eff. action due to fermions:
(all terms of dim ≤ 6):



$$
\begin{aligned}
\Gamma_{\text{eff}} = \; & \frac{\Lambda^4}{\Lambda_{\text{NC}}^4} \int \frac{d^4x}{(2\pi)^2} \Big(g^{\alpha\beta} D_\alpha \varphi^i D_\beta \varphi_i \\
& - \tfrac{1}{2} \Lambda_{\text{NC}}^4 \big(\bar\theta^{\mu\nu} F_{\nu\mu} \bar\theta^{\rho\sigma} F_{\sigma\rho} + (\bar\theta^{\sigma\sigma'} F_{\sigma\sigma'})(F\bar\theta F\bar\theta) \big) \\
& - 2\bar\theta^{\nu\mu} F_{\mu\alpha} g^{\alpha\beta} \partial_\nu \varphi^i \partial_\beta \varphi_i + \tfrac{1}{2} (\bar\theta^{\mu\nu} F_{\mu\nu}) g^{\alpha\beta} \partial_\beta \varphi^i \partial_\alpha \varphi_i + \text{h.o.} \Big) \\
& + \frac{\Lambda^2}{\Lambda_{\text{NC}}^4} \int \frac{d^4x}{(2\pi)^2} \Big(-\tfrac{11}{2} F_{\rho\eta} \Box_g F_{\sigma\tau} \bar G^{\rho\sigma} \bar G^{\eta\tau} - 12 \Box_g \varphi^i \Box \varphi_i \\
& \quad + \tfrac{1}{2} \Lambda_{\text{NC}}^4 (\bar\theta^{\mu\nu} F_{\mu\nu}) \bar\Box_G (\bar\theta^{\rho\sigma} F_{\rho\sigma}) \quad + \ldots \Big) \\
& + \frac{\Lambda^6}{\Lambda_{\text{NC}}^8} \int \frac{d^4x}{(2\pi)^2} (\ldots) + \ldots
\end{aligned}
$$

(all of this is due to UV/IR mixing !)

(D. Blaschke, H.S., M. Wohlgenannt JHEP 1103 (2011)) re-assembled as matrix model: $X^a = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} -\bar{\theta}^{\mu\nu} A_\nu \\ \phi^i \end{pmatrix}$

$$\Gamma_L[X] = \mathrm{Tr} \frac{L^4}{\sqrt{\frac{1}{2}H^2 - H^{ab}H_{ab} + \frac{1}{L^2}\mathcal{L}_{\rm curv}[X] + \ldots}} \sim \int d^4x\, \Lambda^4(x)\sqrt{g(x)}$$

$$H^{ab} = [X^a, X^c][X^b, X_c], \qquad H = H^{ab}\eta_{ab}$$

(D. Blaschke, H.S. M. Wohlgenannt JHEP 1103 (2011))

$SO(10)$ manifest, spontaneously broken

$\Rightarrow$ highly non-trivial predictions for (NC) gauge theory

effective generalized matrix model
= powerful new tool for (NC) gauge theory and gravity
full $SO(9,1)$ in $\mathcal{N} = 4$ SYM ?!

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(1-loop) finiteness of IKKT model

background field method $X^a \to X^a + Y^a$:

$$
\begin{aligned}
\Gamma_{1-\text{loop}} &= \frac{1}{2}\mathrm{Tr}\left(\log(\mathbb{1} + \Sigma^{(10)}_{ab}\Box^{-1}[\Theta^{ab},.]) - \frac{1}{2}\left(\log(\mathbb{1} + \Sigma^{(16)}_{ab}\Box^{-1}[\Theta^{ab},.]\right)\right) \\
&= O(\mathrm{Tr}(\Sigma_{ab}\Box^{-1})^4), \qquad \text{due to } \mathcal{N} = 4 \\
\Box &= [X^a,[X^a,.]] \\
\Theta^{rs} &= [X^r, X^s], \qquad \Sigma_{rs} ... SO(9,1) \text{ generator}
\end{aligned}
$$

fully $SO(9,1)$ covariant (IKKT)

background $\mathbb{R}^4_\theta$: $\equiv \mathcal{N} = 4$ SYM on $\mathbb{R}^4_\theta$, no UV div.

$SO(9,1)$ invariant formalism, broken spontaneously through $\mathbb{R}^4_\theta$
NC essential.

(work in progress, D. Blaschke, H.S.)

higher-order terms $\mathcal{L}_{\rm curv}[X]$: curvature

$$
\begin{aligned}
H^{ab} &:= [X^a, X^c][X^b, X_c] \\
T^{ab} &:= H^{ab} - \tfrac{1}{4}\eta^{ab} H, \qquad H := H^{ab}\eta_{ab}\,, \\
\Box X &:= [X^b, [X_b, X]]
\end{aligned}
$$

result:

for 4-dim. $\mathcal{M} \subset \mathbb{R}^D$ with $g_{\mu\nu} = G_{\mu\nu}$:

$$
Tr\left(2T^{ab}\Box X_a \Box X_b - T^{ab}\Box H_{ab}\right) \sim \frac{2}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{2\sigma} R
$$

$$
\begin{aligned}
& Tr([[X^a, X^c], [X_c, X^b]][X_a, X_b] - 2\Box X^a \Box X^a) \\
& \quad \sim \frac{1}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{\sigma}\left(\tfrac{1}{2} e^{-\sigma}\theta^{\mu\eta}\theta^{\rho\alpha} R_{\mu\eta\rho\alpha} - 2R + \partial^\mu\sigma\partial_\mu\sigma\right)
\end{aligned}
$$

Blaschke, H.S. CQG 27 (2010)

(cf. Arnlind, Hoppe, Huisken 2010, 2011)

cf. Einstein-Hilbert, pre-geometric

issues for (emergent) gravity

- complicated dynamics, not well understood
- bare M.M. action:

  NC $U(1)$ gauge fields $\partial^\mu F_{\mu\nu} = 0 \quad \Rightarrow \quad R_{\mu\nu}[\bar{G} + h] = 0$

  (Rivelles 2002)

  compactif. extra dim $\Rightarrow$ additional Ricci-flat d.o.f.
- 1-loop

  $\Rightarrow$induced E-H action,

  also modification of bulk metric:

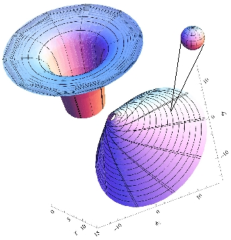
  e.g. $AdS^5 \times S^5$ near stack of branes
- Minkowski signature:

  either complexified $\theta^{\mu\nu}$

  or G, g have different causality structures illustrative examples:

- 1) “harmonic branch”
  near-realistic cosmolog. solutions (big bounce)
  (NC) minimal surfaces $\mathcal{M} \subset \mathbb{R}^D$ D. Klammer, H.S. PRL 102 (2009)
  insensitive to vacuum energy, SN Ia without fine-tuning !

- 2) ”Einstein branch“
  Example: Schwarzschild geometry
  Blaschke, H.S. CQG 27 (2010)
  embedding $\mathcal{M}^4 \subset \mathbb{R}^{10}, \quad \theta^{\mu\nu}(x)$



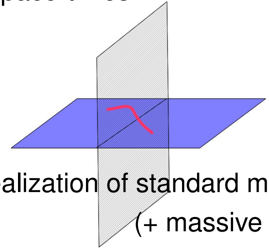
Relation with particle physics

- intersecting brane solutions (with A. Chatzistavrakidis, G. Zoupanos)

  chiral fermions at intersection = 4D space-times

  (as in string theory)

$$\begin{pmatrix} X^a_{(11)} & \psi_{(12)} \\ \psi_{(21)} & X^a_{(22)} \end{pmatrix}$$



- stacks of intersecting branes → realization of standard model

  (+ massive stuff)

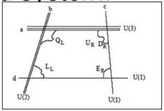
  cf. string theory

  clear-cut, predictive framework

  1-loop → intersecting branes can form bound system!

  A. Chatzistavrakidis, H.S., G. Zoupanos

  cf. H. Grosse, F. Lizzi, H.S. Phys.Rev. D81 (2010)



summary and outlook

- matrix models $Tr\Big([X^a, X^b][X_a, X_b] + \overline{\Psi} \not{D} \Psi\Big)$

  dynam. NC spaces (gravity), gauge theory & matter
- NC gauge theory ($\mathcal{N} = 4$ SYM) $\leftrightarrow$ "stringy" (IKKT)

  $\rightarrow$ candidate for quantum theory of fund. interactions
- QFT methods for gravity / "string theory"!
  (integration over geometries)
- intersecting branes $\rightarrow$ particle physics:
  brane-world scenarios
- ... lots of interesting issues!

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$U(1)$ gauge fields as gravitons (fix embedding)

$$G^{\mu\nu}(x) = \overline{\eta}^{\mu\nu} - h^{\mu\nu} \qquad (+O(F^2))$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

find

$$h_{\mu\nu} = \bar{\eta}_{\nu\nu'}\bar{\theta}^{\nu'\rho}\, F_{\rho\mu} + \bar{\eta}_{\mu\mu'}\bar{\theta}^{\mu'\eta}\, F_{\eta\nu} - \frac{1}{2}\bar{\eta}_{\mu\nu}\,(\bar{\theta}^{\rho\eta}F_{\rho\eta})$$

... linearized metric fluctuation

e.o.m:

$$\begin{aligned}
[X^\mu,[X^\nu,X^{\mu'}]]\eta_{\mu\mu'} &= 0 \\
\Rightarrow\ \partial^\mu F_{\mu\nu} &= 0 \\
\Rightarrow\ R_{\mu\nu}[G] &= 0 \quad (\partial^\mu h_{\mu\nu} = 0 \ldots \text{harm. gauge})
\end{aligned}$$

cf. Rivelles [hep-th/0212262]

while $R_{\mu\nu\rho\eta} \neq 0$

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

i.e.: NC $U(1)$ on $\mathbb{R}^4_\theta$ as gravitons

Geometrical e.o.m.

assume effective action

$$S = \int d^4x \sqrt{|g|}\,(-2\Lambda^4 + \Lambda_4^2 R) + S_{\rm matter}$$

e.o.m.

$$\begin{aligned}\delta S &= \int d^4x \sqrt{|g|}\,\delta g_{\mu\nu}(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu})\\ &= -2\int \delta\phi^i \partial_\mu(\sqrt{|g|}\,(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu})\partial_\nu\phi^i)\end{aligned}$$

since $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi^i$

1. “Einstein branch”

$$\Lambda^4 g^{\mu\nu} + \Lambda_4^2 \mathcal{G}^{\mu\nu} = 8\pi T^{\mu\nu}$$

2. “harmonic branch”

$$\Lambda^4 \Box_g \phi = (8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu})\nabla_\mu\partial_\nu\phi$$

prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$, even for $\Lambda \gg 0$!