Covariant Loop Gravity

carlo rovelli

I. definition of the theory

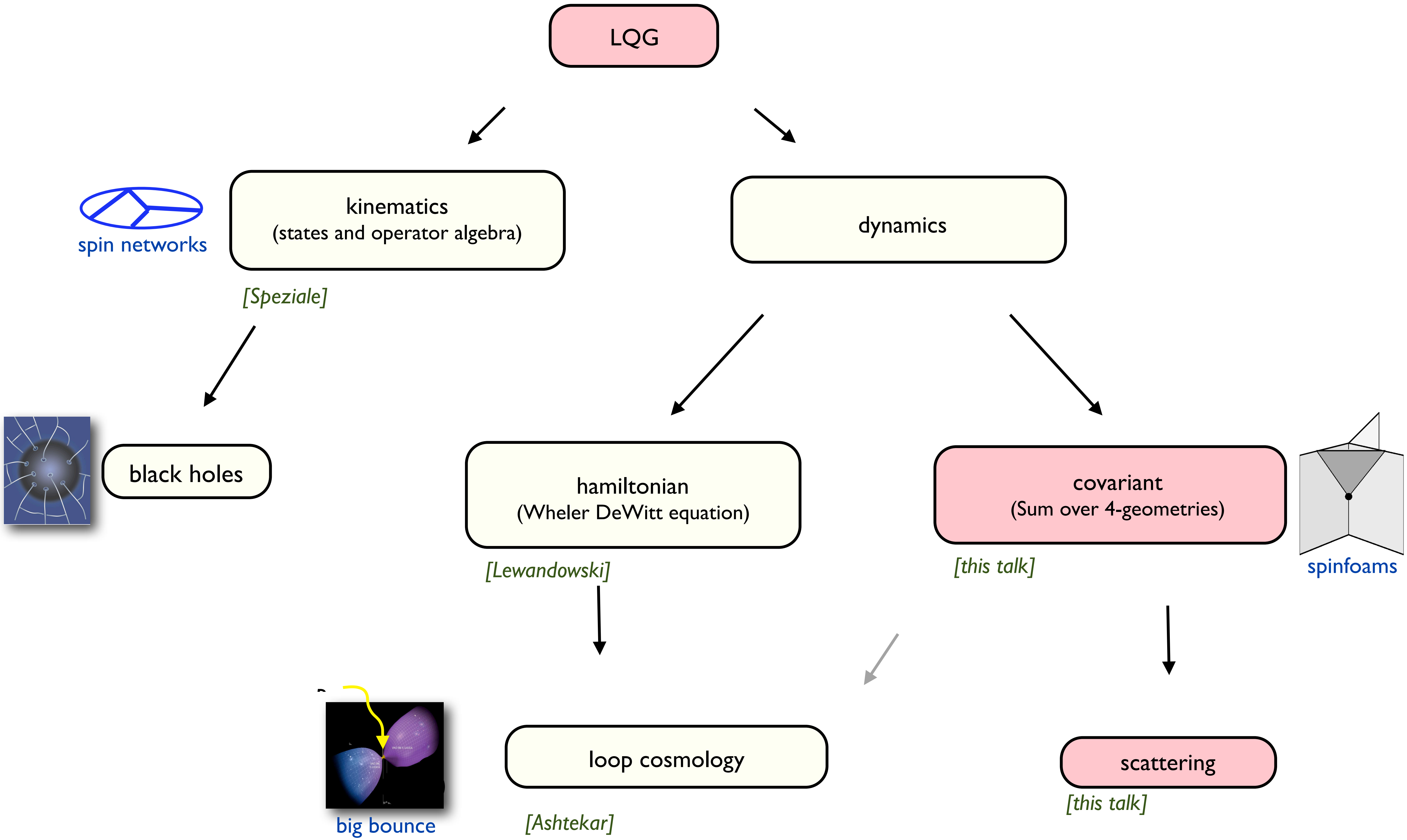
II. properties

III. how to extract physics from a background-independent theory

IV. results

[Recent review: CR: ``Zakopane lectures in loop gravity", arXiv:1102.3660]

I. loop quantum gravity



Aim:

I. Define a consistent quantum field theory for the gravitational field (coupled to ordinary matter)

II. Apply to early universe, black holes, Planck scale scattering ...

III. Understanding quantum spacetime

Ingredients:

I. The classical limit of the theory is General Relativity

II. Non perturbative QFT (in the sense of lattice QCD)

III. Take the symmetry of GR as guiding principle (“background independence”)

Not addressed:

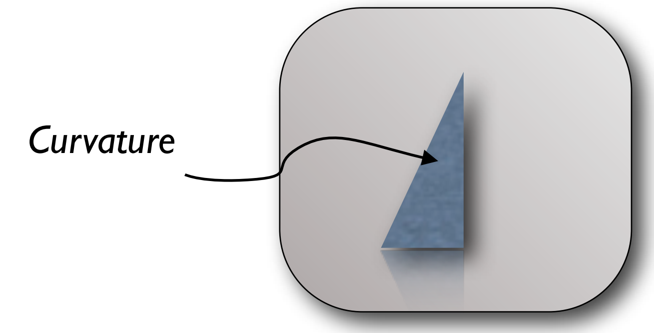
I. Unification of interactions

II. Measurement problem

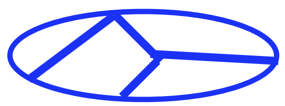
III. Quantum theory of closed systems

I. history of the main ideas

- 1957, Misner — $Z(q) = \int_{\partial g = q} Dg \;\; e^{iS_{EH}[g]}$
- 1961, Regge — Regge calculus → *truncation of GR on a manifold with d-2 defects*

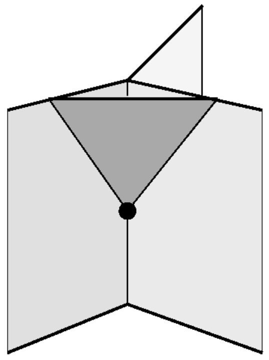


- 1967, Wheeler, DeWitt — W-DeW equation
- 1971, Penrose — Spin-geometry theorem → *spin network*



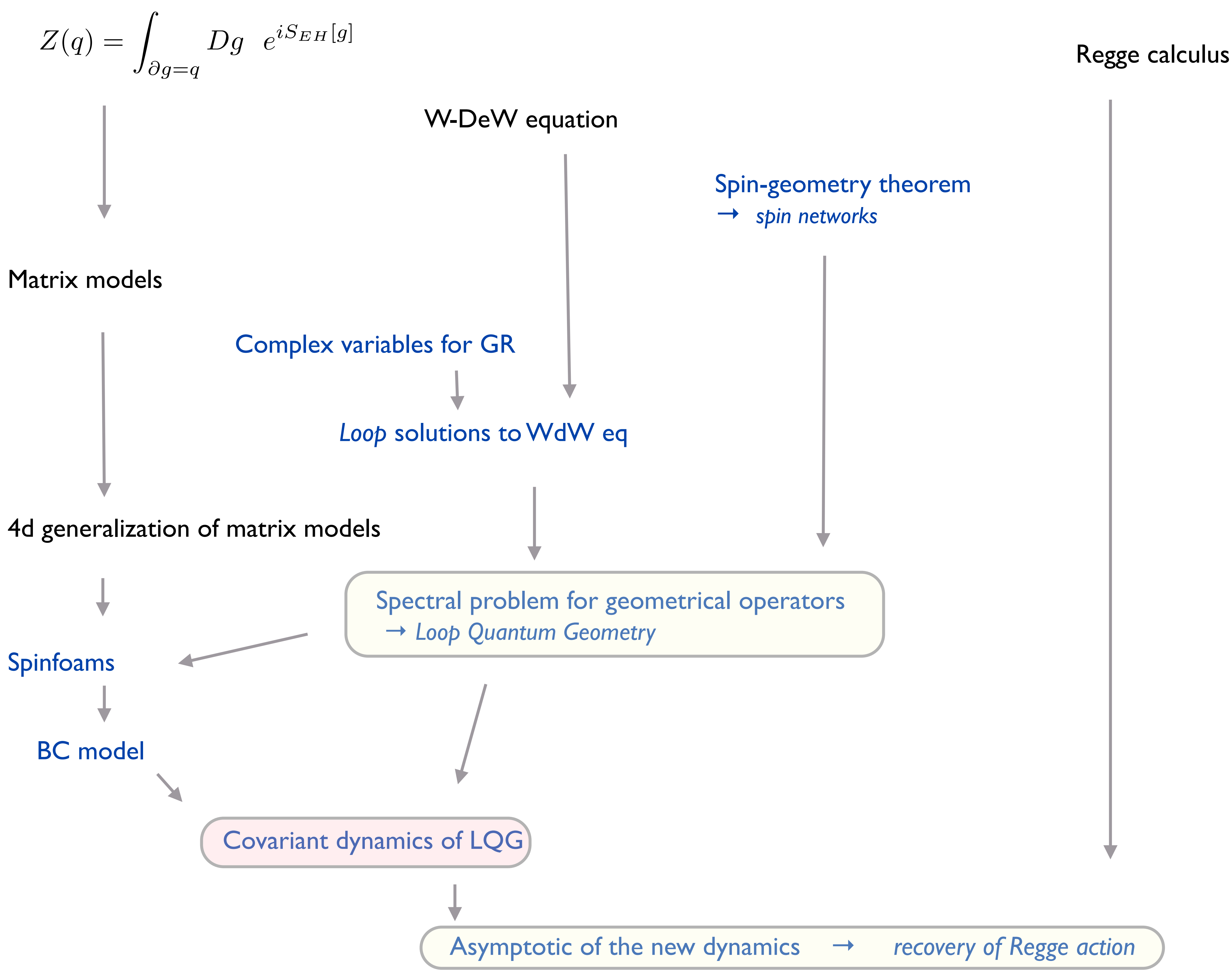
a "spin network"

- 1970-80, David, Brezin, Parisi, Kazakov, Itzkinson, Zuber, ... — Matrix models
- 1988, Ashtekar — Complex variables for GR
- 1988, Jacobson, Smolin CR Ashtekar — *Loop* solutions to WdW eq and loop representation
- 1988, Atiyah — General covariant QFT as TQFT
- 1992, Ooguri — 4d generalization of matrix models-GFT
- 1994, Smolin CR, Ashtekar Lewandowski — Spectral problem for geometrical operators → *Quantum geometry*
- 1994, Reisenberger, CR, Baez — Spinfoams
- 1999, Barrett Crane — BC model
- 2008, Engle, Pereira, CR, Speziale, Livine, Freidel, Krasnov — Covariant dynamics of LQG

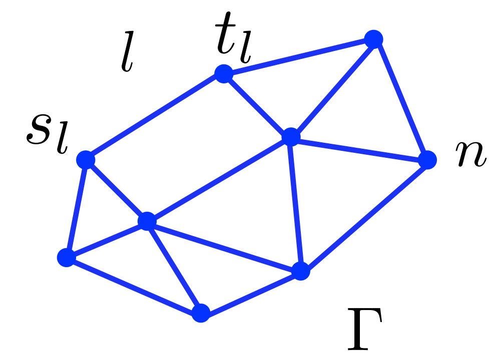


a "spinfoam"

I. relations between the main ideas

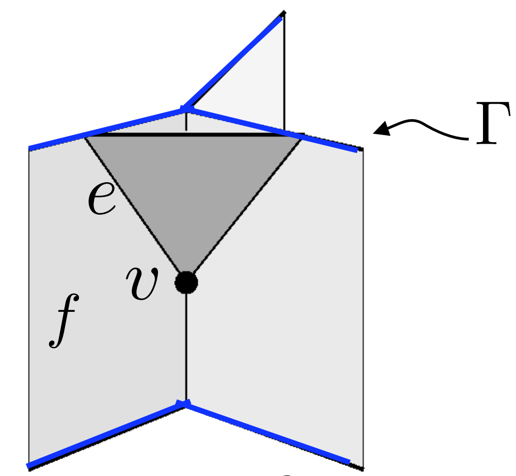


Graph:
$$\Gamma = \{N, L\}, \quad L \subset N \times N$$
$$n \in N, \quad l \in L$$
$$l = (n, n') := (s_l, t_l)$$



Graph
(nodes, links)

Two-complex:
$$\mathcal{C} = \{V, E, F\}, \quad E \subset V \times V, \quad F \subset P(V)$$
$$v \in V, \quad e \in E, \quad f \in F$$
$$e = (v, v') := (s_e, t_e)$$
$$f = (e, e', ..., e^n) \quad \text{cyclic}$$

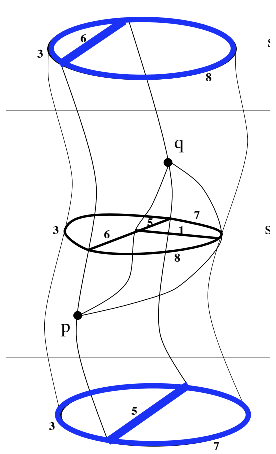


2-complex
(vertices, edges, faces)

SU(2) unitary representations: $|j; m\rangle \in \mathcal{H}_j, \qquad j \in N/2, m = -j, ..., j$

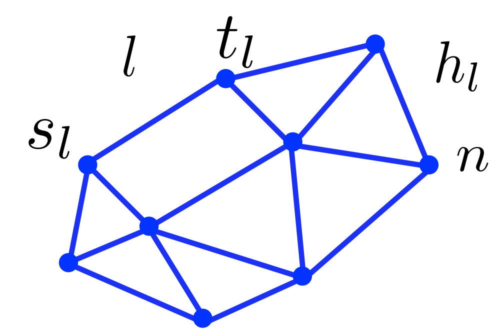
SL(2,C) unitary representations: $|k, \nu; j, m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu}$

$$k \in N/2, \quad \nu \in R$$



Hilbert space:

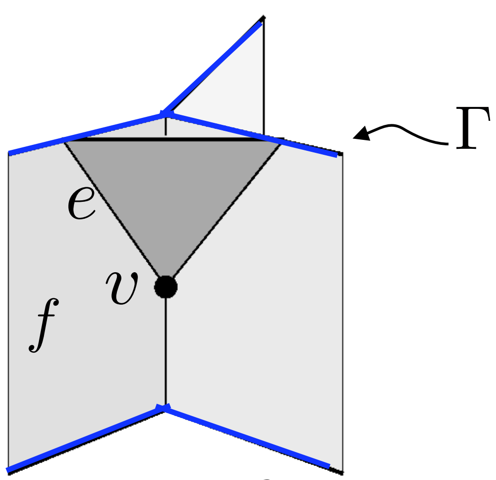
$$\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N] \ni \psi(h_l)$$
$$\mathcal{H} = \lim_{\Gamma\to\infty} \mathcal{H}_\Gamma$$



Graph
(nodes, links)
Γ

Transition amplitudes:

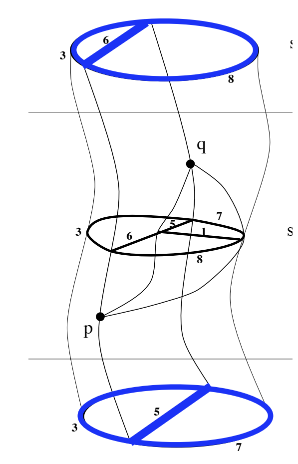
$$Z_\mathcal{C}(h_l) = \int_{SL2C} dg_{ev} \int_{SU2} dh_{ef} \sum_{j_f} N_{\{j_f\}} \prod_f (2j_f+1)\, \chi^{\gamma j_f, j_f}\Big(\prod_{e\in\partial f} g_{ef}^{\epsilon_{lf}}\Big) \prod_{e\in\partial f} \chi^{j_f}(h_{ef})$$



2-complex $\mathcal{C}$
(vertices, edges, faces)

$$Z(h_l) = \lim_{\mathcal{C}\to\infty} Z_\mathcal{C}(h_l).$$

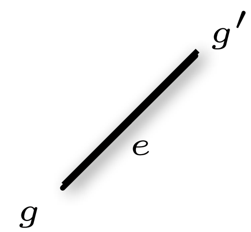
[Barrett Crane 1999,
Engle Pereira CR, Livine Speziale, Freidel Krasnov 2008,
Lewandowski et al 2010]



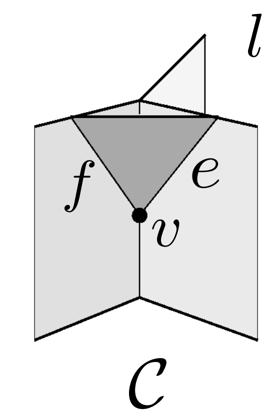
I. the theory: definition (pure gravity)

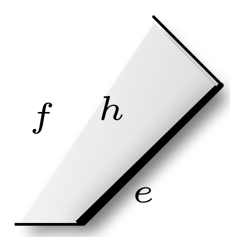
$$Z_{\mathcal{C}}(h_l) = \int_{SL2C} dg_{ev} \int_{SU2} dh_{ef} \sum_{j_f} N_{\{j_f\}} \prod_f (2j_f+1)\, \chi^{\gamma j_f, j_f}\Big(\prod_{e\in\partial f} g_{ef}^{\epsilon_{lf}}\Big) \prod_{e\in\partial f} \chi^{j_f}(h_{ef})$$

Feynman rules:

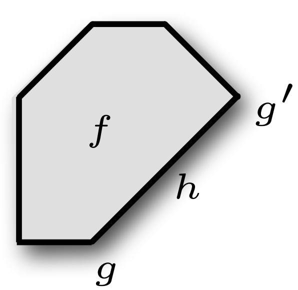
(i) 

$$\longmapsto \int_{SL2C} dg' \int_{SL2C} dg'$$



(ii) 

$$\longmapsto \int_{SU2} dh\, \chi^{j_f}(h)$$

(iii) 

$$\longmapsto \sum_j (2j+1)\, \chi^{\gamma j, j}\Big(\prod_{e\in\partial f} ghg'^{-1}\Big)$$

(face amplitude)

(iv) Drop one dg integral per vertex

(v) Combinatorial factor N_{j_l} = number of symmetries

(vi) For each external edge (link) $ghg'^{-1} \rightarrow h_l$

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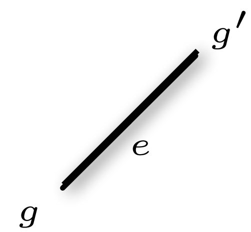
(i) 

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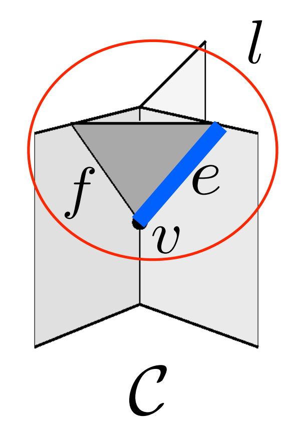
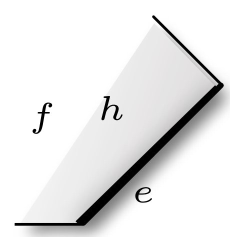


(ii) 

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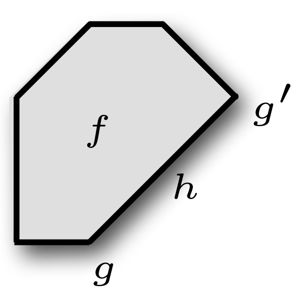
(iii) 

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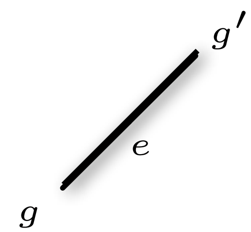
(i) 

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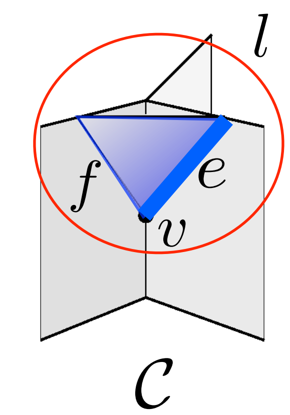
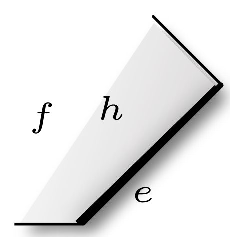


(ii) 

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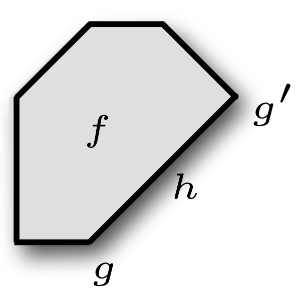
(iii) 

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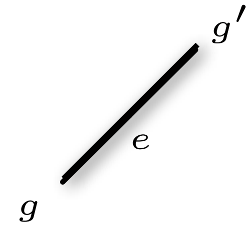

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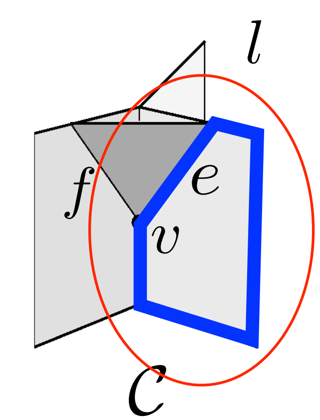

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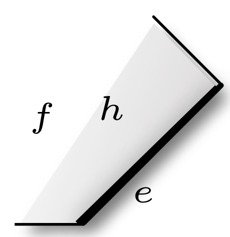

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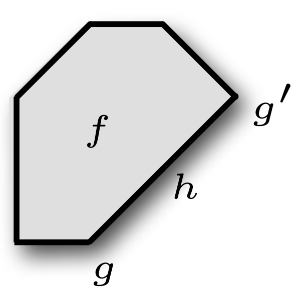

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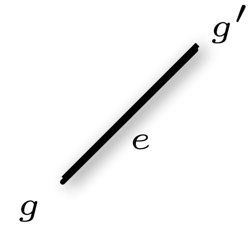
(i) 

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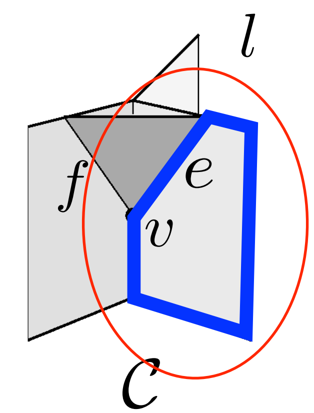
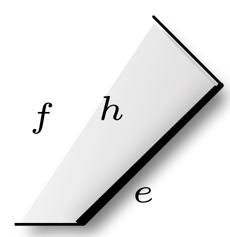


(ii) 

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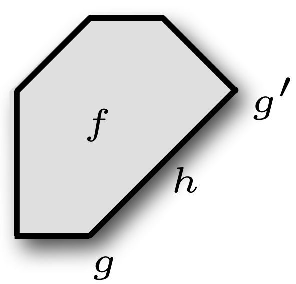
(iii) 

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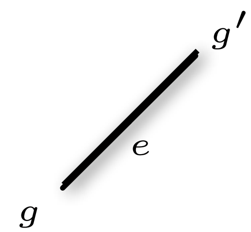

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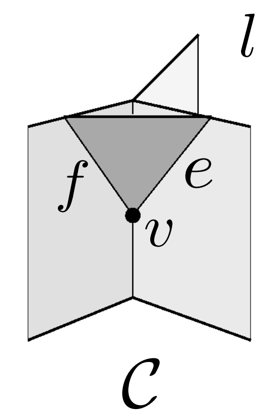

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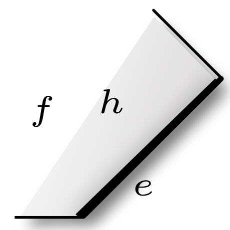

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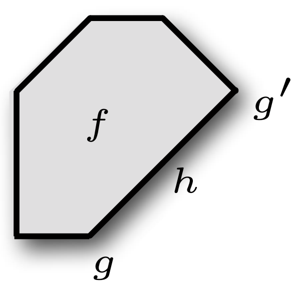

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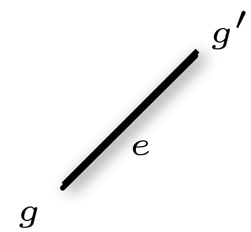

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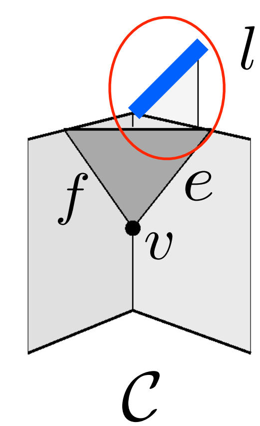

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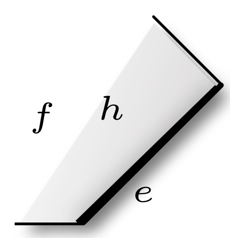

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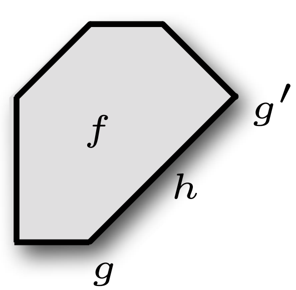

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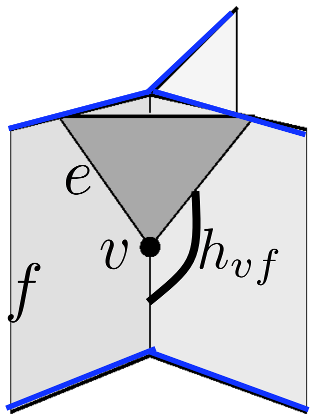
Hilbert space:

$$\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N] \otimes (C^4)^N \otimes L^2[G^L/G^N]$$

Fermions *Yang Mils*

Transition amplitudes:

$$Z_\mathcal{C} = \sum_{\{c\}} \sum_{j_f} \int_{SL(2,C)} dg_{ve} \int_{SU(2)} dh_{ef} \int_G dU_{ve}$$
$$\prod_f d_{j_f}\ \chi^{\gamma j_f, j_f}\Big(\prod_{e\in\partial f}(g_{es_e} h_{ef} g_{et_e}^{-1})^{\epsilon_{lf}}\Big) \prod_{e\in\partial f} \chi^{j_f}(h_{ef})$$
$$\prod_c (-1)^{|c|}\ \chi^{\frac{1}{2}}\Big(\prod_{e\in c_n}(g_{es_e} U_{es_e} U^\dagger_{et_e} g^\dagger_{et_e})^{\epsilon_{ec}}\Big).$$



Fermion variable *Fermion loops* *Yang Mills group variable*

[Han, Wieland, Perini Magliaro Bianchi, CR ... 2010]

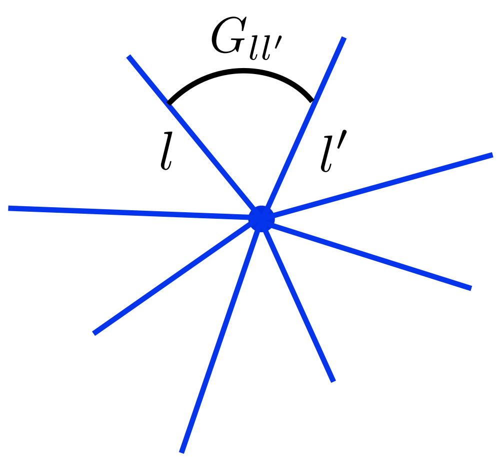
Cosmological constant: Replace SL2C and SU2 with a quantum deformations, with (real) deformation parameter q *[Roche Noui, Fairbairn Moesburger, Han. .-2010]*

i. Quantum states, transition amplitudes
ii. 4 dimensions, Lorentzian
iii. Couples with the Standard Model
iv. Includes a cosmological constant

i. 3d geometry
ii. Ultraviolet finiteness
iii. Infrared finiteness
iv. Relation to GR
v. Lorentz covariance
vi. Continuous limit
vii. Semiclassical limit

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

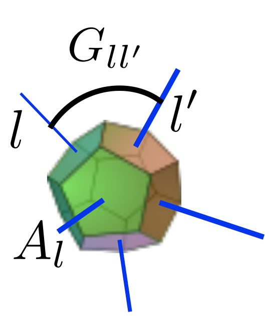
Derivative operator: $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$ $\sum_{l\in n} \vec{L}_l = 0$



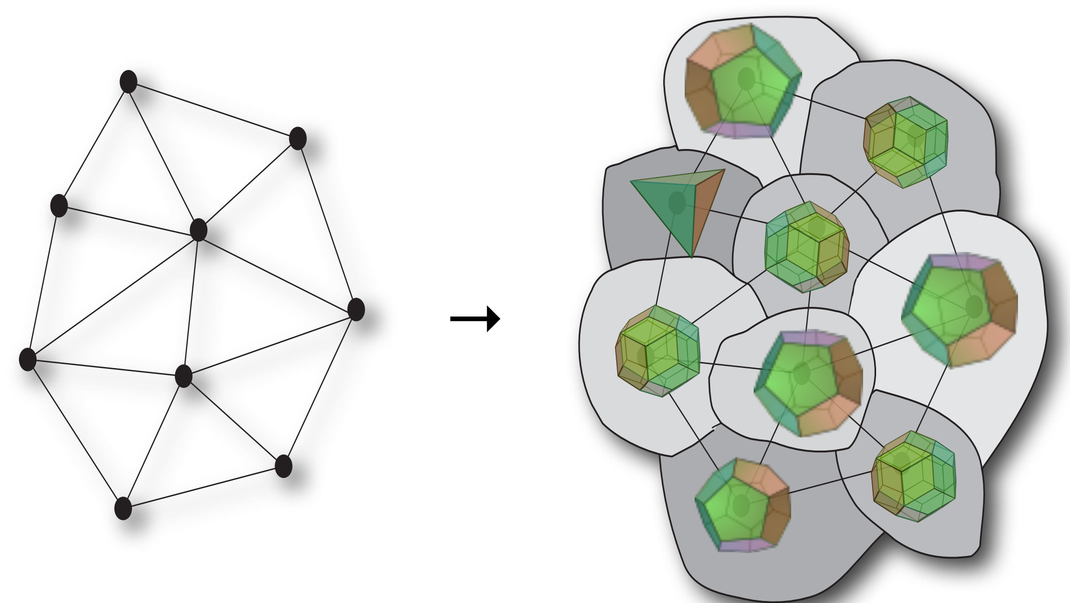
The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ satisfies $\sum_{l\in n} G_{ll'} = 0$

Is precisely the Penrose metric operator on the graph

It satisfies 1971 Penrose spin-geometry theorem, and 1897 Minkowski theorem: semiclassical states have a geometrical interpretation as polyhedra.

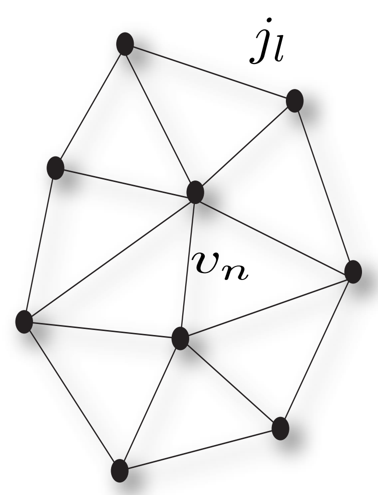


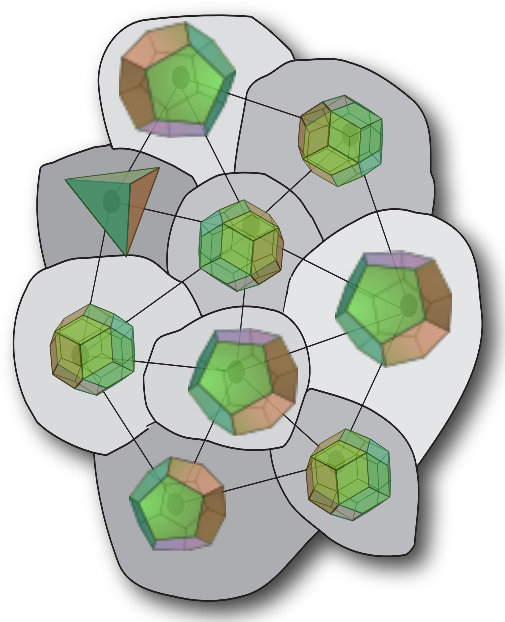
Polyhedron



area $A_l^2 = G_{ll}$ volume $V_n^2 = \frac{2}{9}\,\vec{L}_{l_1} \cdot (\vec{L}_{l_2} \times \vec{L}_{l_3})$

- Area and volume (A_l, V_n) form a complete set of commuting observables → basis $|\Gamma, j_l, v_n\rangle$





Nodes: discrete quanta of volume ("quanta of space") with quantum number v_n .

Links: discrete quanta of area, with quantum number j_l .

Geometry is quantized:

(i) eigenvalues are discrete
(ii) the operators do not commute
(iii) a generic state is a quantum superposition

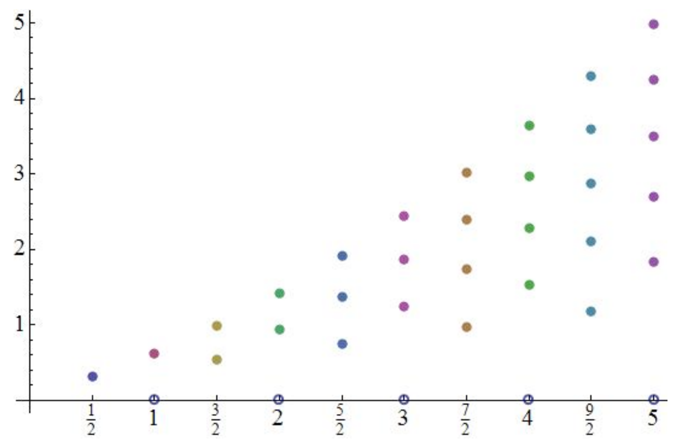
→ coherent states theory (based on *Perelomov 1986* SU(2) coherent state techniques)

→ States in $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$ describe quantum geometries:

not quantum states <u>in</u> spacetime

but rather quantum states <u>of</u> spacetime

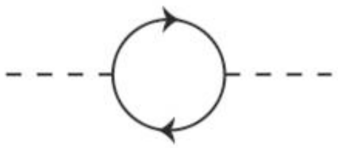
- Area eigenvalues $A = 8\pi\gamma\hbar G\ \sqrt{j_l(j_l+1)}$
- There is an area gap $a_o = 8\pi\gamma\hbar G\ \frac{\sqrt{3}}{2}$ and the volume eigenvalues are finite and discrete:



- Using this basis, the amplitude reads $$Z_{\mathcal{C}} = \sum_{j_f, v_e} N_{\{j_f\}} \prod_f (2j_f+1) \prod_v A_v(j_f, v_e)$$

→ Ultraviolet finiteness of the transition amplitudes

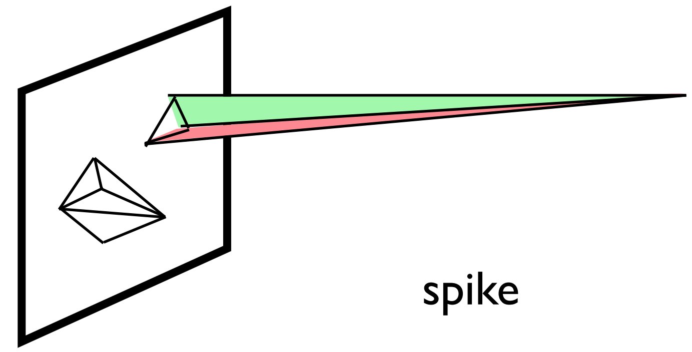
Infrared divergences (large j):



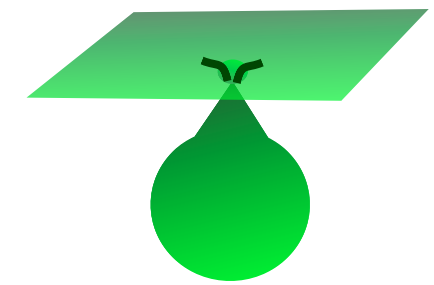
loop

→

bubble



spike



→ Ultraviolet finiteness of the transition amplitudes

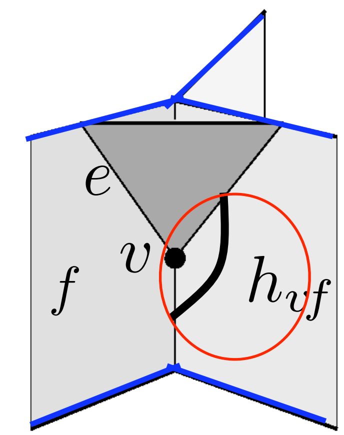
With the cosmological constant, the amplitudes $Z_{\mathcal{C}}(h_l)$ are finite *[Han. Fairbairn Moesbeurger.-2010]*

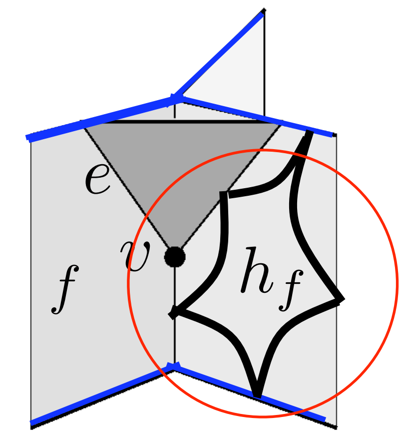
For any two-complex without boundaries $\mathcal{C}$, the function of two real variables

$$Z_{\mathcal{C}}(q,\gamma)$$

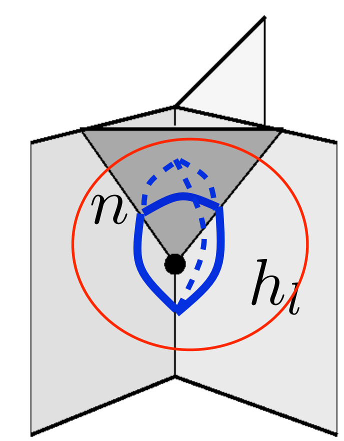
is finite. This family of function and their limit $Z(q,\gamma) = \lim_{\mathcal{C}\to 0} Z_{\mathcal{C}}(q,\gamma)$ characterize the theory.

$$Z_{\mathcal{C}} = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A_v(h_{vf}).$$

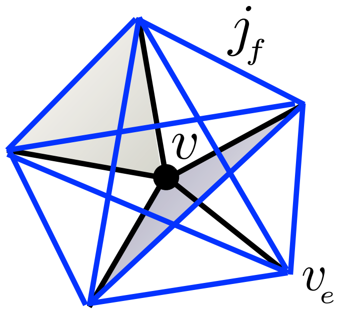




vertex amplitude: $A_v(\psi)$



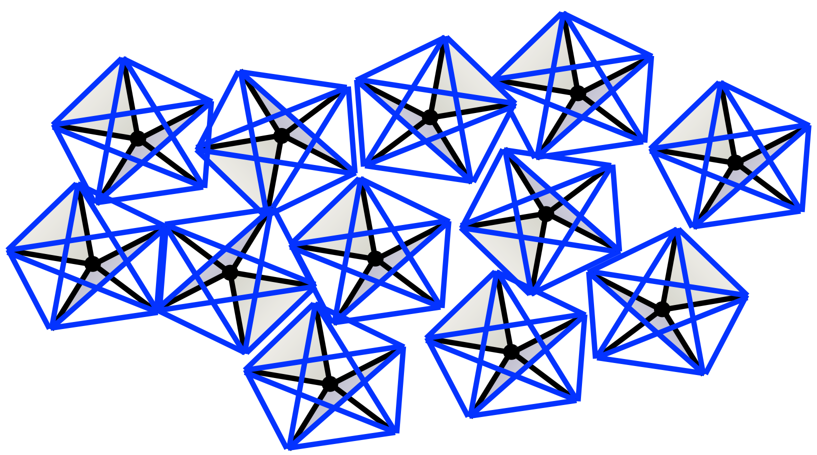
i. Fix ψ_q to determine a given piecewise flat geometry q on the boundary of a flat 4d spacetime region. Then : $S_{\text{Regge}}[q]$



Large j limit: $A_v(\psi_q) \sim e^{iS_{\text{Regge}}[q_{j_l}, v_n]}$

[Barrett et al 2008-2010]

ii. On a given $\mathcal{C}$, the sum over variables that dominates the amplitude is dominated by configurations that correspond to classical Regge geometries, weighted by the exponential of the Regge action



[Freidel Conrady 2008, Bianchi, Satz 2006, Magliaro Perini, 2011]

Therefore $Z_{\mathcal{C}}(h_l)$ defines a Regge like truncation of *Misner-Hawking* 's $Z(q) = \int_{\partial g = q} Dg \;\; e^{iS_{EH}[g]}$

First background independent quantum gravity theory: Ponzano-Regge model in 3d.

Based on the surprising discovery by Ponzano and Regge that

$$Z_{\mathcal{C}} = \sum_{j_f} \prod_f (2j_f+1) \prod_v A_v(j_f)$$

$$A_v = \{6j\} \sim e^{iS_{\text{Regge}}}$$

$$A_v(\psi_q) = \psi_q(\mathbb{1})$$

Here is the analog result in 4d:

$$Z_{\mathcal{C}} = \sum_{j_f, v_e} N_{\{j_f\}} \prod_f (2j_f+1) \prod_v A_v(j_f, v_e)$$

$$A_v(\psi_q) \sim e^{iS_{\text{Regge}}}$$

$$A_v(\psi) = (f\psi)(\mathbb{1})$$

$$f = P_{SL(2,C)} \circ Y_\gamma$$

$$Z = \int dh_{vf} \prod_f \delta(h_f) \prod_v A_v.$$

vertex amplitude:

$$A_v(\psi) = (f\psi)(\mathbb{1})$$

$$f = P_{SL(2,C)} \circ Y_\gamma$$

SU(2) unitary irrep: $|j;m\rangle \in \mathcal{H}_j$

SL(2,C) unitary irrep: $|k,\nu;j',m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j'=k,\infty} \mathcal{H}^{j'}_{k,\nu}$

Dupuis-Livine map:

$$\nu = \gamma j, \quad k = j' = j$$

$$y_\gamma : \quad \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j}$$

$$|j;m\rangle \mapsto |j,\gamma j;j,m\rangle$$

$$Y_\gamma : \quad L_2[SU(2)] \to F[SL(2,C),C]$$

$$\psi(h) \quad \mapsto (Y_\gamma\psi)(g), \qquad h \in SU(2), g \in SL(2,C)$$

$$D^{(j)}_{mm'}(h) \mapsto D^{(j,\gamma j)}_{jm,jm'}(g)$$

Main property: $\vec{K} + \gamma \vec{L} = 0$ on the image of Y_γ

boost generator $\vec{K} = J \cdot t$

rotation generator $\vec{L} = J^* \cdot t$

$h \triangleright t = 0,$

$h \in SO(3) \subset SO(3,1)$

$$Z = \int dh_{vf} \prod_f \delta(h_f) \prod_v A_v.$$

$$A_v(\psi) = (f\psi)(\mathbb{1})$$

Suppose we drop Y_γ . Then Z becomes the partition function of BF theory where B is recognized as the generator of SL2C.

$$S[A,B] = \int B \wedge F[A]$$

Therefore the theory can be identified with a BF theory where B satisfies the additional equation.

$$\vec{K} + \gamma \vec{L} = 0$$

$$\vec{K} = B \cdot t$$

$$\vec{L} = B^* \cdot t$$

Solution

$$B = (e \wedge e)^* + \frac{1}{\gamma} e \wedge e$$

Plugging this into the action:

$$S[A,e] = \int [(e \wedge e)^* + \frac{1}{\gamma} e \wedge e] \wedge F[A]$$

Solving the eq of m for A:

$$S[e] = \int [(e \wedge e)^* + \frac{1}{\gamma} e \wedge e] \wedge R[\omega(e)]$$

$$= \int e\, e_I^\mu e_J^\nu R_{\mu\nu}^{IJ} + \frac{1}{\gamma} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$$

$$= \int \sqrt{-g}\ R[g]$$

II. vertex amplitude: relation with GR, II

$$Z = \int dh_{vf} \prod_f \delta(h_f) \prod_v A_v.$$

$$A_v(\psi) = (f\psi)(\mathbb{1})$$

$$f = P_{SL(2,C)} \circ Y$$

$$Y : \; SU(2)\text{irrep} \to SL(2,C)\text{irrep}$$
$$|j;m\rangle \mapsto |\gamma j, j; j, m\rangle$$

On the image of Y , $\vec{K} = \gamma \vec{L}$ (Simplicity constraint)

BF theory + $\vec{K} = \gamma \vec{L}$ = General Relativity

II. vertex amplitude: relation with GR, q deformed case (euclidean).

$$Z_{\mathcal{C}} = \int dh_{vf} \prod_f \delta(h_f) \prod_v A_v . \qquad A_v(\psi) = Ev_q(f\psi)$$

→ The vertex with cosmological constant is the Chern Simon expectation value of boundary spin network.

$$Ev_q(\psi) = \int \Psi[A^\pm] \, e^{\frac{2\pi i}{h_+} S_{Chern-Simon}[A^+] - \frac{2\pi i}{h_-} S_{Chern-Simon}[A^-]} \, DA^\pm$$

$$q^\pm = q^{\frac{\pm 8}{(1\pm\gamma)^2}} = e^{ih_\pm}, \qquad q = e^{i\Lambda l_P^2}$$

→ Vassiliev invariants associated with the graph bounding the vertex

(related to the quantum group $SU_{q^+}(2) \otimes SU_{q^+}(2)$)

- $\mathcal{H}_\Gamma$ in the time gauge (Lapse=1, Shift=0)
- Manifest Lorentz covariance: $\mathcal{K}$ is mapped by be mapped by Y_γ to a space of SL(2,C) functions, determined by their restriction on SU(2).
- These are square-integrable in the SU(2) scalar product, but not in the SL(2,C) one. (cfr Gupta-Bleuler). *[Speziale CR -2010]*
- The theory is locally SL(2,C)-invariant in the bulk, and yields states in $\mathcal{K}$ on the boundary.
- Covariant LQG is manifestly Lorentz-covariant

The common idea that a minimal length breaks Lorentz covariance is wrong !

It would be true in a classical theory. It is NOT true in a quantum theory:
the minimal length appears as an eigenvalue, and

eigenvalues do not transform continuously with continuous symmetry!

cfr angular momentum theory !

[Speziale CR]

(I) As written, the theory has no dimensional scales.

It has an intrinsic scale, at which it lives (recall $a_o = \frac{\sqrt{3}}{2}$). This is the physical scale L_{loop} of the theory.

In dimensional units, L_{loop} has a value in centimeters. So, in cm:

From canonical quantization: $$L^2_{loop} = 8\pi\gamma G\hbar = 8\pi\gamma L^2_{Planck}$$

$$a_o = \frac{\sqrt{3}}{2}\, 8\pi\gamma\hbar G$$

(II) q or the cosmological constant Λ is a second scale.

(III) The Immirzi parameter γ .

Nothing corresponds to the QCD $a \to 0$ continuum limit:

$$QCD: \begin{cases} a & \rightarrow & 0 \\ \mathcal{C} & \rightarrow & \infty \end{cases}$$

$$LQG: \quad \mathcal{C} \rightarrow \infty$$

II. scales and "continuum limit"

ħ

G

Planck length: 10^{-33} cm. The theory is defined at this scale, which provides an intrinsic cut-off, which makes the theory UV finite.

There are no degrees of freedom below the Planck scale (background independence)

*Recovering the continuum limit is **not** taking a short distance scale cut off to zero.*

The theory is different from approaches that assume degrees of freedom at any scale, and taking a cut off to zero. (CDT, asymptotic safety...).

The continuum limit and the large-scale limit are different limits, and should not be confused !

→ Continuum limit

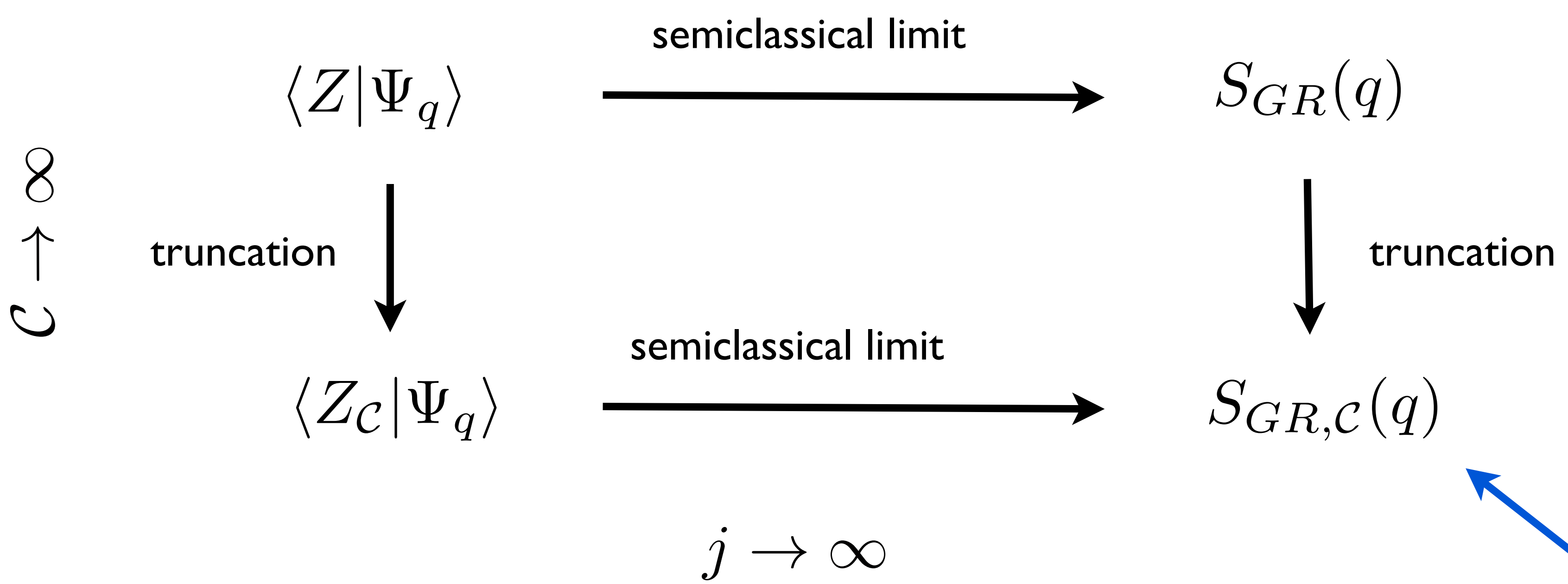
Recovery of all degrees of freedom $\mathcal{C} \to \infty$

→ Semiclassical limit

High quantum numbers $j \to \infty$

→ Large distance limit

Fix the truncation, disregard Planck scale effects



Hamilton function of a Regge truncation of GR on $\mathcal{C}$.

Regime where small -$\mathcal{C}$ it is good:

$$\frac{1}{\sqrt{R}} \gg \ell \gg L_{\text{Planck}}$$

i. Quantum states, transition amplitudes

ii. 4 dimensions

iii. Lorentzian

iv. Couples with the Standard Model

v. Includes a cosmological constant

vi. Infrared finite (on every C)

vii. Ultraviolet finite (on every C)

i. Simple vertex form

ii. Lorentz covariance manifest

iii. Continuous limit $\mathcal{C} \to \infty$

iv. Large scale limit $j \to \infty$

v. Recovery of Regge truncation of GR

i. How to compute physics (background independence)?

ii. What can be computed ?

Hamilton function $$S(q,t,q',t') = \int_t^{t'} dt\; L(q(t),\dot q(t))$$

Hamilton's "boundary logic": $p(q,t,q',t') = \dfrac{\partial S(q,t,q',t')}{\partial q}$ $\qquad (q,q')_{t,t'} \to (p,p')_{t,t'}$

Notice also $$E(q,t,q',t') = -\frac{\partial S(q,t,q',t')}{\partial t}$$

Treats (q,t) on equal footing $\underbrace{(q,t}_{q_i},q',t') \to \underbrace{(p,E}_{p_i},p',E')$

Covariant form of S $$S(q_i,q_i') = \int_\tau^{\tau'} d\tau\; L(q_i,\dot q_i)$$

→ Dynamics is the relative evolution of a set of variables, not the evolution of these variables in time. Hamilton dynamics captures this relational dynamics.

III. boundary formalism: QM and QFT

- Quantum theory

$$W(q,t,q',t') = \langle q|e^{iH(t'-t)}|q'\rangle = \langle q,t|q',t'\rangle \sim e^{\frac{i}{\hbar}S(q,t,q',t')}$$

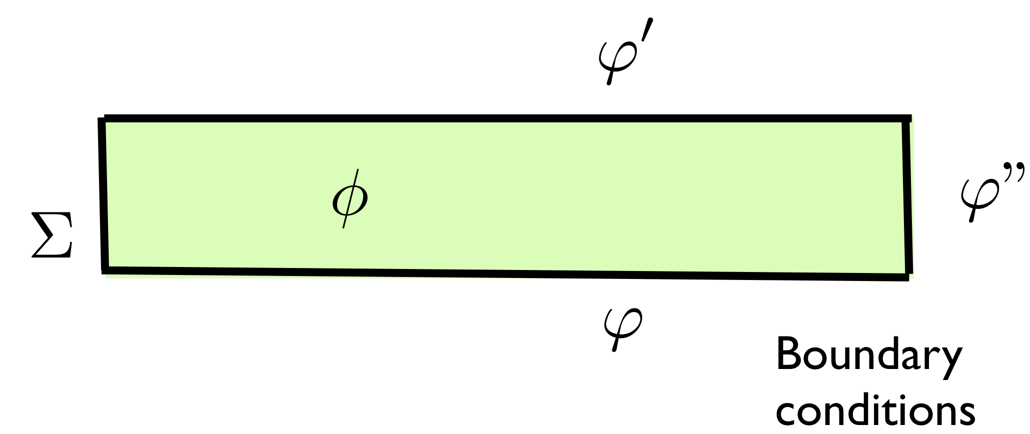
$$= \int_{q,t,q',t'} Dx(t)\ \ e^{\frac{i}{\hbar}S[x(t)]}$$

Evolution operator

Hamilton function !

- Field theory

$$W[\varphi_b,\Sigma] = \int_{\varphi_b,\Sigma} D\phi\ \ e^{iS[\phi]}$$



- General covariant field theory

$$W[\varphi_b,\Sigma] = W[\varphi_b]$$

- For the gravitational theory: φ_b gives the geometry of the boundary

Semiclassical limit

$$W[\varphi_b] \;\rightarrow\; e^{\frac{i}{\hbar G} S_{GR}[\varphi_b]} + \textit{correction in } \hbar G$$

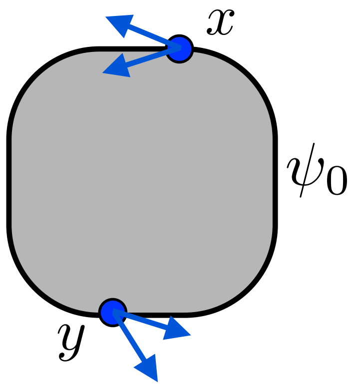
Hamilton function of GR

Field propagator → Particle propagator: $\langle 0|\phi(\vec{x}', t')\phi(x,t)|0\rangle = \langle 0|\phi(\vec{x}')\; e^{iH(t'-t)}\; \phi(x)|0\rangle$

$$= \int d\varphi\, d\varphi'\; \underbrace{W[\varphi, t, \varphi', t']}_{\text{Field propagator}}\; \underbrace{\varphi(\vec{x})\,\varphi'(\vec{x}')}_{\text{Field insertion}}\; \underbrace{\overline{\Psi_0[\varphi]}\,\Psi_0[\varphi']}_{\text{Vacuum boundary state}} \qquad = \langle W \mid \phi(\vec{x})\phi'(\vec{x}') \mid \Psi_0\rangle$$

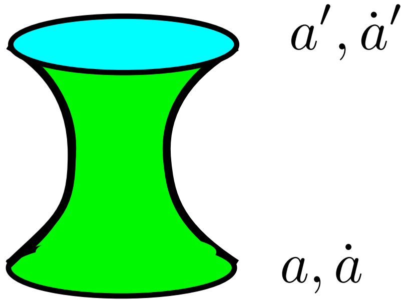
(i) n-point functions. The background enters in the choice of a "background" boundary state

$$\frac{\langle Z|G_{l_a l_b} G_{l_c l_d}|\psi_0\rangle}{\langle Z|\psi_0\rangle} \sim \langle 0|g_{ab}(x) g_{cd}(y)|0\rangle$$



In principle this technique allows generic *n*-point functions to be computed, and compared with Effective Quantum GR, and *corrections* to be computed.

(ii) cosmology. Transition amplitude → Hamilton function

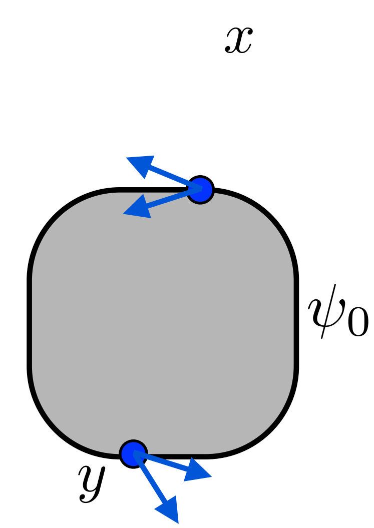


Classical Hamilton function $S(a,a') = \frac{2}{3}\sqrt{\frac{\Lambda}{3}}({a'}^3 - a^3)$

$$W(a,a') \rightarrow e^{\frac{i}{\hbar}S(a,a')}$$

$$\langle Z|\psi_{a\dot{a}} \otimes \psi_{a\dot{a}'}\rangle$$

(1) Gravitational waves. Starting from $Z_{\mathcal{C}}(h_l)$, it is possible to compute the two point function of the metric on a background. The background enters in the choice of a "background" boundary state ψ_0



$$\frac{\langle Z_{\mathcal{C}}|G_{l_a l_b} G_{l_c l_d}|\psi_0\rangle}{\langle Z_{\mathcal{C}}|\psi_0\rangle} \sim \langle 0|g_{ab}(x) g_{cd}(y)|0\rangle$$

This can be computed at first order in the *expansion* in the number of vertices.

$$\langle W|E^a_n \cdot E^b_n \; E^c_m \cdot E^d_m|j_{ab}, \Phi_a(\vec{n})\rangle =$$

$$\int \prod_{a=1}^{5} dg_a^+ dg_a^- \;\; A_i^{na} A_i^{nb} \;\; A_i^{nc} A_i^{nd} \;\; e^{\sum_{ab,\pm} 2j^{\pm}_{ab} \log\langle -\vec{n}_{ab}|(g_a^{\pm})^{-1} g_b^{\pm}|\vec{n}_{ba}\rangle}$$

$$A_i^{na} = \gamma j^{\pm}_{na} \frac{\langle -\vec{n}_{an}|(g_a^{\pm})^{-1} g_n^{\pm} \sigma^i|\vec{n}_{na}\rangle}{\langle -\vec{n}_{an}|(g_a^{\pm})^{-1} g_n^{\pm}|\vec{n}_{na}\rangle}$$

$$\longrightarrow \quad \langle h_{\mu\nu}(x) h_{\rho\sigma}(y)\rangle = \frac{-1}{2|x-y|^2}(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\rho\sigma}).$$

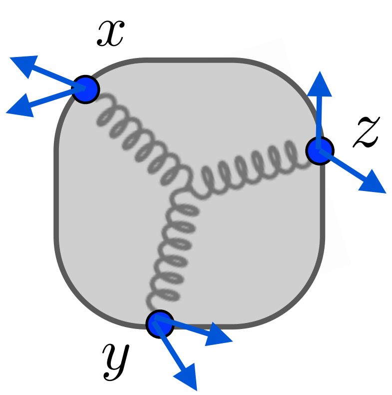
Result: The free graviton propagator is recovered in the Lorentzian theory

[Bianchi Magliaro Perini 2009, Ding 2011]

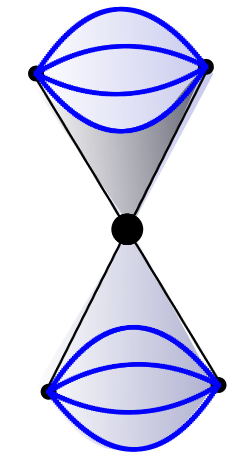
(II) Scattering.

New Result: The Regge n-point function is recovered in the large j limit (euclidean theory)

[Zhang, CR 2011]



IV. results



(III) Cosmology. Starting from $Z_{\mathcal{C}}(h_l)$, it is possible to compute the transition amplitude between homogeneous isotropic geometries

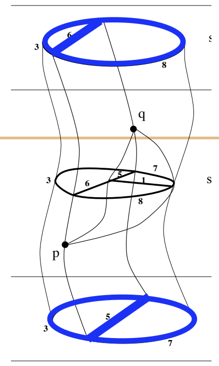
$$W(z_i, z_f) = \int_{SO(4)^4} dG_1^i\, G_2^i\, dG_1^f\, G_2^f \prod_{l^i} P_t(H_l(z_i)\,,\, G_1^i G_2^{i\,-1}) \prod_{l^f} P_t(H_l(z_f)\,,\, G_1^f G_2^{f\,-1})$$

$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} tr\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y\right].$$

$$\rightarrow \qquad e^{\frac{i}{\hbar}\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a'^3 - a^3)} = e^{\frac{i}{\hbar}S(a,a')}$$

Result: The expanding Friedmann dynamics and the DeSitter Hamilton function are recovered

[Bianchi Vidotto Krajewski CR 2010]



(I) Loop quantum gravity transition amplitudes $Z_{\mathcal{C}}(h_l)$ (spinfoams)

(II) Quantum 3-geometry (spin networks, quanta of space)

(III) Lorentzian dynamics. Lorentz covariance manifest.

(IV) Matter couplings: fermions and Yang Mills fields. Cosmological constant.

(V) UV finite, IR finite at each order in $\mathcal{C}$.

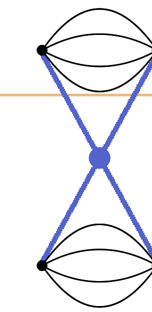
(VI) Background independent QFT

(VII) Boundary technique gives well defined observables.

(VIII) Indications that the $\hbar \to 0$ is general relativity

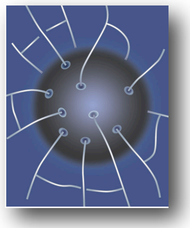
- ☑ *Ooguri's BF+ (BF → GR constraints)*
- ☑ *Asymptotics of the vertex and of the full amplitude*
- ☑ *Lorentzian 2 point function, Euclidean n-point function*
- ☑ *De Sitter solution*

main open issues



(i) More solid arguments that the classical limit is GR

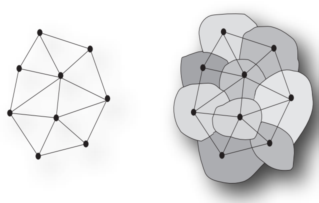
(ii) Scaling *[Rivaseau, Oriti and collaborators]*

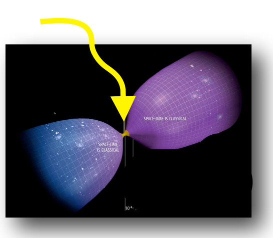


(iii) Is the expansion in $\mathcal{C}$ meaningful? (Lower terms dominate?Do we need to renormalize them?)

(iv) Study the family of functions $Z_{\mathcal{C}}(q,\gamma)$ and $Z(q,\gamma) = \lim_{\mathcal{C}\to\infty} Z_{\mathcal{C}}(q,\gamma)$

(v) Compute higher corrections. Observable consequences? Cosmology? *[Ashtekar et al, Barrau et al]*





Several issues are open, but this is

a potentially possible

theory of quantum gravity.