ma per seguir virtute e canoscenza

SISSA

GRAVITY DUALS OF 2D SUSY GAUGE THEORIES

BASED ON:

- 0909.XXXX with E. Conde and A.V. Ramallo (Santiago de Compostela)

[See also 0810.1053 with C. Núñez, P. Merlatti and A.V. Ramallo]

Daniel Areán
Zürich, September 2009

OUTLINE

- INTRODUCTION. AdS/CFT and its generalisations

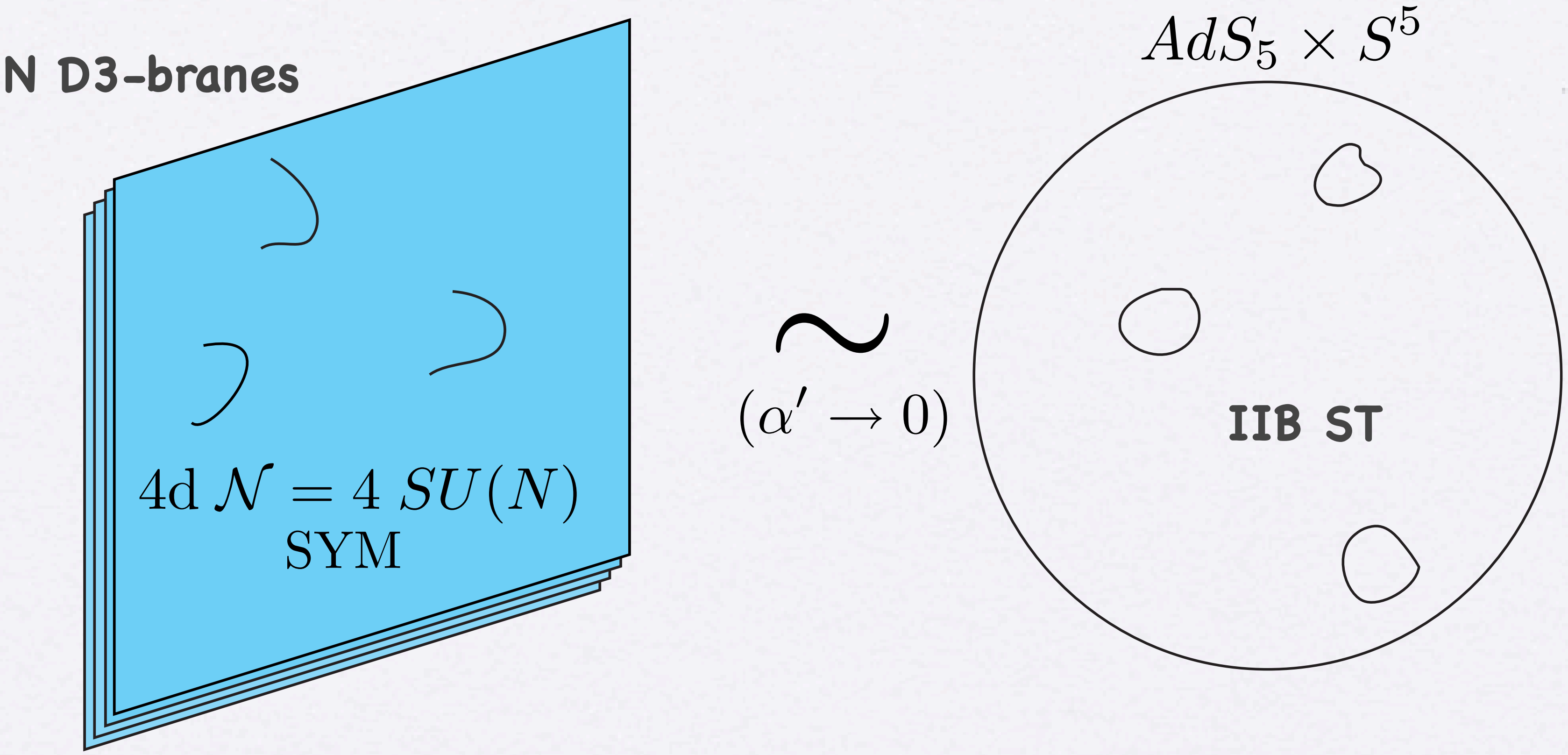
- GRAVITY DUAL OF 2d N=(1,1) from wrapped branes
    - Brane setup
    - 10d SUGRA ansatz
    - Gauged SUGRA approach (7d)
    - Solution → Coulomb branch

- ADDING FLAVOR
    - Flavor D5s
    - Backreaction → smearing
    - Flavored solution

- GRAVITY DUAL OF 2d N=(2,2) from wrapped branes

- SUMMARY

AdS / CFT Correspondence



N D3-branes

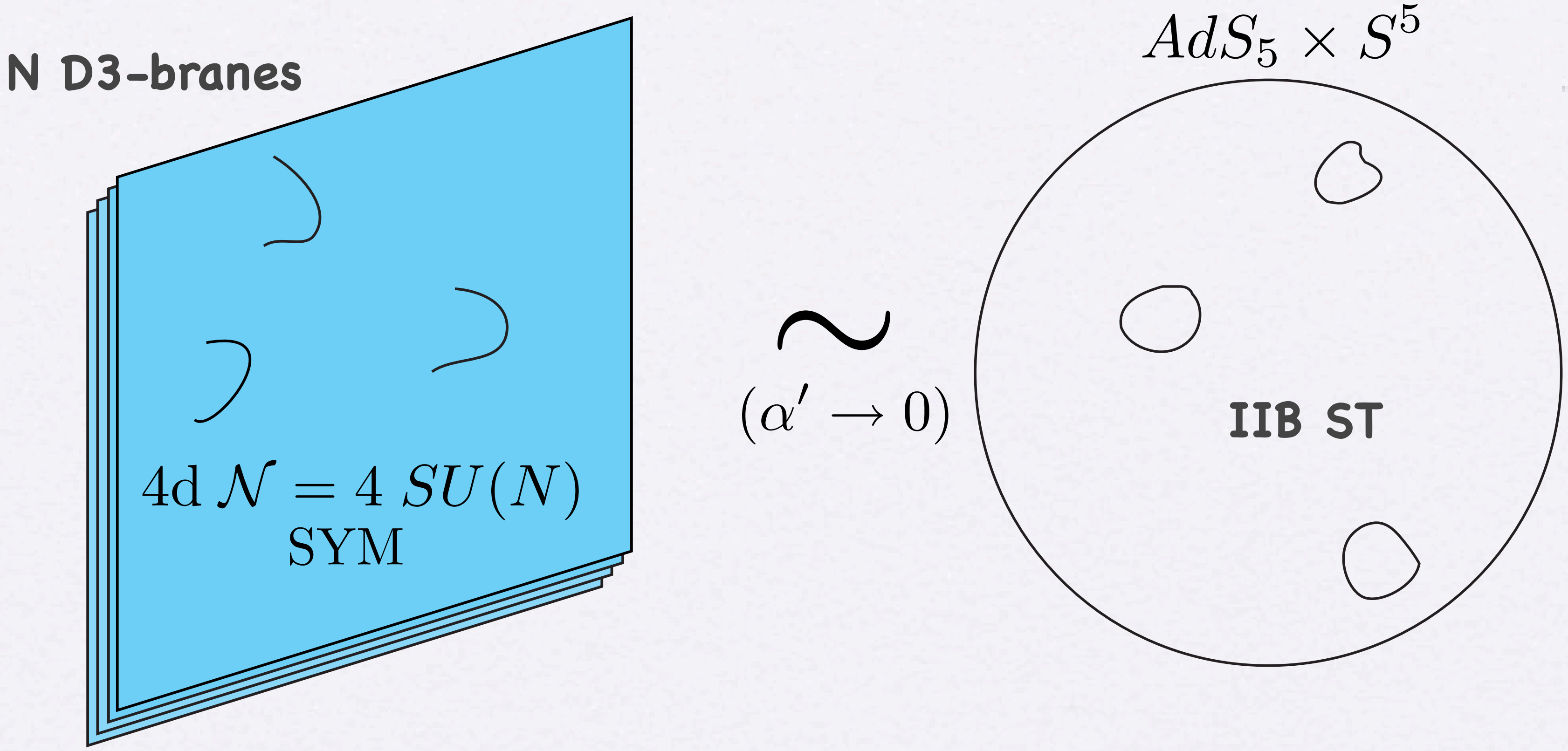
4d $\mathcal{N}=4\ SU(N)$
SYM

$\sim$
$(\alpha' \to 0)$

$AdS_5 \times S^5$

IIB ST

AdS / CFT Correspondence



GENERALISE

- ★ d = 2
- ★ 2 (4) SUSYs
- ★ ~~Conformal~~
- ★ Add Flavor

$\longrightarrow$

$$\boxed{2d \quad \begin{matrix} \mathcal{N}=(2,2) \\ \mathcal{N}=(1,1) \end{matrix} \quad \text{SYM} \ + \ N_f \ \text{flavors}}$$

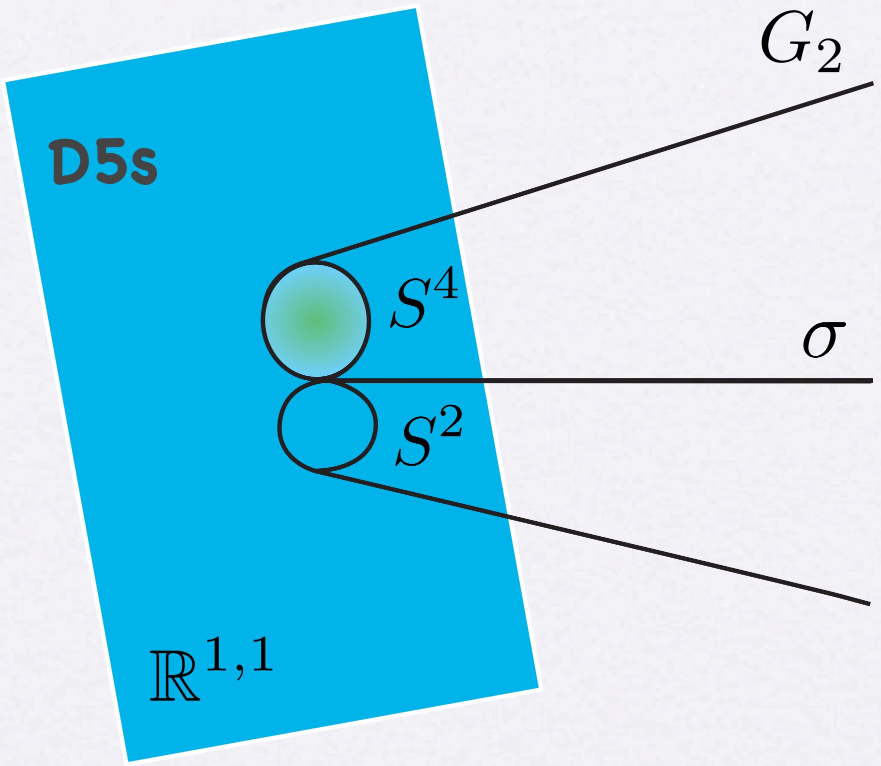
★ USE WRAPPED BRANES

(4d: Maldacena & Núñez, Gauntlett et al, Bigazzi et al)

(3d: Chamseddine & Volkov, Maldacena & Nastase, Schvellinger & Tran, Gomis & Russo, Gauntlett et al)

DUAL TO N=(1,1) SYM FROM WRAPPED D5s

★ BRANE SETUP



$\times \quad \mathbb{R}\ (\rho)$

			G_2							
	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	–	–	○	○	○	○	·	·	·	·

$N_3\ :\ (\sigma, \theta, \phi)$

DUAL TO N=(1,1) SYM FROM WRAPPED D5s

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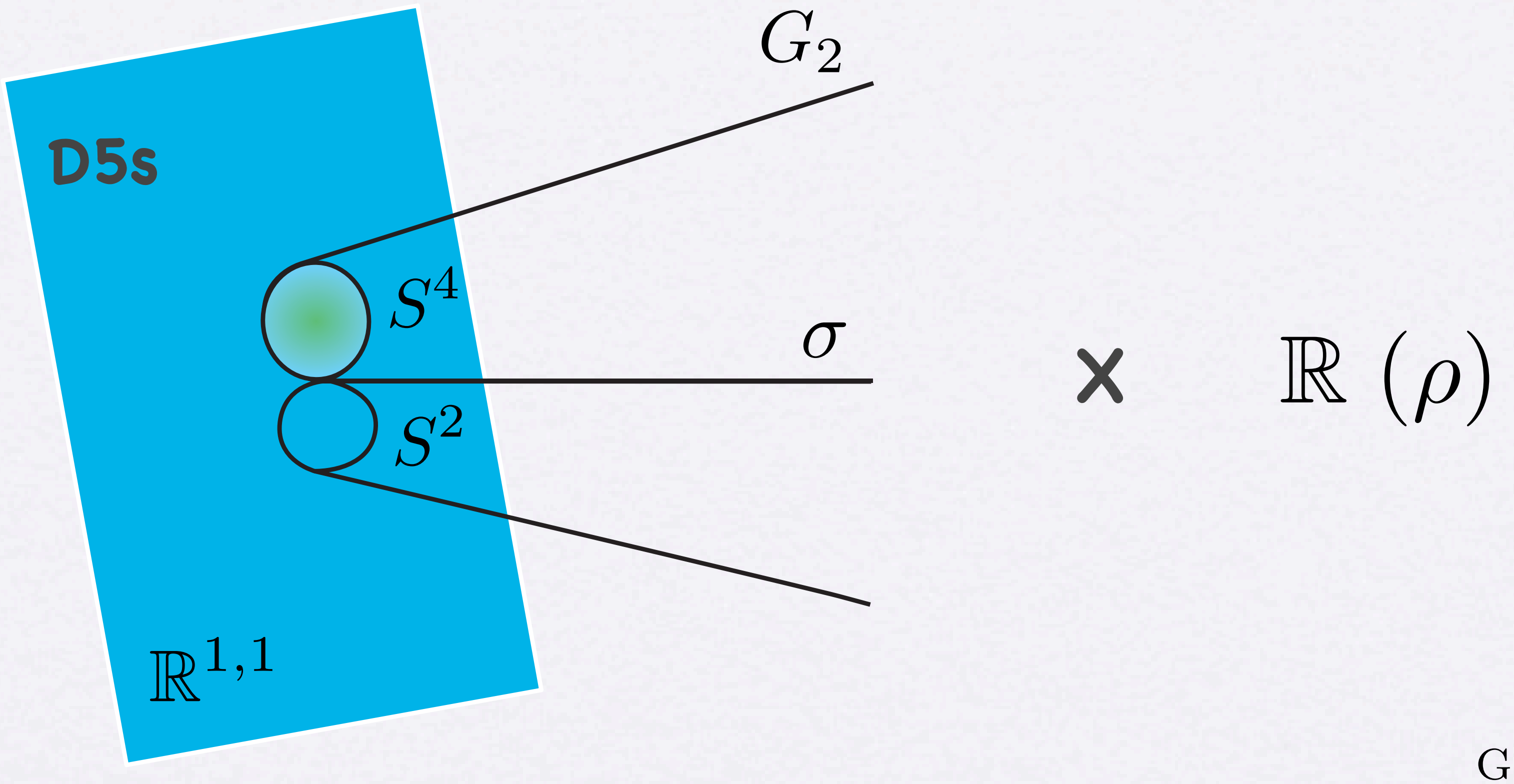


$\overbrace{\qquad\qquad\qquad}^{G_2}$

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	—	—	○	○	○	○	·	·	·	·

$N_3\ :\ (\sigma, \theta, \phi)$

- ◆ G_2 → 1/8 SUSY
- ◆ D5s (on a calibrated C_4) → 1/2 SUSY

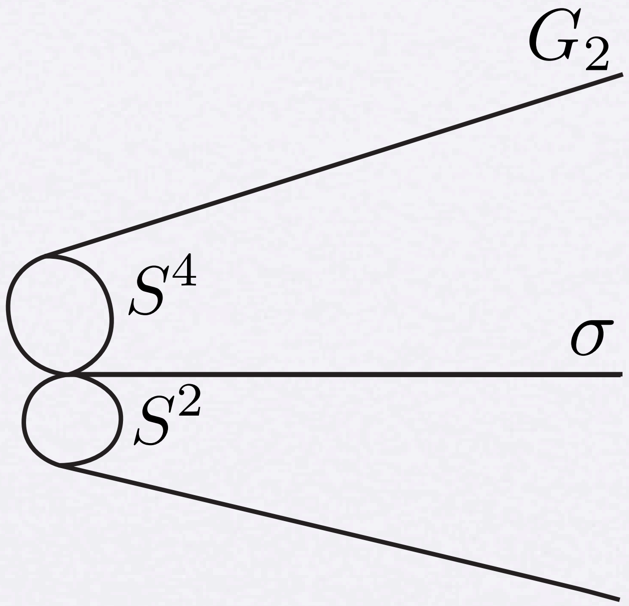
→ **2 SUSYS**

★ SUGRA ANSATZ

			G_2							
	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	–	–	○	○	○	○	·	·	·	·

$N_3 \;:\; (\sigma, \theta, \phi) \qquad \mathbb{R}\ (\rho)$

- **(resolved)**G_2 **cone:** $ds_7^2 = \frac{(d\sigma)^2}{1-\frac{a^4}{\sigma^4}} + \frac{\sigma^2}{2}\, d\Omega_4^2 + \frac{\sigma^2}{4}\left(1-\frac{a^4}{\sigma^4}\right)\left[(E^1)^2+(E^2)^2\right]$ **(Bryant, Salamon)** **(Gibbons, Page, Pope)**



★ SUGRA ANSATZ

G_2

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	○	○	○	○	$\cdot$	$\cdot$	$\cdot$	$\cdot$

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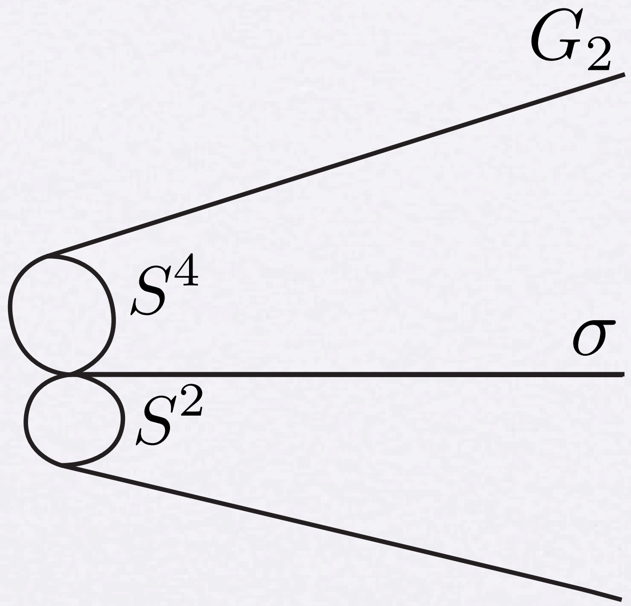

- S^4**:** $d\Omega_4^2 = \dfrac{4}{(1+\xi^2)^2}\left[d\xi^2 + \dfrac{\xi^2}{4}\left((\omega^1)^2+(\omega^2)^2+(\omega^3)^2\right)\right]$

- **fibered** S^2**:**

$$E^1 = d\theta + \frac{\xi^2}{1+\xi^2}\left(\sin\phi\,\omega^1 - \cos\phi\,\omega^2\right)$$

$$E^2 = \sin\theta\left(d\phi - \frac{\xi^2}{1+\xi^2}\,\omega^3\right) + \frac{\xi^2}{1+\xi^2}\,\cos\theta\left(\cos\phi\,\omega^1 + \sin\phi\,\omega^2\right)$$

$$\overbrace{\qquad S^4 \qquad N_3 \qquad}^{\mathrm{G}_2}$$

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	○	○	○	○	·	·	·	·

$N_3\ :\ (\sigma, \theta, \phi)$ $\quad\mathbb{R}\ (\rho)$

- **10d metric** $ds^2 = \boxed{e^{\Phi}}\Big[dx_{1,1}^2 + \frac{\boxed{z}}{m^2}\,d\Omega_4^2\Big] + \frac{e^{-\Phi}}{m^2 \underline{z^{\frac{4}{3}}}}\left[d\sigma^2 + \sigma^2\left((E^1)^2 + (E^2)^2\right)\right] + \frac{e^{-\Phi}}{m^2}(d\rho)^2$

- **3-form** $F_3 = dC_2\ ,\quad C_2 = \boxed{g_1}\,E^1 \wedge E^2 + \boxed{g_2}\left(\mathcal{S}^{\xi} \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\right)$

			G_2							
	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	–	–	○	○	○	○	·	·	·	·

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$$\boxed{\textbf{SUSY}} \xrightarrow{\textbf{N=(1,1)}} \boxed{\textbf{BPSs}} \longrightarrow$$

$z(\rho, \sigma)$	**SIZE OF $\mathcal{C}_4$**
$\Phi(\rho, \sigma)$	**DILATON**
$g_i(\rho, \sigma)$	**3-FORM FLUX**

G$_2$

	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	–	–	○	○	○	○	·	·	·	·

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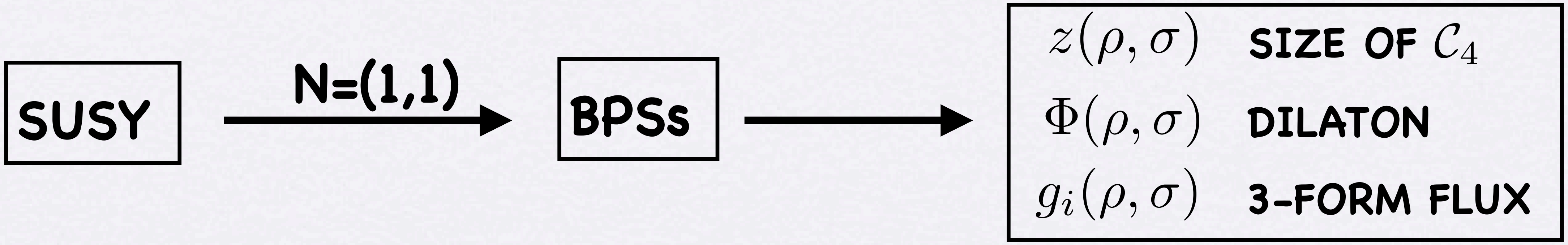

SUSY → **N=(1,1)** → **BPSs** →

$z(\rho,\sigma)$	**SIZE OF $\mathcal{C}_4$**
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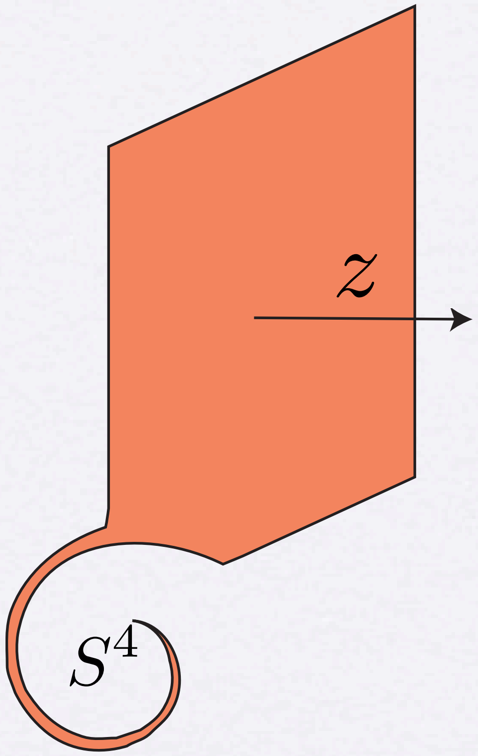
$g_i(\rho,\sigma)$	**3-FORM FLUX**

- **BPSs are PDEs ☹, 7d Gauged SUGRA → SOLUTION ☺**

★ GAUGED SUGRA APPROACH ⟶ LINEAR DISTRIBUTION OF D5S

◆ Take 7d SO(4) Gauged SUGRA ⟶ Domain wall problem



- 1d problem → BPSs easy $\xrightarrow{\text{Uplift}}$ 10d solution in terms of $\boxed{c}$

$$\begin{array}{l} \rho \rightarrow \mathbb{R} \perp (\mathbb{R}^{1,1}, G_2) \\ \sigma \rightarrow G_2 \end{array} \longleftrightarrow (z, \psi)$$

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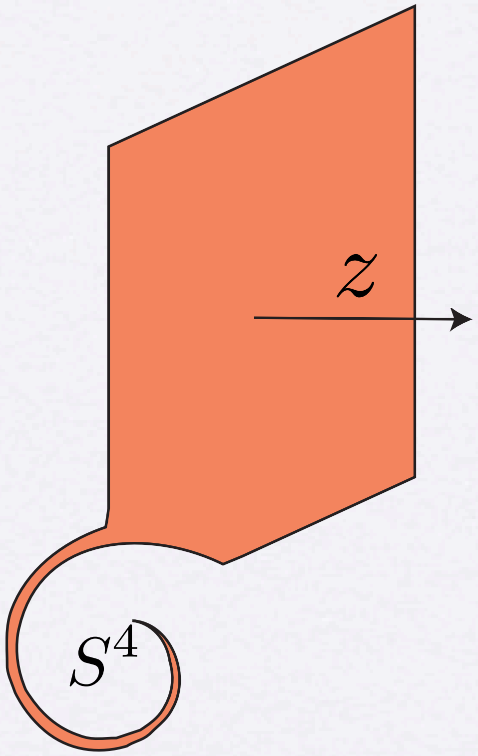

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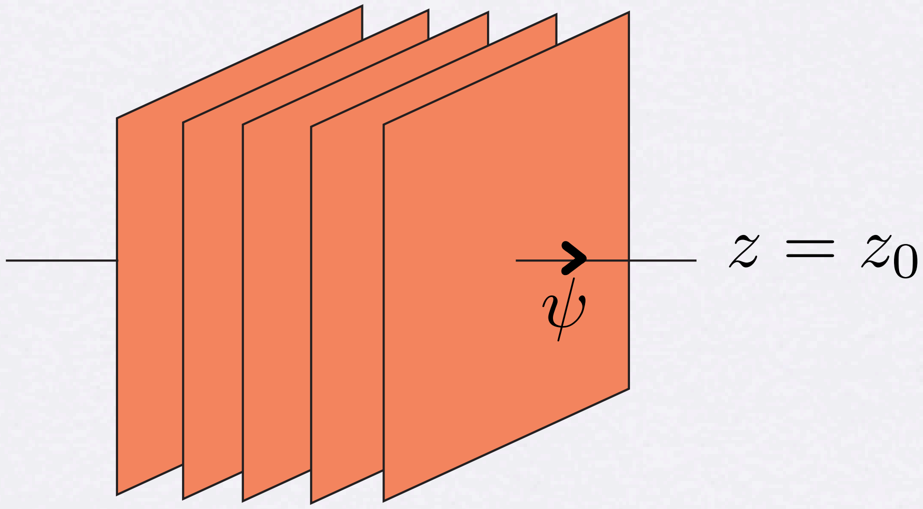
- ◆ UV (z→∞): $ds^2 \to \boxed{\text{D5s along } \mathbb{R}^{1,1}\times S^4}$ [➡ Linear dilaton]

- ◆ IR (for c<-1):
  - Singularity (good) at $z = z_0$
  - Linear distribution (ψ)

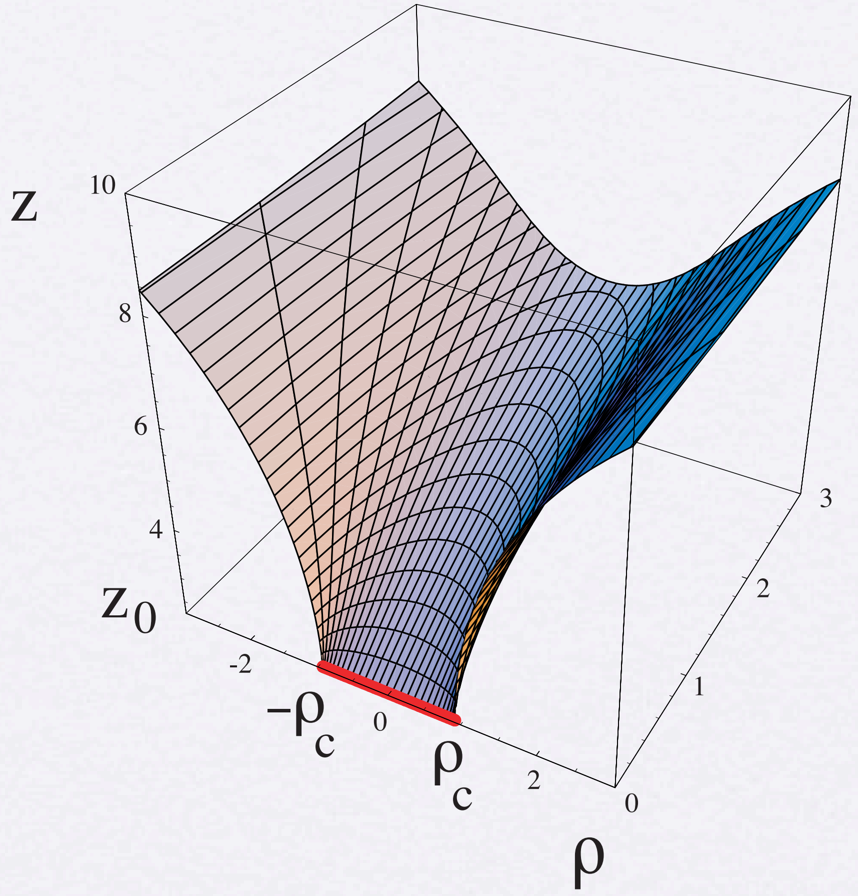


			G_2							
	$\mathbb{R}^{1,1}$		S^4				N_3			$\mathbb{R}$
D5	$-$	$-$	○	○	○	○	$\cdot$	$\cdot$	$\cdot$	$\cdot$

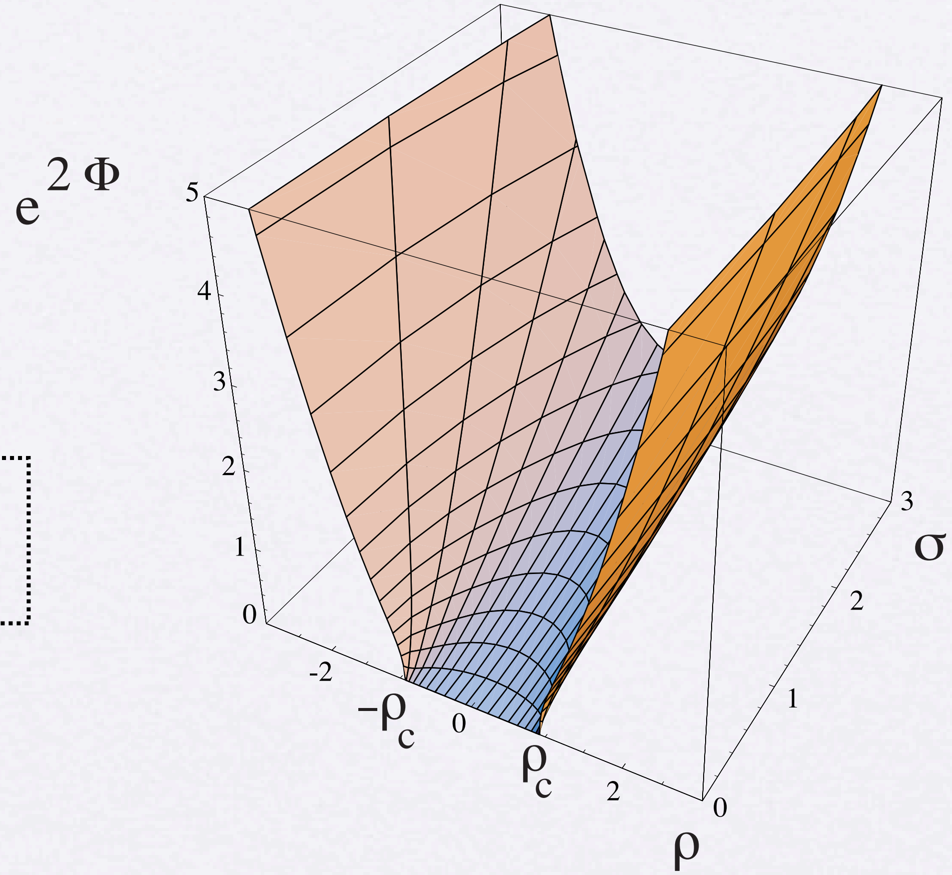
$N_3\ :\ (\sigma,\theta,\phi)\qquad \mathbb{R}\,(\rho)$

- **Changing vbles.** $(z,\psi)\rightarrow(\rho,\sigma)$

➥ **Analytic (implicit) sol. for** $z(\rho,\sigma)$



$$\boxed{c=-\frac{3}{2}}$$



G_2

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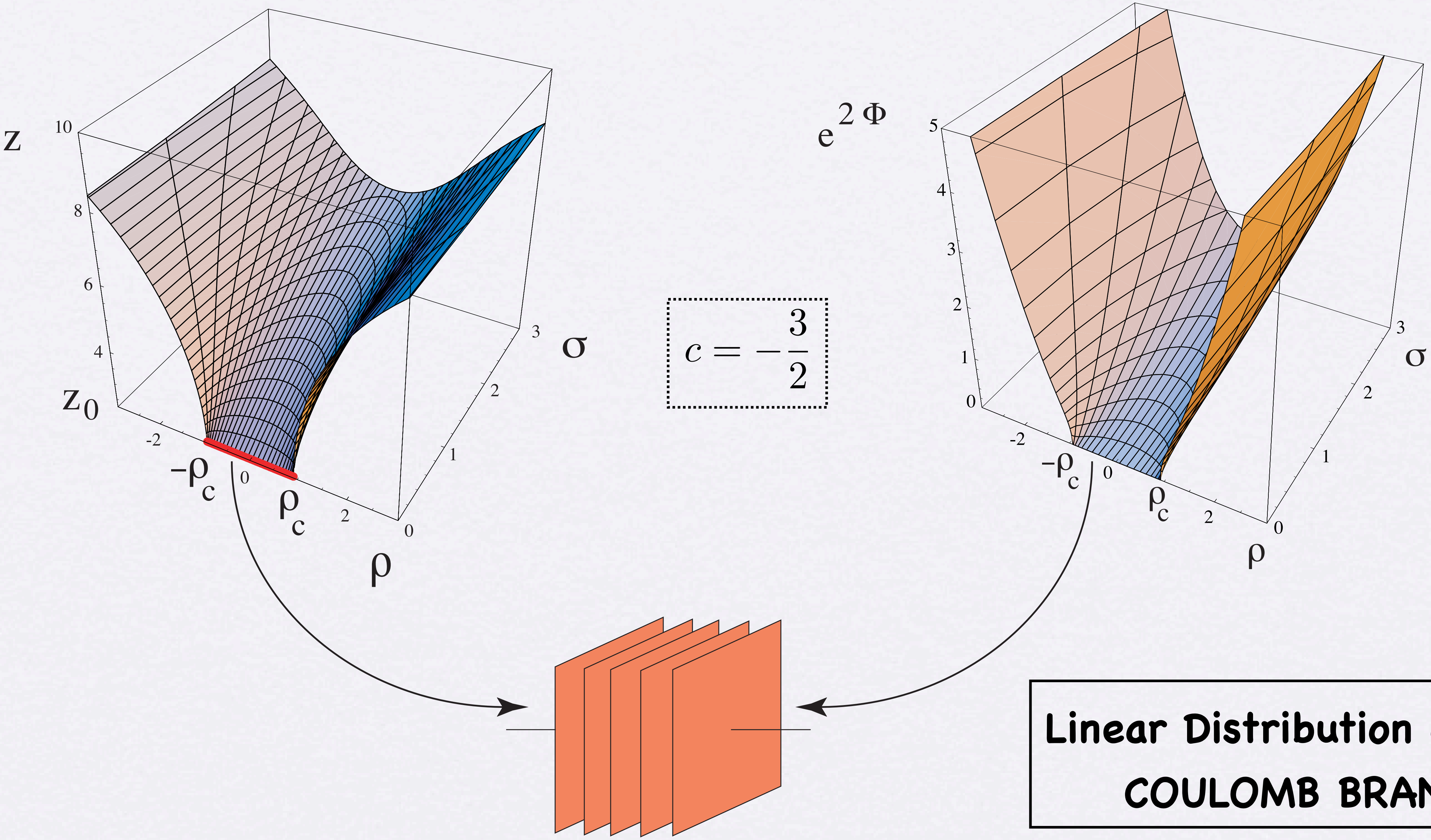


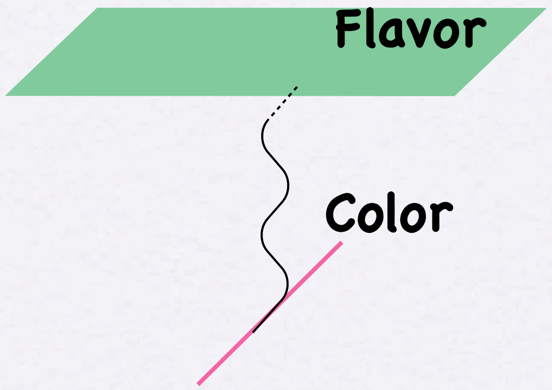
$$c = -\frac{3}{2}$$

$$(z = z_0, \psi) \rightarrow (|\rho| < \rho_c, \sigma = 0)$$

Linear Distribution of D5s

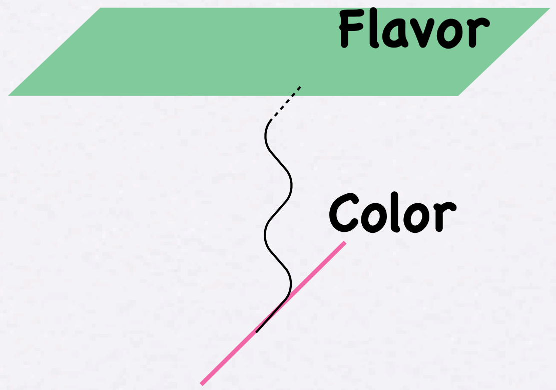
COULOMB BRANCH

★ ADDING FLAVOR



- Add an open string sector → FLAVOR BRANES

★ ADDING FLAVOR



◆ Add an open string sector → FLAVOR BRANES

- Brane setup

Flavor D5s

- Non-compact $\mathcal{C}_4 \subset G_2$
- At fixed $\rho = \rho_Q$

→

★ Global Sym: flavor

★ $m_Q \sim \rho_Q$

★ Same SUSY

★ ADDING FLAVOR

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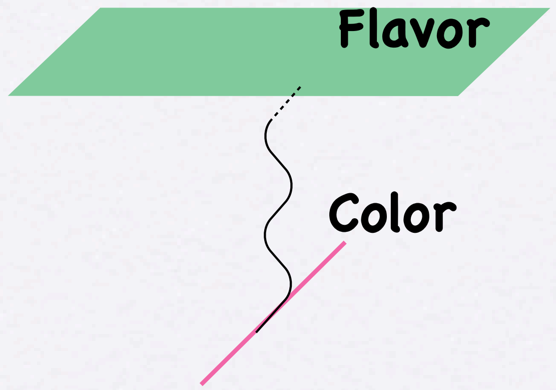

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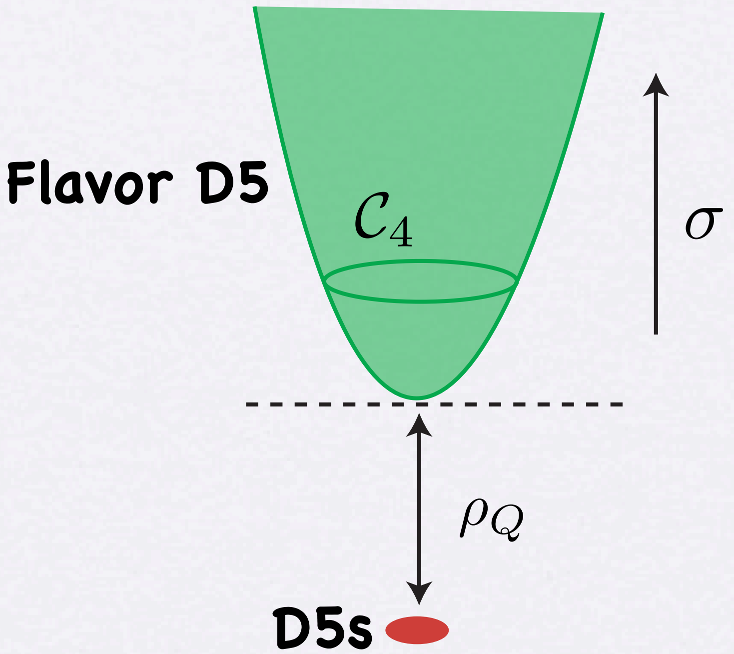
- Probe approximation $N_f \ll N_c \,, \; N_c \to \infty$

(Karch & Randall, Karch & Katz)

↕

Quenched flavor in the large N_c limit.

→



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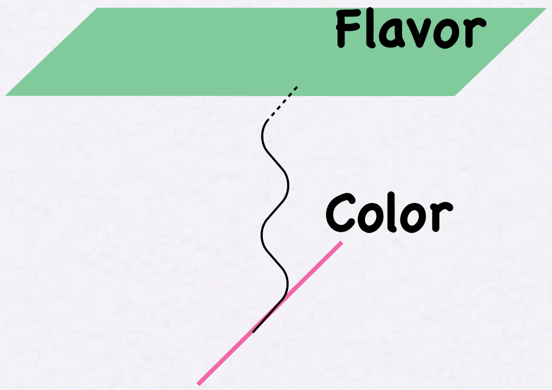

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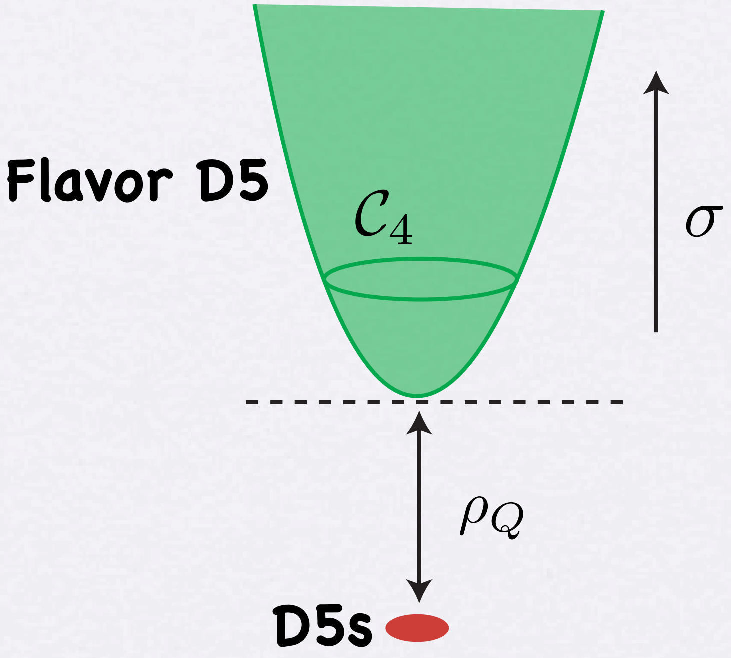


- Backreaction $N_f \sim N_c$

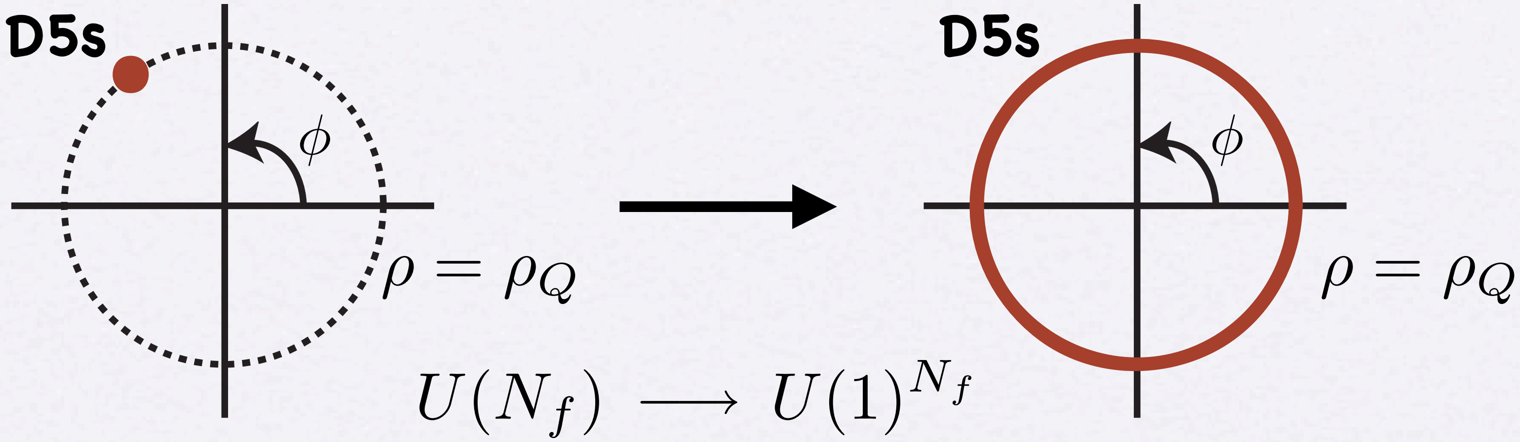
$N_f \, , \; N_c \to \infty$
N_f / N_c fixed

→ Veneziano limit
Quarks loops included

- **Computing the backreaction is difficult** $\boxed{S = S_{IIB} + S_{DBI}^{\text{flavor}} + S_{WZ}^{\text{flavor}}}$

➥ **Smearing**

(Bigazzi et al, Casero et al)



$$U(N_f) \longrightarrow U(1)^{N_f}$$

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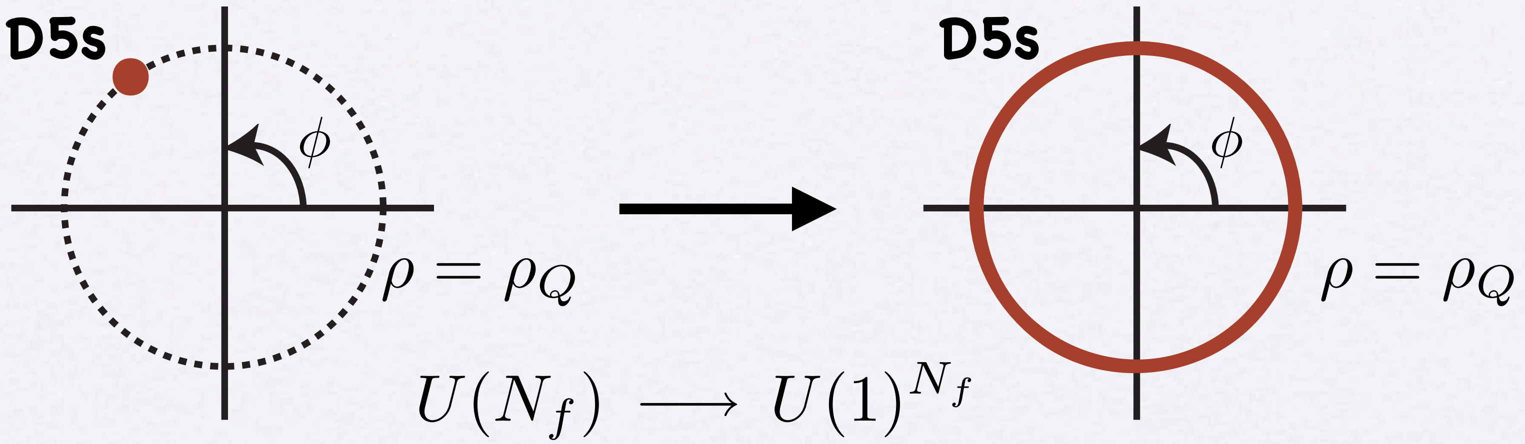

$$U(N_f) \longrightarrow U(1)^{N_f}$$

$$S^{flavor}_{WZ} = T_5 \sum^{N_f} \int_{\mathcal{M}_6^{(i)}} \hat{C}_6 \Longrightarrow -T_5 \int_{\mathcal{M}_{10}} \enclose{circle}{\Omega} \wedge C_6 \longrightarrow \boxed{dF_3 \; = \; 2\kappa_{10}^2 \, T_5 \, \Omega}$$

~~**Bianchi identity**~~

➥ **Ω + metric →** $\boxed{\textbf{Flavored BPSs}}$

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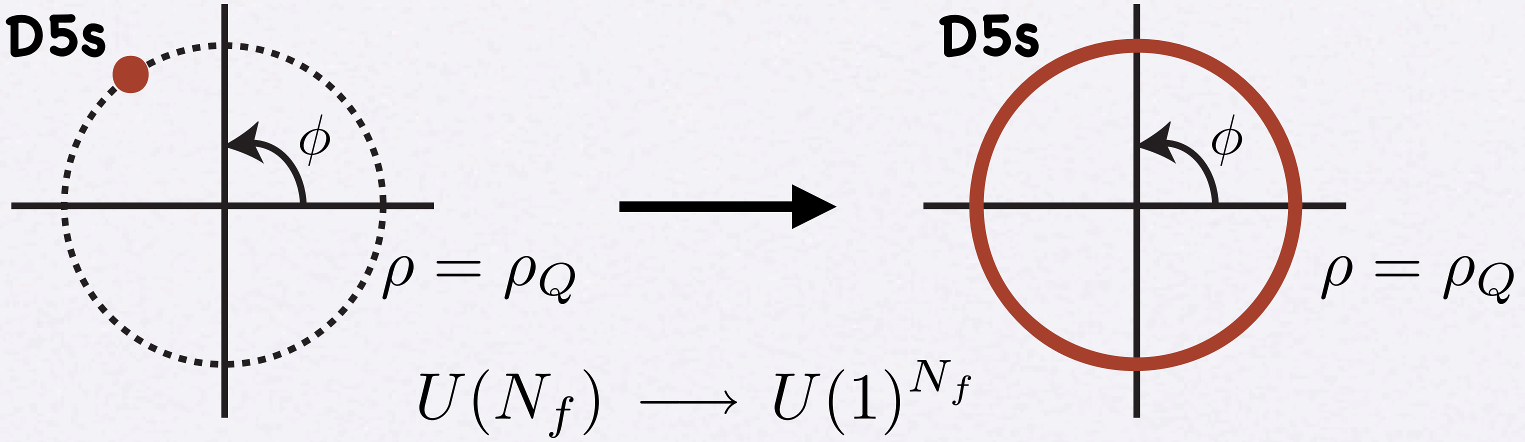

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~~Bianchi identity~~

➥ Ω + metric → **Flavored BPSs**

- D5 embeddings (κ-symmetry) → Ω , this is hard!!

- D5-branes at $\rho = \rho_Q$
- Same SUSY (2)
- No new deformations of g_{ab}

→ **generic Ω** /

- Consistent BPSs (➡ EoM)
- Color ∩ Flavor = ∅

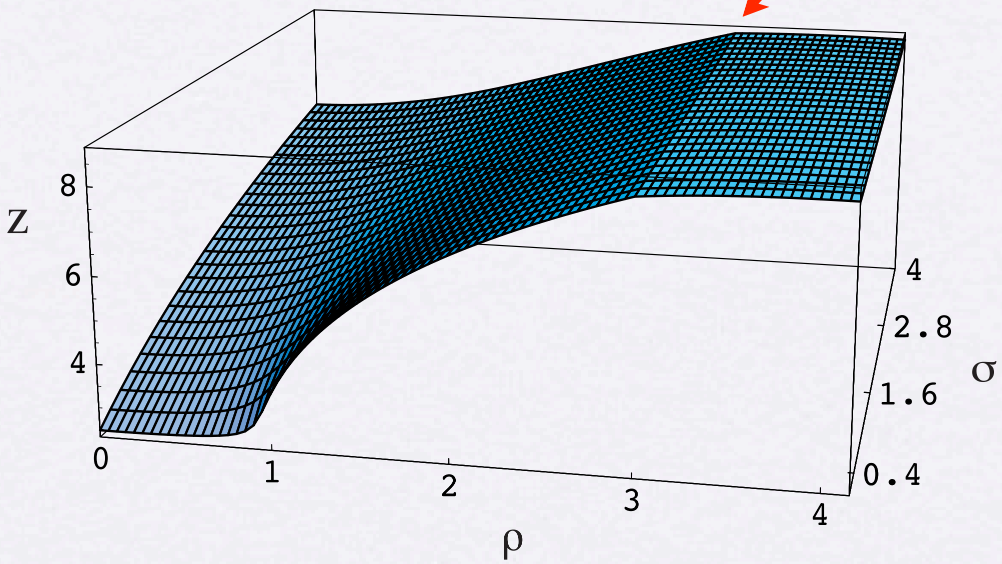
◆ Particular charge distribution / homogeneous charge distribution along ⊥ $\mathbb{R}^3$

- Numerical solution with $z,\ \phi,\ g_i$ continuous at $\rho = \rho_Q$

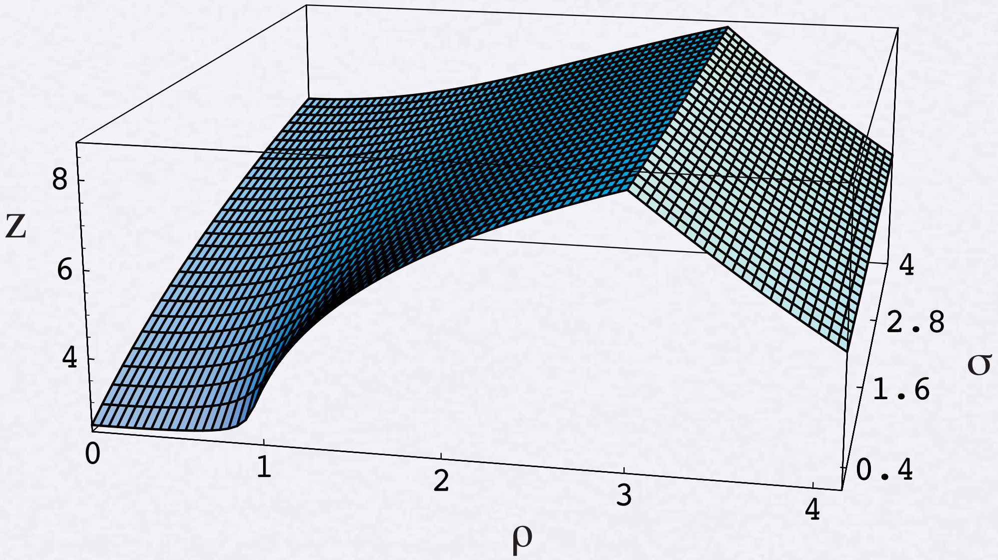
- Coincides with the unflavored for $\rho < \rho_Q$

$\rho = \rho_Q$

$$x \equiv \frac{18\pi\, n_f}{N_c} = 0.2$$



$$x = 1$$



- Flavor contributes as expected [$1/g_{YM}^2 \sim z^2(\rho, \sigma = 0)$]

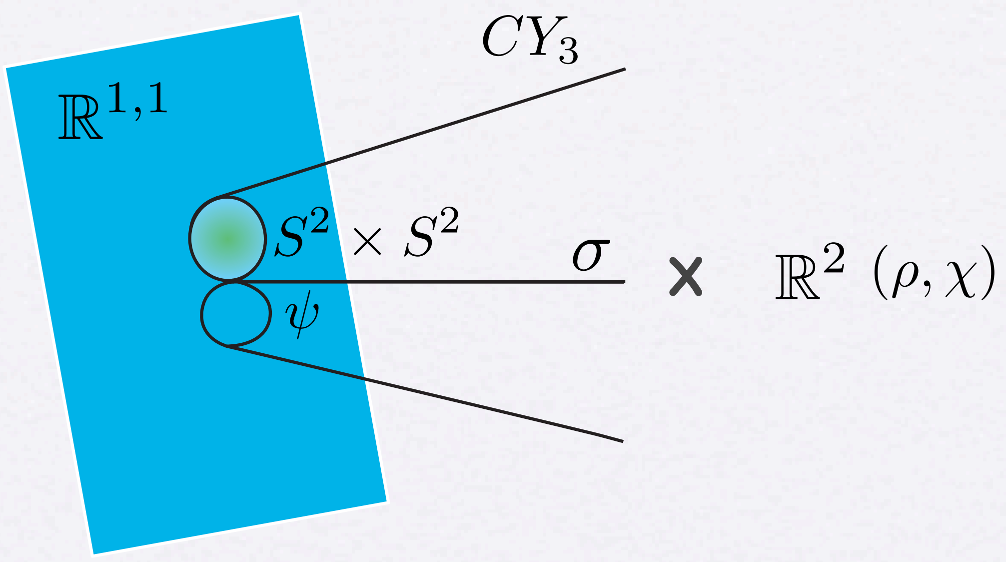
★ SUGRA DUALS OF 2D THEORIES WITH N=(2,2) SUSY

- D5s on a 4-cycle of a CY3 ~ 2d N = (2,2)

◆ CY3 → 1/4 SUSY
◆ D5s → 1/2 SUSY

→

2 SUSYS



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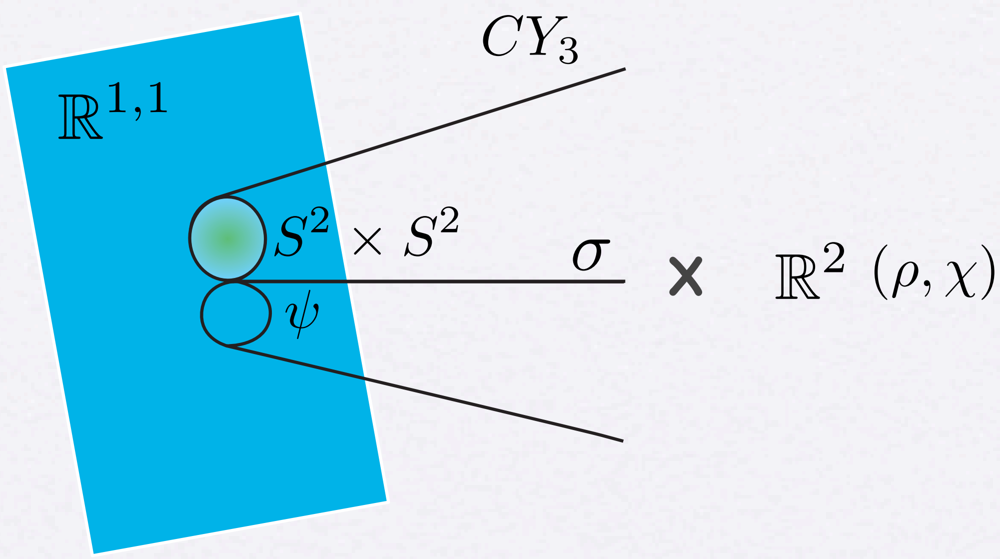

- 10d Ansatz
  - Metric → z (ρ,σ) & ϕ(ρ,σ)
  - 3-form → g(ρ,σ)

→ **BPSs**

- Analyt. sol ✓
- EoM ✓

[7d Gauged SUGRA]

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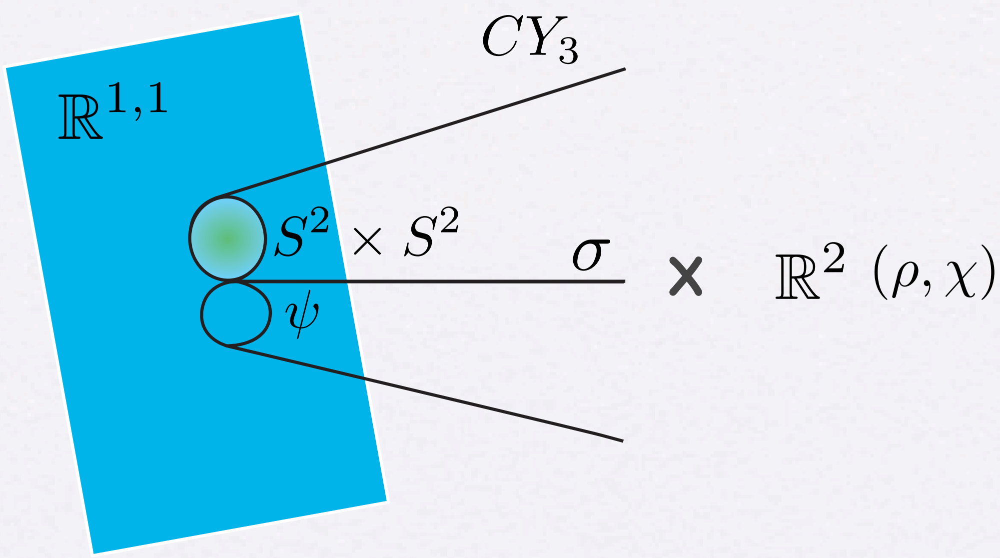

◆ CY3 → 1/4 SUSY
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⟶ 2 SUSYS

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⟶ BPSs

- Analyt. sol ✓
- EoM ✓

[7d Gauged SUGRA]

- Flavoring → D5s on a non-compact 4-cycle → Embeddings found

➥ Ω constructed → new BPSs → (Numeric) Flavored background

★ SUMMARY / TO TRY

- Gravity duals of 2d N=(1,1) & (2,2) SUSY theories from wrapped D5s ✓
- Large number of flavors via backreacting flavor D5s ✓
- Explore the F.T. (a little) → color probe brane ✓ (E-r relation missing)
- Higgs branch → Color & flavor branes recombining
- Alternative setup → D3s on a 2-cycle of a CY3. Better UV.
- Non-singular background?
- Less SUSY → D5s on a 4-cycle of a Spin(7)