On the Classification of Brane Tilings

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Zürich, September 11, 2009
1. Brane Tilings for D3 branes
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3. Our Algorithm
4. Tilings Generated
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Motivation for Tilings

- First developed to help understand the SUSY gauge theory living on D3 branes probing Toric Calabi-Yau singularities
- Gauge theory dual of Type IIB string theory on $AdS_5 \times X_5$
- Tiling gives gauge symmetry as well as superpotential data of theory living on D3 branes
- Tiling easily computed with knowledge of either gauge theory or Calabi-Yau singularity
So ... What is a Brane Tiling (Dimer Model)?

- Periodic Bipartite Tiling on the Plane
- Each white (black) node represents a positive (negative) superpotential term
- Each face corresponds to a gauge group
- Each edge represents a bifundamental chiral field
- Tilings correspond to Supersymmetric Quiver Gauge Theories
What is a Quiver Gauge Theory

- A quiver gauge theory is a special supersymmetric gauge theory that has a matter content that can be represented by a graph called a quiver
- A quiver is simply a directed graph
- Nodes of the quiver represent gauge groups
- Edges of the quiver represent bifundamental chiral superfields
- Superpotential information is not encoded in the quiver
Brane Tilings and Quiver Gauge Theories

- One can easily read off the quiver gauge theory with knowledge of the tiling
- Periodic quiver is graph dual to brane tiling
Some Features of Brane Tilings

- Can find vacuum moduli space of the theory via the fast forward algorithm (FFA)

- Space can be identified with the CY singularity probed by D3 branes. Best described using the language of toric geometry
- Inverse algorithm also exists to find tiling (and gauge theory) corresponding to generic toric CY singularities
Warning!

- Not all periodic bipartite tilings of the plane correspond to consistent brane tilings in 3+1 dimensions
- Failure of current methods
Brane Setup

- Recent work shows that brane tilings can also be used to describe supersymmetric quiver Chern-Simons (CS) theories.
- These theories are thought to describe M2 branes probing the singular tip of toric CY 4-fold singularities.
Similarities between the two interpretations

- Periodic Bipartite Tiling on the Plane
- Each white (black) node represents a positive (negative) superpotential term
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Differences between the two interpretations

- Each face represents a Chern Simons term
- A set of CS levels must be chosen
- There is no *known* consistency condition
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Differences between the two interpretations

- There are many simple tilings that have not been studied so far and may be relevant for M2 branes
- A classification of tilings is important
Our Aim

- We would like an algorithm that generates brane tilings
- The algorithm should be computationally feasible
- The generation should be exhaustive
The Algorithm

Generate ‘Irreducible’ Quivers satisfying ‘Calabi-Yau’ Condition

↓

Generate ‘Toric’ Superpotentials

↓

Check For Tiling
Irreducibility

- An ‘Irreducible’ gauge theory is one that has no nodes in the quiver of order two
- Any reducible quiver gauge theory can be formed by adding such nodes to an irreducible quiver

Our Algorithm
Tilings Generated
Conclusion
Calabi-Yau Condition

- Nodes of quivers corresponding to brane tilings must have equal numbers of incoming and outgoing arrows.
- This is known as the ‘Calabi-Yau condition’ and corresponds to an anomaly cancellation condition in 3+1 dimensions.
- Without this observation, our algorithm would be computationally infeasible.
A theory satisfying the toric condition has each field appearing in the superpotential exactly twice - once in a positive term and once in a negative term.

We also insist upon having no superpotential terms of order 2.
Order Parameters

It is fairly easy to find good parameters to order our generation of brane tilings. Suitable parameters turn out to be:

- $N_T$ - the number of superpotential terms
- $G$ - the number of gauge groups (or nodes in the quiver)

The number of fields is related to these two parameters by the Euler condition $E = N_T + G$. 

![Diagram of a brane tiling](image_url)
We would like to perform an exhaustive search of all (irreducible) quivers given a pair of order parameters \((N_T, G)\).

To achieve this we make the following observation:

- A quiver diagram satisfies the Calabi Yau (in-out) condition iff it can be formed from a sum of cycles.
Generation of Superpotentials

- Each term in the superpotential is gauge invariant
- Can be written in terms of cycles
- These cycles have already been generated in the algorithm

\[ \begin{align*}
1 &\quad 2 \\
3 &\quad 4
\end{align*} \]

= \[ \begin{align*}
1 &\quad 2 \\
3 &\quad 4
\end{align*} \] + \[ \begin{align*}
1 &\quad 2 \\
3 &\quad 4
\end{align*} \] + \[ \begin{align*}
4 &\quad 3
\end{align*} \]

- Compute positive then negative superpotential terms
Try to combine superpotential terms into a fundamental domain of the periodic quiver
**Tiling Check**

- Attempt to use this candidate fundamental domain to tile the plane
Comments

- Exhaustive
- Computationally cheap - can compute all tiles with 6 superpotential terms easily (well ... fairly easily)
Two Superpotential Terms

[Diagram showing two superpotential terms with labels 1 and 2 on different parts of the network.]

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On the Classification of Brane Tilings
Four Superpotential Terms
Six Superpotential Terms (1)

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On the Classification of Brane Tilings
Six Superpotential Terms (2)
Conclusion

- Brane tilings are a tool that have allowed us to find a large class of SCFTs with AdS duals
- Can be useful to describe D3 and M2 branes
- Our algorithm allows an exhaustive generation of brane tilings
- Inconsistent tilings generated are thought to be useful in the M2 brane story
- More relationships between tilings can be explored (e.g. Higgsing M2-brane Theories hep-th/0908.4033)
- If nothing else you can generate some really pretty pictures to impress your friends