On the Classification of Brane Tilings

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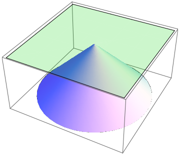
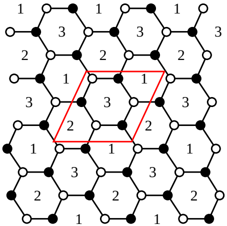
Motivation for Tilings

- First developed to help understand the SUSY gauge theory living on D3 branes probing Toric Calabi-Yau singularities
- Gauge theory dual of Type IIB string theory on $AdS_5 \times X_5$
- Tiling gives gauge symmetry as well as superpotential data of theory living on D3 branes
- Tiling easily computed with knowledge of either gauge theory or Calabi-Yau singularity



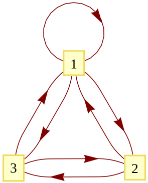
So . . . What is a Brane Tiling (Dimer Model)?

- Periodic Bipartite Tiling on the Plane
- Each white (black) node represents a positive (negative) superpotential term
- Each face corresponds to a gauge group
- Each edge represents a bifundamental chiral field
- Tilings correspond to Supersymmetric Quiver Gauge Theories



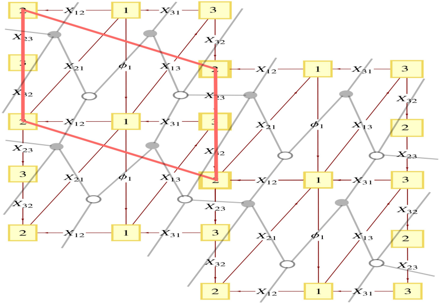
What is a Quiver Gauge Theory

- A quiver gauge theory is a special supersymmetric gauge theory that has a matter content that can be represented by a graph called a quiver
- A quiver is simply a directed graph
- Nodes of the quiver represent gauge groups
- Edges of the quiver represent bifundamental chiral superfields
- Superpotential information is not encoded in the quiver



Brane Tilings and Quiver Gauge Theories

- One can easily read off the quiver gauge theory with knowledge of the tiling
- Periodic quiver is graph dual to brane tiling



Some Features of Brane Tilings

- Can find vacuum moduli space of the theory via the fast forward algorithm (FFA)

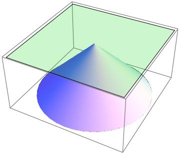
- Space can be identified with the CY singularity probed by D3 branes. Best described using the language of toric geometry
- Inverse algorithm also exists to find tiling (and gauge theory) corresponding to generic toric CY singularities

Warning!

- Not all periodic bipartite tilings of the plane correspond to consistent brane tilings in 3+1 dimensions
- Failure of current methods

Brane Setup

- Recent work shows that brane tilings can also be used to describe supersymmetric quiver Chern-Simons (CS) theories
- These theories are thought to describe M2 branes probing the singular tip of toric CY 4-fold singularities



Similarities between the two interpretations

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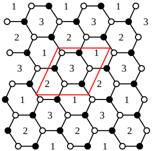

Differences between the two interpretations

- Each face represents a Chern Simons term
- A set of CS levels must be chosen
- There is no *known* consistency condition

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Differences between the two interpretations

- There are many simple tilings that have not been studied so far and may be relevant for M2 branes
- A classification of tilings is important

Our Aim

- We would like an algorithm that generates brane tilings
- The algorithm should be computationally feasible
- The generation should be exhaustive

The Algorithm

Generate ‘Irreducible’ Quivers satisfying ‘Calabi-Yau’ Condition

$\downarrow$

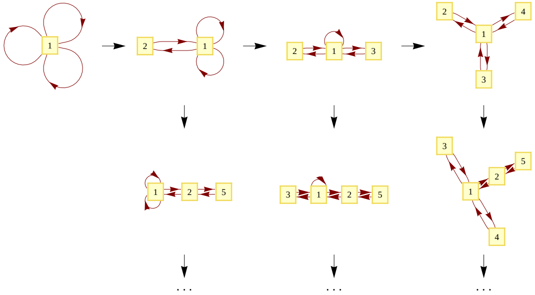
Generate ‘Toric’ Superpotentials

$\downarrow$

Check For Tiling

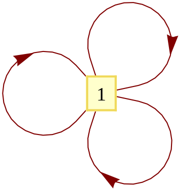
Irreducibility

- An 'Irreducible' gauge theory is one that has no nodes in the quiver of order two
- Any reducible quiver gauge theory can be formed by adding such nodes to an irreducible quiver



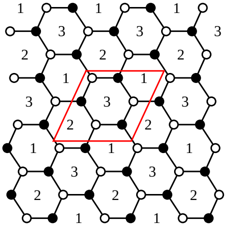
Calabi-Yau Condition

- Nodes of quivers corresponding to brane tilings must have equal numbers of incoming and outgoing arrows.
- This is known as the 'Calabi-Yau condition' and corresponds to an anomaly cancellation condition in 3+1 dimensions
- Without this observation, our algorithm would be computationally infeasible



Toric Condition

- A theory satisfying the toric condition has each field appearing in the superpotential exactly twice - once in a positive term and once in a negative term
- We also insist upon having no superpotential terms of order 2

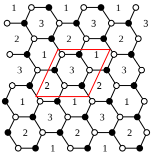


Order Parameters

It is fairly easy to find good parameters to order our generation of brane tilings. Suitable parameters turn out to be:

- N_T - the number of superpotential terms
- G - the number of gauge groups (or nodes in the quiver)

The number of fields is related to these two parameters by the Euler condition $E = N_T + G$.

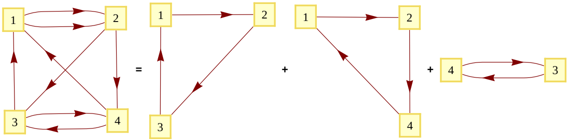


Generation of Quivers

We would like to perform an exhaustive search of all (irreducible) quivers given a pair of order parameters (N_T, G)

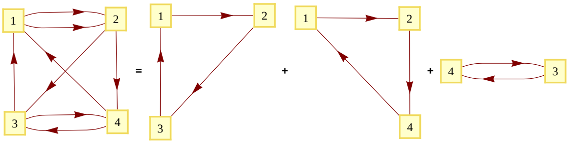
To achieve this we make the following observation:

- A quiver diagram satisfies the Calabi Yau (in-out) condition iff it can be formed from a sum of cycles



Generation of Superpotentials

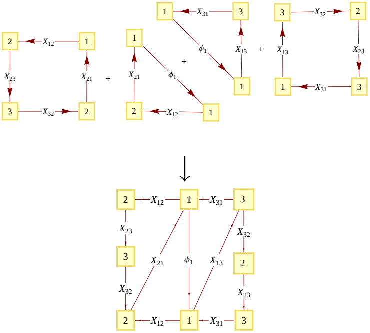
- Each term in the superpotential is gauge invariant
- Can be written in terms of cycles
- These cycles have already been generated in the algorithm



- Compute positive then negative superpotential terms

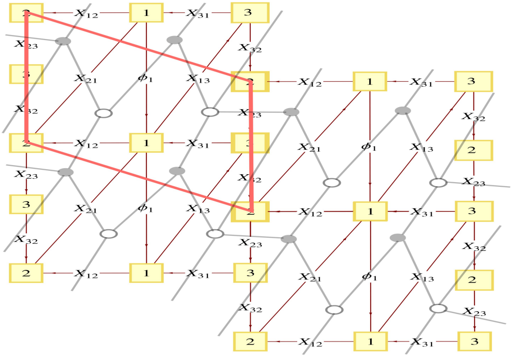
Tiling Check

- Try to combine superpotential terms into a fundamental domain of the periodic quiver



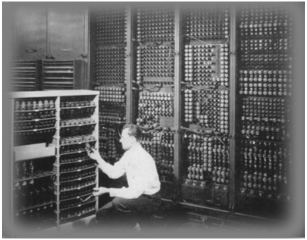
Tiling Check

- Attempt to use this candidate fundamental domain to tile the plane

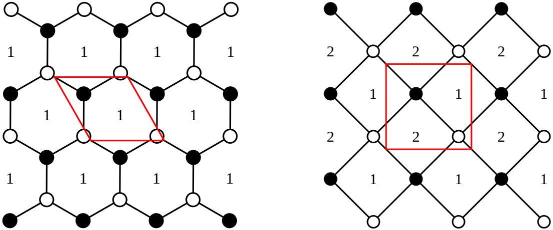


Comments

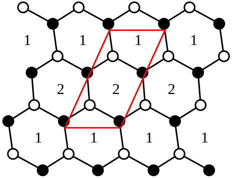
- Exhaustive
- Computationally cheap - can compute all tiles with 6 superpotential terms easily (well . . . fairly easily)

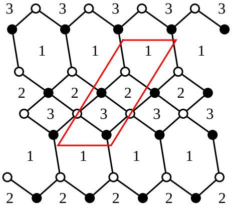


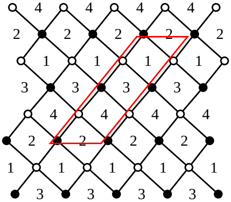
Two Superpotential Terms

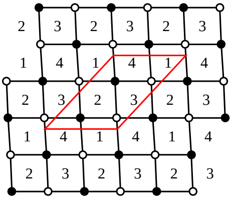


Four Superpotential Terms

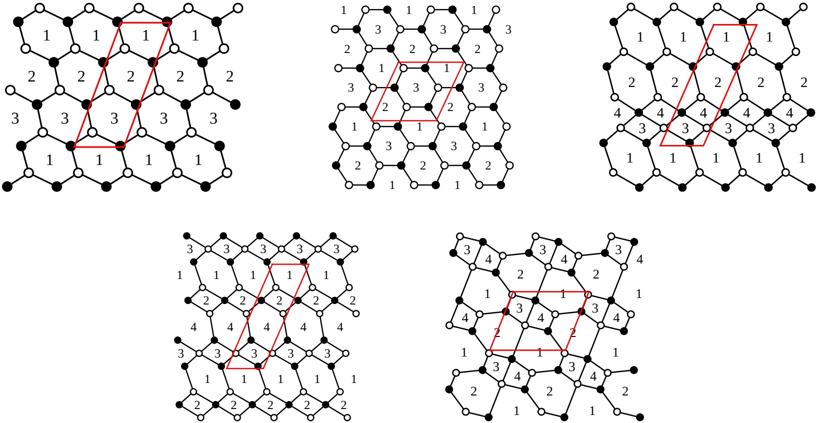




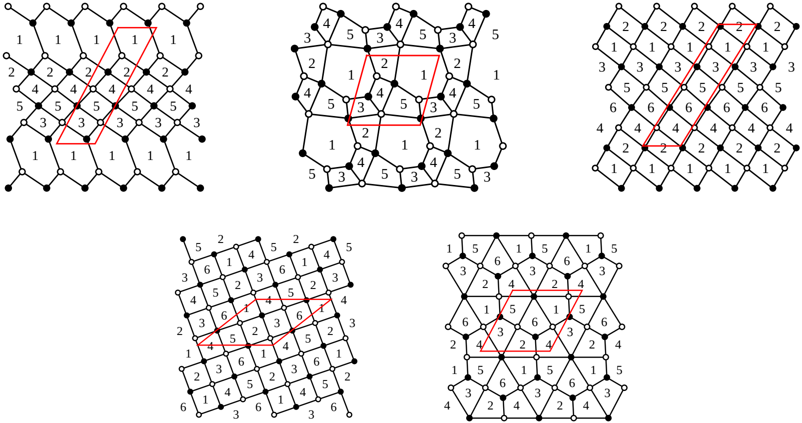




Six Superpotential Terms (1)



Six Superpotential Terms (2)



Conclusion

- Brane tilings are a tool that have allowed us to find a large class of SCFTs with AdS duals
- Can be useful to describe D3 and M2 branes
- Our algorithm allows an exhaustive generation of brane tilings
- Inconsistent tilings generated are thought to be useful in the M2 brane story
- More relationships between tilings can be explored (e.g. Higgsing M2-brane Theories hep-th/0908.4033)
- If nothing else you can generate some really pretty pictures to impress your friends