Applications of string theory to the very hot and the very cold

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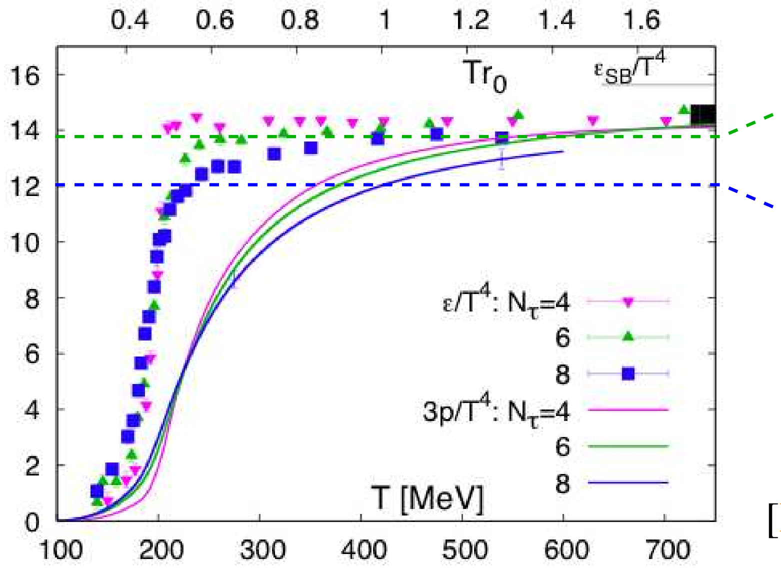
1. The very hot: heavy-ion collisions

$T_{\rm peak} \approx 300\,{\rm MeV}$ for central RHIC collisions, about $200{,}000$ times hotter than the core of the sun, and about 1.7 times bigger than $T_c \approx 180\,{\rm MeV}$ where QCD deconfines.

First natural question: What is the equation of state? Lattice gives pretty reliable answers (except T_c is hard to pin down in ${\rm MeV}$).



$$\epsilon/\epsilon_{\rm free} = 0.88 \;\leftrightarrow\; \lambda_{SYM} = 5.5$$

$$\epsilon/\epsilon_{\rm free} = 0.77 \;\leftrightarrow\; \lambda_{SYM} = 6\pi$$

[Bazavov et al. 2009]

1.1. Equation of state and bulk viscosity

Authors of [Kharzeev and Tuchin 2008; Karsch et al. 2008] suggest a way to translate EOS into a prediction for bulk viscosity:

$$\zeta = \frac{1}{9\omega_0}\left[T^5 \frac{\partial}{\partial T}\frac{\epsilon - 3p}{T^4} - 16\epsilon_{\rm vac}\right] + (\text{quark terms})\,. \tag{1}$$

(1) comes out of a low-energy theorem ("sum rule") for $\theta \equiv T^\mu_\mu$:

$$G^E(0,\vec{0}) = \int d^4x\,\langle\theta(x)\theta(0)\rangle = \left(T\frac{\partial}{\partial T} - 4\right)\langle\theta(0)\rangle + (\text{quark terms})\,, \tag{2}$$

plus observation that $\langle\theta(0)\rangle = \epsilon - 3p + 4\epsilon_{\rm vac}$, plus (crucially) the *assumption* of a low-frequency parametrization

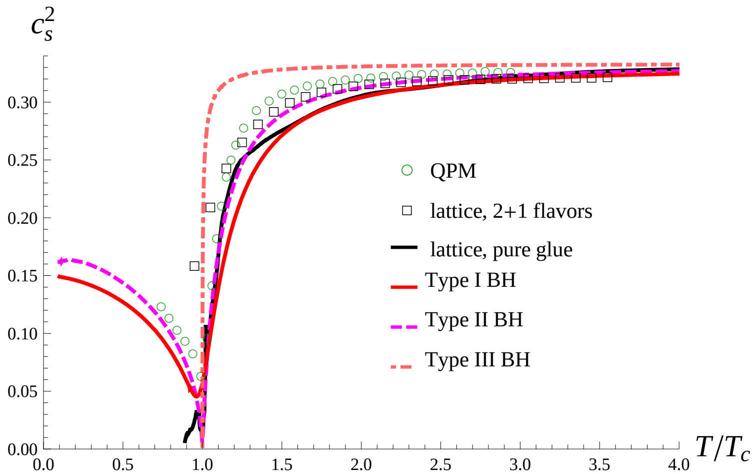
$$\rho(\omega,\vec{0}) = \frac{9\zeta\omega}{\pi}\frac{\omega_0^2}{\omega_0^2+\omega^2} \qquad\qquad \omega_0 \sim 1\,\text{GeV} \tag{3}$$

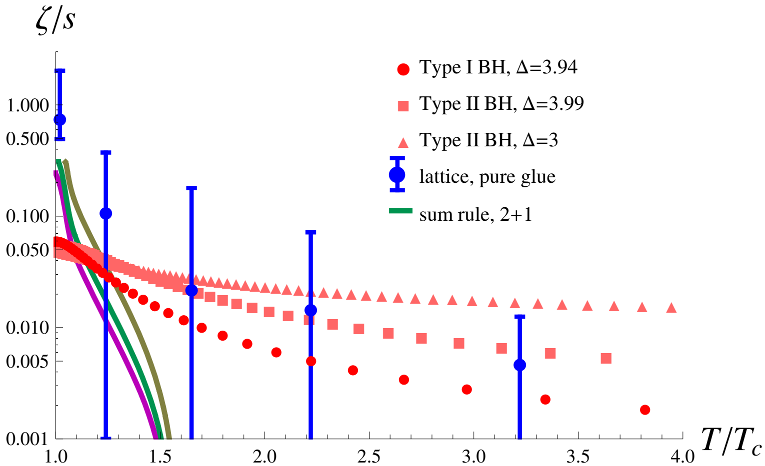
for the spectral measure of the two-point function of T^μ_μ.

Because (3) is *ad hoc*, it seems worthwhile to obtain ζ using strongly coupled methods and compare with (1).

The results [Gubser and Nellore 2008; Gubser et al. 2008ab]: ζ rises near T_c, but not so much as (1) predicts.

- Type I: smooth cross-over: quasi-realistic.
- Type II: nearly second order, $c_s^2 \to 0$ at T_c.
- Type III: No BH below T_c, like [Gursoy et al. 2008b].





- Sharper behavior of c_s^2 gives sharper ζ/s.
- Large ζ at T_c is hard to arrange with a reasonably realistic EOS.
- Poses a challenge for "soft statistical hadronization" proposal of [Karsch et al. 2008].

The method:

Reproduce the lattice EOS using

$$\mathcal{L} = \frac{1}{2\kappa_5^2}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] . \tag{4}$$

$V(\phi)$ can be adjusted to match dependence of

$$\text{speed of sound:} \quad c_s^2 \equiv \frac{dp}{d\epsilon} \tag{5}$$

on T. Then adjust κ_5^2 to get desired ϵ/T^4 at some high scale (say $3\,\mathrm{GeV}$). A quasi-realistic EOS comes from

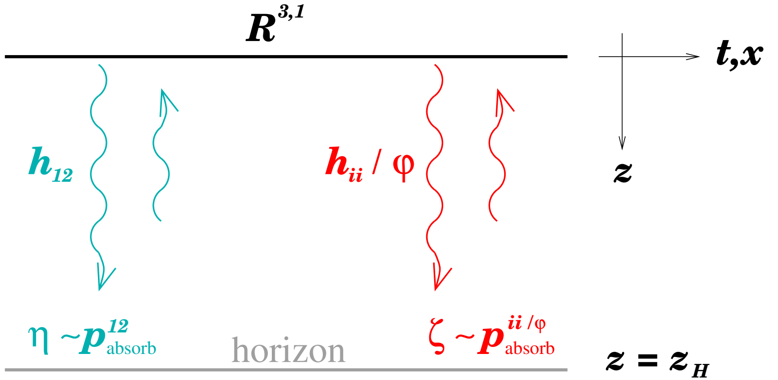
$$V(\phi) = \frac{-12\cosh\gamma\phi + b\phi^2}{L^2} \qquad \gamma = 0.606\,, \quad b = 2.057\,. \tag{6}$$

Authors of [Gursoy and Kiritsis 2008; Gursoy et al. 2008ab] took same starting point (4) further: an appropriate $V(\phi)$, with $V \sim -\phi^2 e^{\sqrt{\frac{2}{3}}\phi}$, gives a Hawking-Page transition to confinement; logarithmic RG in UV; glueball with $m^2 \sim n$, as in linear confinement; and favorable comparison with thermodynamic and transport quantities [Gursoy et al. 2009ab].

Once conformal invariance is broken, we can investigate bulk viscosity [Gubser et al. 2008ba], following a number of earlier works, e.g. [Parnachev and Starinets 2005; Buchel 2005 2007]:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} \int d^3x \, dt \, e^{i\omega t} \theta(t) \langle [T^\mu{}_\mu(t,\vec{x}), T^\nu{}_\nu(0,0)] \rangle \,. \tag{7}$$

Shear viscosity relates to absorption probability for an h_{12} graviton. Bulk viscosity relates to absorption of a mixture of the h_{ii} graviton and the scalar ϕ.



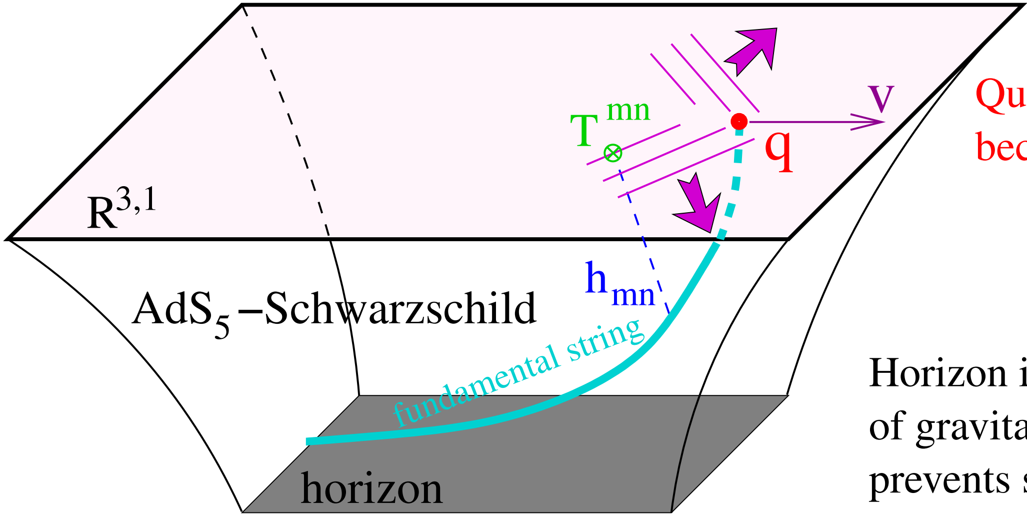
$$ds^2 = e^{2A(r)} \left(-h(r)dt^2 + d\vec{x}^2\right) + e^{2B(r)} \frac{dr^2}{h(r)} \qquad \phi = \phi(r)\,. \tag{8}$$

In a gauge where $\delta\phi = 0$, let's set $h_{11} = e^{-2A}\delta g_{11} = e^{-2A}\delta g_{22} = e^{-2A}\delta g_{33}$. Then

$$h_{11}'' = \left(-\frac{1}{3A'} - 4A' + 3B' - \frac{h'}{h}\right) h_{11}' + \left(-\frac{e^{-2A+2B}}{h^2}\omega^2 + \frac{h'}{6hA'} - \frac{h'B'}{h}\right) h_{11} \tag{9}$$

1.2. Drag force on heavy quarks

The results: [Herzog et al. 2006; Gubser 2006a]



Quark can't slow down because $m = \infty$

Horizon is "sticky" because of gravitational redshift: prevents string from moving.

$$\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}}{2} T_{SYM}^2 \frac{v}{\sqrt{1-v^2}} = -\frac{p}{\tau_Q} \qquad \tau_Q = \frac{2m_Q}{\pi T_{SYM}^2 \sqrt{\lambda}}$$

$\tau_{\text{charm}} \approx 2\,\text{fm}$ $\tau_{\text{bottom}} \approx 6\,\text{fm}$ if $T_{QCD} = 250\,\text{MeV}$

The method:

Consider a more general problem of embedding a string in a warped background [Herzog 2006; Gursoy et al. 2009b; Gubser and Yarom 2009]:

$$\begin{aligned} ds^2 &= -e^{2A(r)} h(r) dt^2 \\ &\quad + e^{2A(r)} d\vec{x}^2 + \frac{dr^2}{h(r)} \end{aligned} \qquad X^\mu(\tau, r) = \begin{pmatrix} \tau + \zeta(r) \\ v\tau + v\zeta(r) + \xi(r) \\ 0 \\ 0 \\ r \end{pmatrix}, \tag{10}$$

Using classical equations of motion and a gauge choice for ζ, find

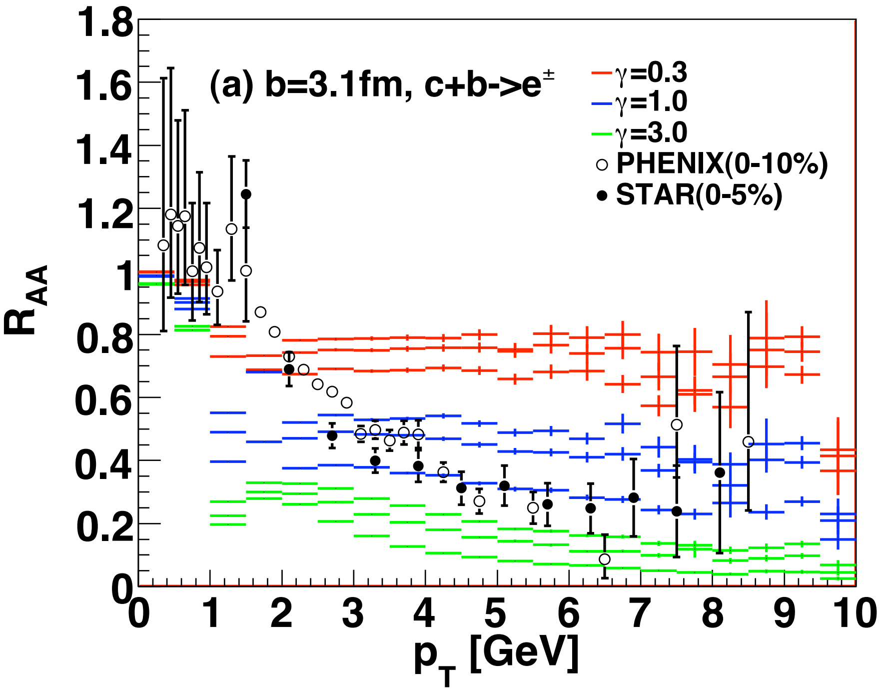
$$\xi'(r) = -\frac{\pi_\xi}{he^A} \sqrt{\frac{h - v^2}{he^{4A}/(2\pi\alpha')^2 - \pi_\xi^2}} \qquad \zeta'(r) = \frac{v\xi'}{h - v^2}, \tag{11}$$

where $\pi_\xi = \partial \mathcal{L}_{\text{string}} / \partial \xi'$. To make $\xi'(r)$ everywhere real, we must choose

$$\pi_\xi = -\frac{\sqrt{h(r_*)} e^{2A(r_*)}}{2\pi\alpha'} \qquad \text{where} \quad h(r_*) = v^2 \,. \tag{12}$$

F_{drag} can be argued to be precisely $(\pi_\xi, 0, 0)$.

A recent study shows that these equilibration times are at least roughly consistent with R_{AA} of non-photon electrons:



$$\vec{F}_{\rm drag} = -\gamma \frac{T^2}{m_Q} \vec{p}$$

$\gamma \approx 2$ based on AdS/CFT

Colored triples show different freezeout assumptions

Analysis should work for $p_T \gtrsim 3\,\text{GeV}$. [Akamatsu et al. 2008]

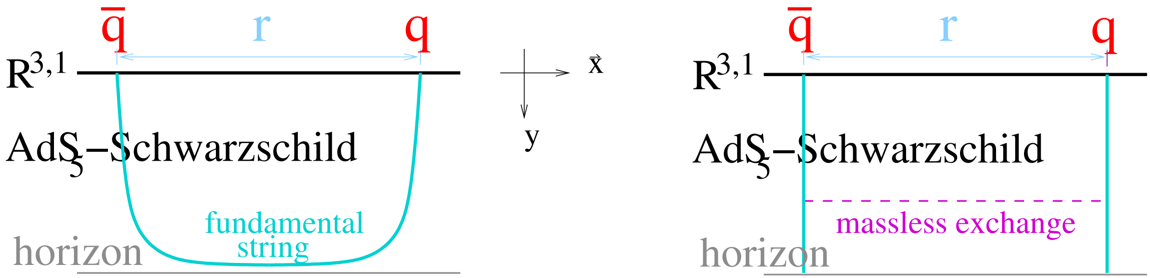
To get this $\gamma \approx 2$, have to match SYM and QCD at fixed energy density, and also set $\lambda \equiv g_{YM}^2 N = 5.5$ to approximately match the static q-$\bar{q}$ force calculated from the lattice [Gubser 2006c].

A bit more detail on why $g_{YM}^2 N \approx 5.5$ based on matching string theory to lattice q-$\bar{q}$ potential:

- Lattice people define an effective coupling:

$$\alpha_{q\bar{q}}(r,T) \equiv \frac{3}{4} r^2 \frac{\partial F_{q\bar{q}}}{\partial r} \,. \tag{13}$$

- Analogous quantity in string theory receives contributions from two configurations:

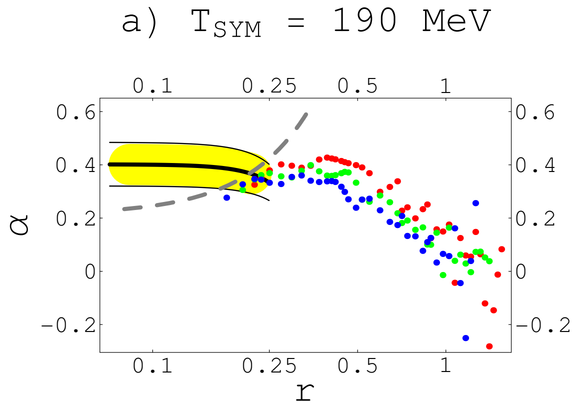


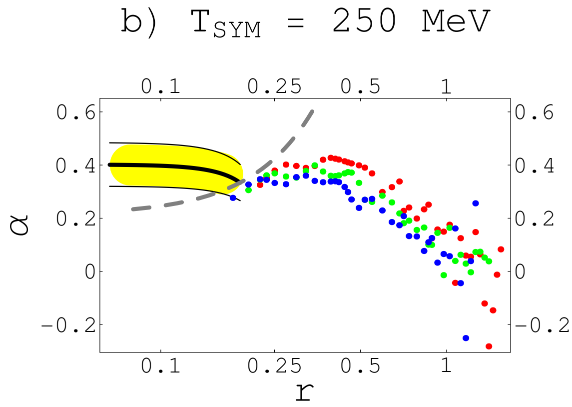
- Simplest approximation to U-curve contribution is zero temperature result:

$$\alpha_{\rm SYM}(T\!=\!0) \equiv \frac{3}{4} r^2 \frac{\partial V_{q\bar{q}}}{\partial r} = \sqrt{g_{YM}^2 N} \frac{3\pi^2}{\Gamma(1/4)^4} \,. \tag{14}$$

$T \neq 0$ results in a bit of Debye screening.

To fix $g_{YM}^2 N \approx 5.5$, compare to lattice at largest r where U-shape dominates.





lattice data from [Kaczmarek and Zantow 2005], $T \approx 250\,\mathrm{MeV}$.

- Overlap of lattice and SYM is a bit better when one compares at fixed *energy density* rather than fixed *temperature*.
- Makes sense: more matter, faster thermal screening.
- $\epsilon_{\mathrm{SYM}} = \epsilon_{\mathrm{QCD}}$ means $T_{\mathrm{SYM}} \approx T_{\mathrm{QCD}}/3^{1/4}$.
- Match between SYM and lattice here is conspicuously imperfect, but I wanted some comparison where leading-order result on SYM side involves $g_{YM}^2 N$.

As with equation of state, the approach is to fix key parameters using comparison with lattice; then use stringy methods to get real-time transport properties.

1.3. Stochastic forces and the Einstein relation

The heavy quark dynamics is described using Langevin:

$$\frac{d\vec{p}}{dt} = -\vec{F}_{\rm drag} + \vec{F}(t) \qquad \langle F_i(t) F_j(0) \rangle = D(p) \delta_{ij} \delta(t) \qquad \Gamma = \frac{D(p)}{2ET} - \frac{1}{2p} \frac{dD(p)}{dp}$$

Direct calculations of stochastic forces [Casalderrey-Solana and Teaney 2006; Gubser 2006b; Casalderrey-Solana and Teaney 2007; Giecold et al. 2009] show that

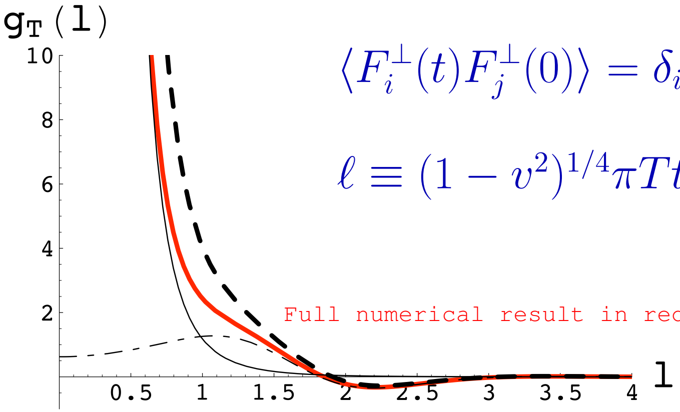
$$\begin{aligned} \langle F^{||}(t) F^{||}(0) \rangle &\approx \kappa_L \delta(t) \qquad & \langle F_i^\perp(t) F_j^\perp(0) \rangle &\approx \kappa_T \delta_{ij} \delta(t) \\ \kappa_L &= \frac{\pi\sqrt{\lambda}}{(1-v^2)^{5/4}} T^3 \delta(t) \qquad & \kappa_T &= \frac{\pi\sqrt{\lambda}}{(1-v^2)^{1/4}} T^3 \delta(t) \,. \end{aligned} \tag{15}$$

Compare to Einstein relation, derived by demanding that Langevin equilibrates to a Boltzmann distribution $p(\vec{k}) \propto e^{-E(\vec{k})/T}$:

$$\kappa_L = -\frac{2|\vec{F}_{\rm drag}|T}{v} = \frac{\pi\sqrt{\lambda}}{(1-v^2)^{1/2}} \tag{16}$$

Einstein relation works only when $v = 0$.

Another point of difficulty: the stochastic forces aren't really white noise. They have instead a scaling form:



$$\langle F_i^\perp(t) F_j^\perp(0) \rangle = \delta_{ij} \pi T^3 \frac{\sqrt{\lambda}}{(1-v^2)^{1/4}} g_T(\ell)$$

$$\ell \equiv (1-v^2)^{1/4} \pi T t, \quad \text{so} \quad t_{\text{correlation}} \to \infty \quad \text{as} \quad v \to 1$$

[Gubser 2006b]

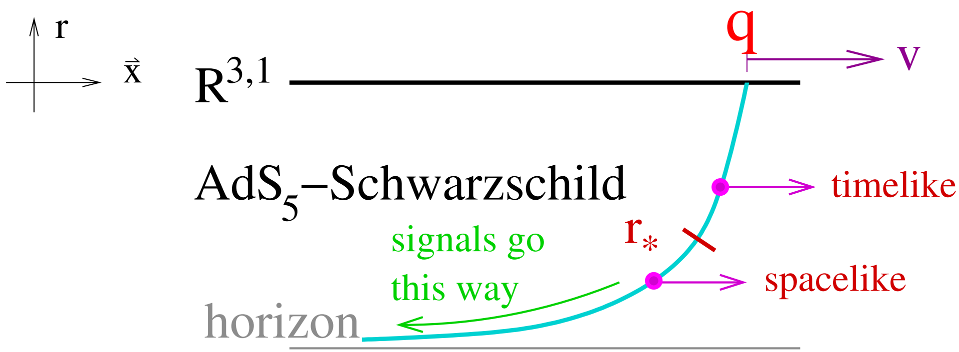
To use Langevin, we need $t_{\text{correlation}} \lesssim t_Q$, i.e.

$$\frac{1}{\sqrt{1-v^2}} \lesssim \frac{4}{\lambda} \frac{m_Q^2}{T^2} \qquad \Longrightarrow \qquad p_T^e \lesssim 20\,\text{GeV} \quad \text{for charm}$$

Obtaining the full scaling form of $\langle F(t) F(0) \rangle$ is involved, but let's at least look at the basic methods...

1.4. The worldsheet horizon

The key insight: $r = r_*$ is a horizon on the worldsheet.



Explicitly, one can show

$$ds^2_{WS} = \gamma_{ab} d\sigma^a d\sigma^b = -e^{2A}(h - v^2) d\tau^2 + \left(\frac{1}{h} + \frac{e^{2A} h \xi'^2}{h - v^2} \right) dr^2$$

$$T_{WS} = \frac{e^{A_*}\sqrt{h'_*}}{4\pi} \left(h'_* + 4v^2 A'_* \right)^{1/2} = T(1 - v^2)^{1/4} \qquad \text{for } AdS_5\text{-Schwarzschild}, \tag{17}$$

where $A_* = A(r_*)$ etc.

Note that τ and t coincide on the boundary, because we can set $\zeta(r) = 0$ there.

$\langle F(t)F(0)\rangle$ is a symmetrized Wightman two-point function based on fluctuations of the string around the trailing string ansatz:

$$\mathcal{L}_{\text{string}} = \text{(trailing string)} + \frac{K_L(r)}{2}(\partial_a \delta x^1)^2 - \sum_{i=2,3} \frac{K_T(r)}{2}(\partial_a \delta x^i)^2 + \mathcal{O}(\delta x^3)$$

$$K_L(r) = -\frac{e^{2A}}{2\pi\alpha'}\frac{\sqrt{h_*}}{h\xi'} \qquad K_T(r) = \frac{e^{6A-2A_*}}{2\pi\alpha'}\frac{h}{\sqrt{h_*}}\xi' \,. \tag{18}$$

Standard AdS/CFT methods give retarded correlator $G^{\text{ret}}(\omega)$, with infalling boundary conditions at the *worldsheet* horizon:

$$\delta x \sim (r - r_*)^{-i\omega/4\pi T_{WS}} \,. \tag{19}$$

To get the Wightman 2-pt function $G(\omega)$, need a funny version of fluctuation dissipation relation:

$$G(\omega) = -\coth\left(\frac{\omega}{2T_{WS}}\right) \operatorname{Im} G^{\text{ret}}(\omega) \tag{20}$$

Now one can easily show that [Hoyos-Badajoz 2009; Gubser and Yarom 2009]

$$\kappa_T = -\frac{2F_{\text{drag}}T_{WS}}{v} \qquad \kappa_L = \kappa_T \frac{\partial \log |F_{\text{drag}}|}{\partial \log v} \,. \tag{21}$$

2. The very cold: superconductors and superfluids

2.1. The basics of superconducting black holes

In the spirit of [Weinberg 1986], I equate "superconducts" to "spontaneously breaks a $U(1)$ gauge symmetry."

If $m_{\rm eff}^2$ for a complex scalar ψ is negative enough, we'll get $\langle\psi\rangle \neq 0$, breaking the $U(1)$ of its phase.

The setup we'll consider is [Gubser 2008; Hartnoll et al. 2008]

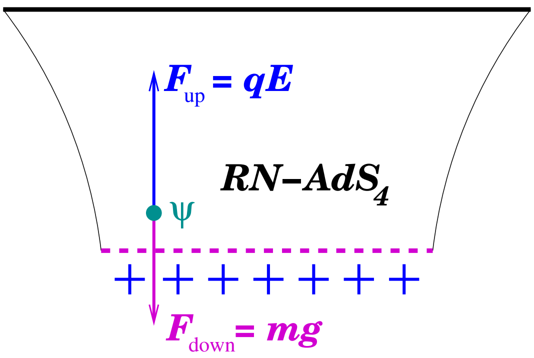
$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - V(|\psi|)\right] . \qquad (22)$$

If we assume $A_{(1)} = \Phi dt$ and look at $|\psi|^2$ terms, we see that

$$m_{\rm eff}^2 = m^2 + q^2\Phi^2 g^{tt} \qquad \text{where} \qquad m^2 \equiv \frac{1}{2}V''(0) . \qquad (23)$$

Since $g^{tt} < 0$, we can make $m_{\rm eff}^2$ very negative with very big q. $\Phi \to 0$ at horizon in order for Φdt to be well-behaved, so $m_{\rm eff}^2 \to m^2$ at horizon.

Below some temperature, quanta of ψ are driven upward from horizon: recall $T = g/2\pi$.

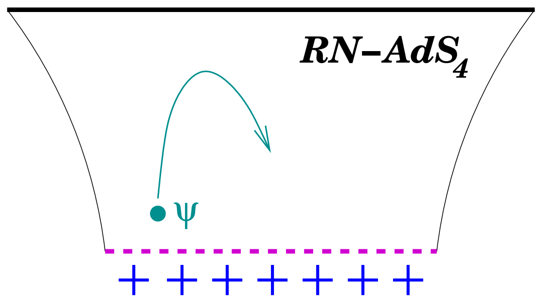


Condensate spontaneously breaks $U(1)$ gauge symmetry, so this is a superconductor: s-wave since ψ is a scalar.

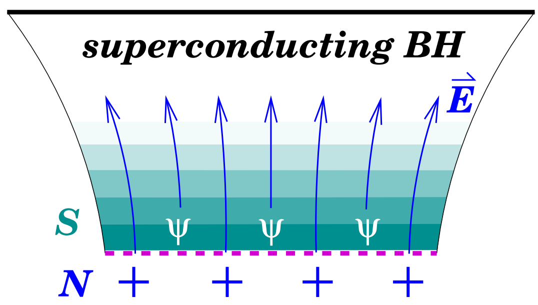
Some fraction of charge remains behind the horizon.

But what is the ground state configuration? No black hole horizon?

ψ quanta can never escape from AdS_4, so they fall back toward horizon.



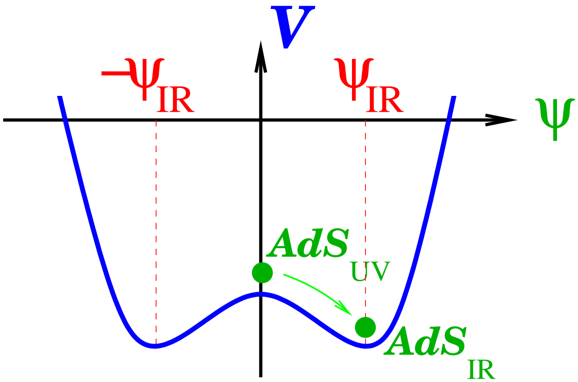
Expected end state has an "atmosphere" of ψ quanta condensed above the horizon.

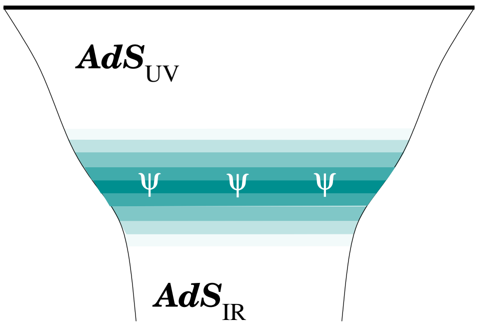


2.2. A candidate ground state

A ground state was suggested [Gubser and Rocha 2009] in AdS_4 for

$$V(|\psi|) = -\frac{6}{L^2} + m^2|\psi|^2 + \frac{u}{2}|\psi|^4 \qquad m^2 < 0, \quad u > 0 \tag{24}$$





- A domain wall between $AdS_{\rm UV}$ and $AdS_{\rm IR}$ involving only scalars is a *holographic RG flow*, and describes dynamics of $\mathcal{L}_{\rm CFT} + m_{\rm soft}^{4-\Delta_\psi} \mathcal{O}_\psi$.

- Here I do *not* deform by $\mathcal{O}_\psi$. A scale is set by $U(1)$ charge density ρ in CFT. One finds a *different* domain wall from $AdS_{\rm UV}$ to $AdS_{\rm IR}$.

- $F_{\mu\nu} \to 0$ in $AdS_{\rm IR}$. *All* the charge is carried by the domain wall.

Ansatz for charged domain wall:

$$\begin{aligned} ds^2 &= e^{2A}(-h dt^2 + dx^2 + dy^2) + \frac{dr^2}{h} \\ A_{(1)} &= \Phi(r) dt \qquad\qquad \psi = \psi(r) \end{aligned} \tag{25}$$

Full equations of motion:

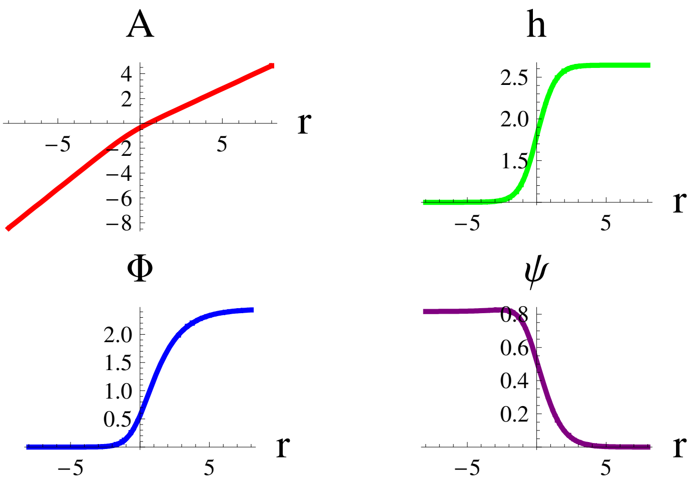
$$\begin{aligned} A'' &= -\frac{1}{2}\psi'^2 - \frac{q^2}{2h^2 e^{2A}}\Phi^2\psi^2 \leq 0 \\ h'' + 3A'h' &= e^{-2A}\Phi'^2 + \frac{2q^2}{he^{2A}}\Phi^2\psi^2 \geq 0 \\ \Phi'' + A'\Phi' &= \frac{2q^2}{h}\Phi\psi^2 \\ \psi'' + \left(3A' + \frac{h'}{h}\right)\psi' &= \frac{1}{2h}V'(\psi) - \frac{q^2}{h^2 e^{2A}}\Phi^2\psi\,, \end{aligned} \tag{26}$$

- "c-theorem:" $A'_{\rm IR} > A'_{\rm UV}$. Radius of $AdS_{\rm IR}$ is *smaller*. As in [Girardello et al. 1998; Distler and Zamora 1999; Freedman et al. 1999].
- "h-theorem:" $h_{\rm IR} < h_{\rm UV}$. Light travels slower in IR as measured by dx/dt.

Non-zero Φ means there is some finite density $\langle J_0 \rangle = \rho$ of a dual charge density.

We prescribe $\psi \sim e^{-\Delta_\psi r}$, dual to some VEV $\langle \mathcal{O}_\psi \rangle$, with *no deformation* of $\mathcal{L}_{\rm CFT}$.

Recovering AdS_4 in the IR (constant ψ, constant h, linear A) means you have *emergent conformal symmetry* in the IR.



- $r \to +\infty$ is the UV, $r \to -\infty$ is the IR.
- Here we chose $L = 1$, $q = 2$, $m^2 = -2$, $u = 3$.
- This solution is essentially unique: related solutions have ψ with nodes.

Null trajectories at constant r have $v(r) \equiv |d\vec{x}/dt| = \sqrt{h(r)}$.

"Index of refraction" $n = v_{\rm UV}/v_{\rm IR} \approx 1.63$ for this setup.

You can also recover Lorentz symmetry but *not* conformal symmetry in IR if $V(|\psi|)$ has no extrema away from $\psi = 0$ [Gubser and Nellore 2009a].

2.3. Embedding in string theory

Focus on AdS_5 embeddings [Gubser et al. 2009ab]. For AdS_4, see also [Gauntlett et al. 2009a; Denef and Hartnoll 2009; Gauntlett et al. 2009b].

$\mathcal{N} = 4$ SYM has $SO(6)$ R-symmetry. Let's pick out a $U(1) \subset SO(6)$ by studying states with

$$\langle J_{12} \rangle = \langle J_{34} \rangle = \langle J_{56} \rangle = \frac{\rho}{\sqrt{3}} \,. \tag{27}$$

The AdS_5 dual is the near-horizon limit of spinning D3-branes. The $d = 5$ description is the Reissner-Nordstrom black hole:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2\kappa^2} \left[R - \frac{1}{4} F_{\mu\nu}^2 + \frac{12}{L^2} + (FFA \text{ Chern-Simons}) \right] \\ ds_5^2 &= e^{2A}(-h dt^2 + d\vec{x}^2) + \frac{dr^2}{h} \qquad A_{(1)} = \Phi dt \\ A &= \frac{r}{L} \qquad h = 1 - \frac{2\epsilon L \kappa^2}{3} e^{-4r/L} + \frac{\rho^2 \kappa^4}{3} e^{-6r/L} \\ \Phi &= \rho\kappa^2 (e^{-2r_H/L} - e^{-2r/L}) \end{aligned} \tag{28}$$

Easily calculate $T = \frac{1}{4\pi} e^{A(r_H)} h'(r_H) \qquad \mu = \lim_{r \to \infty} \Phi(r)$.

5-dimensional perspective:

- $\mathbf{20}$, $\mathbf{10_C}$, and $\mathbf{1_C}$ parametrize $E_{6(6)}/USp(8)$ of $d=5$, $\mathcal{N}=8$ SUGRA [Gunaydin et al. 1986]. Uplift to 10-d only partially known.
- Explicit non-linear action and uplift for just the $\mathbf{20}$ plus $SO(6)$ gauge fields is known [Cvetic et al. 2000].
- The $U(1)$ we've selected, plus the highest-charge member of $\mathbf{10_C}$, plus metric are (almost) all the fields in the $SU(3)$-invariant bosonic sector of $d=5$, $\mathcal{N}=8$:

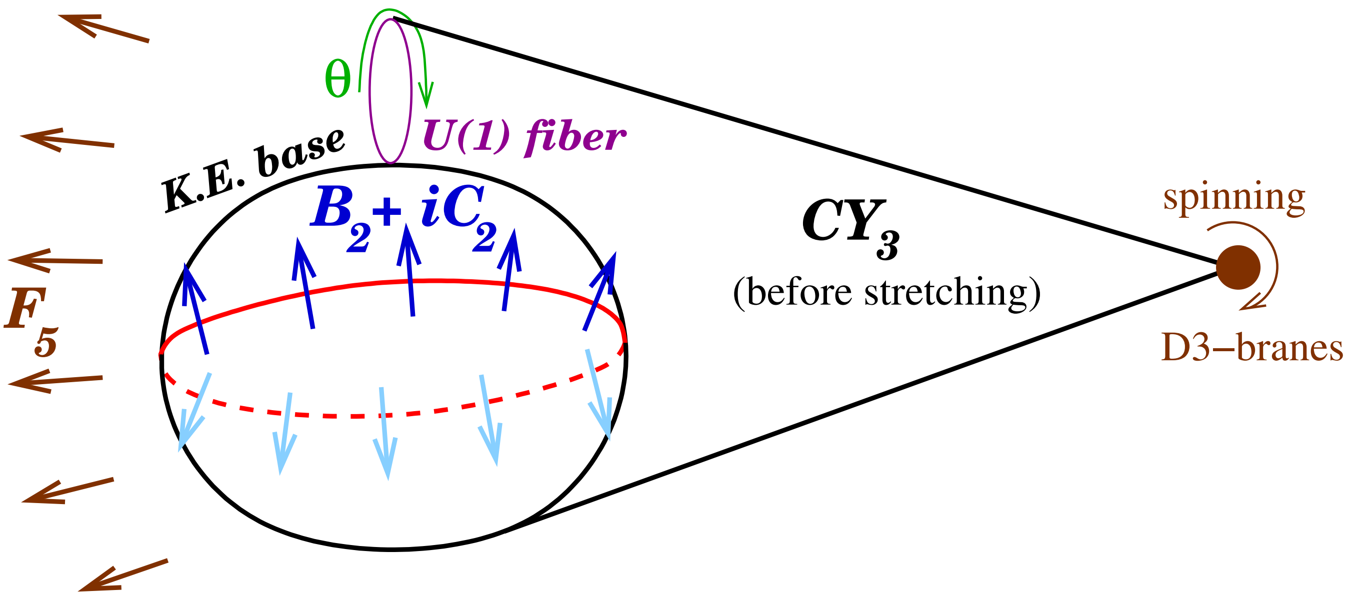
$$\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\left[(\partial_\mu\eta)^2 + \sinh^2\eta\left(\partial_\mu\theta - \frac{\sqrt{3}}{L}A_\mu\right)^2\right] + \frac{3}{L^2}\cosh^2\frac{\eta}{2}(5-\cosh\eta)\,, \tag{29}$$

$\frac{SL(2,\mathbf{R})}{U(1)}$ NLσM

- The non-SUSY vacuum at $\eta=\log(2+\sqrt{3})$ is unstable toward breaking $SU(3)$ [Distler and Zamora 2000], but more sophisticated examples are probably stable.
- A more ornate setup probably flows from $\mathcal{N}=4$ to $\mathcal{N}=1$ superconformal vacuum of [Khavaev et al. 2000], and may be stable.

10-dimensional perspective:

- It helps to view S^5 as a $U(1)$ fibration over $\mathbf{CP}^2$. All results I'll show generalize to SE_5's obtained by replacing $\mathbf{CP}^2$ by a different Einstein-Kahler 2-fold.



- Main trick is to establish some explicit uplift of a sub-theory of $d = 5$, $\mathcal{N} = 8$ SUGRA to type IIB.

To uplift any solution $(ds_M^2, A_{(1)})$ to $\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}^2 + \frac{12}{L^2} + \text{C.S.}$, use [Cvetic et al. 1999 2000]

$$\begin{aligned} & ds_{10}^2 = ds_M^2 + L^2 \sum_{i=1}^{3} |Dz_i|^2 \qquad \sum_{i=1}^{3} |z_i|^2 = 1 \qquad Dz_i \equiv dz_i + \frac{i}{L} A_{(1)} z_i \\ & F_{(5)} = \mathcal{F}_{(5)} + *\mathcal{F}_{(5)} \qquad \mathcal{F}_{(5)} = -\frac{4}{L}\,\text{vol}_M + L^2 (*_M F_{(2)}) \wedge \omega_{(2)}\,, \end{aligned} \tag{30}$$

where $\omega_{(2)}$ is the Kahler form on $\mathbf{CP}^2$.

Now generalize to capture superconducting solutions [Gubser et al. 2009a]: basically, find AdS_5-to-AdS_5 domain walls [Gubser et al. 2009b] similar to quartic example of [Gubser and Rocha 2009].

$SU(3)$ symmetry means we can't squash the $\mathbf{CP}^2$; only stretch the $U(1)$ fiber:

$$ds_5^2 = L^2 \left(ds_{\mathbf{CP}^2}^2 + \cosh^2 \frac{\eta}{2} \zeta_{(1)}^2 \right) \qquad \zeta_{(1)} = \frac{i}{2} \sum_{i=1}^{3} (z_i d\bar{z}_i - \bar{z}_i dz^i) \tag{31}$$

Including spin: $dz_i \to Dz_i \quad \Longrightarrow \quad \zeta_{(1)} \to \zeta_{(1)}^A \equiv \zeta_{(1)} + \frac{1}{L} A_{(1)}$.

The complex scalar $(\eta, \theta) \in \mathbf{10}_{\mathbf{C}}$ describes deformations sourced by $F_{(2)} \equiv B_{(2)} + iC_{(2)}$. A tricky point: How do we choose $F_{(2)}$?

- Consider the CY_3 cone over our SE_5: $ds^2_{CY_3} = dr^2 + r^2 ds^2_{SE_5}$.
- Normalize holomorphic three-form $\Omega_{(3)}$ so that $\Omega_{(3)} \wedge \Omega^*_{(3)} = 8\,\text{vol}_{CY_3}$. $\Omega_{(3)} = dz^1 \wedge dz^2 \wedge dz^3$ when $CY_3 = \mathbf{C}^3$.
- Decompose $\Omega_{(3)} = r^2 dr \wedge \Omega_{(2)} +$ (3-form on base)
- $\boxed{F_{(2)} = iL^2 e^{i\theta} \tanh\frac{\eta}{2}\,\Omega_{(2)}}$

(Related heavy lifting: [Corrado et al. 2002; Pilch and Warner 2001 2002]; also [Romans 1985])

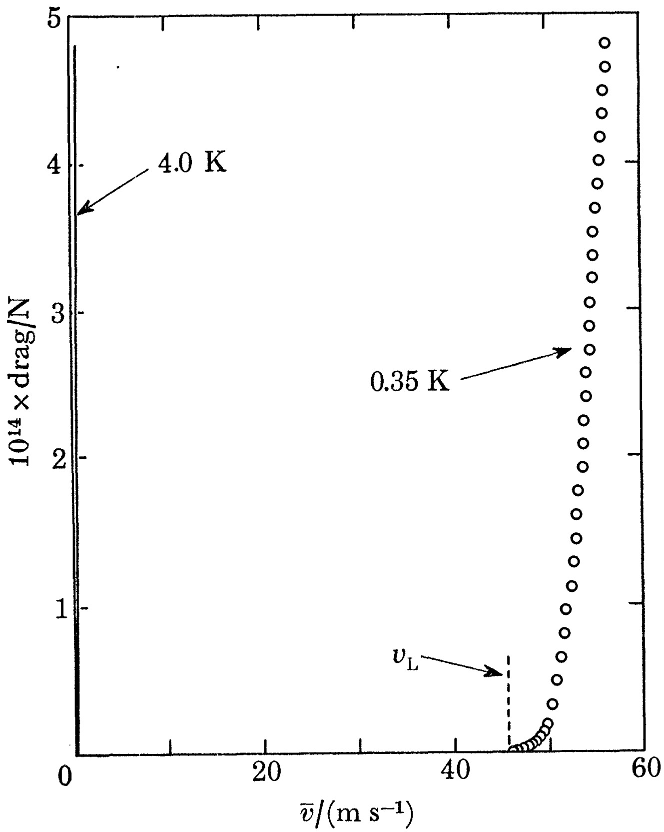
After some further thought, find

$$\begin{aligned} ds^2_{(10)} &= \cosh\frac{\eta}{2} ds^2_M + \frac{L^2}{\cosh\frac{\eta}{2}} ds^2_5 \\ \mathcal{F}_{(5)} &= \cosh^2\frac{\eta}{2}\,\frac{\cosh\eta - 5}{L}\,\text{vol}_M + L^2 (*_M F_{(2)}) \wedge \omega_{(2)} \\ &\quad + L^4 \tanh^2\frac{\eta}{2}\left(d\theta - \frac{3}{L}A_{(1)}\right) \wedge \omega_{(2)} \wedge \omega_{(2)} \end{aligned} \tag{32}$$

2.4. A critical velocity

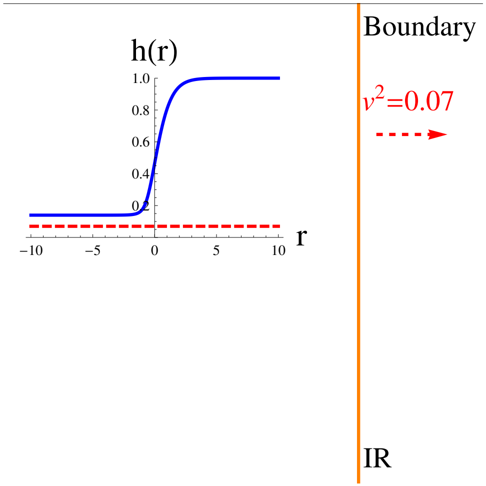
A familiar probe [Allum et al. 1977; Raman et al. 1999] of superfluids is a point particle (e.g. a non-relativistic heavy ion) pulled through it at constant velocity.

- v_L is Landau velocity, above which massless probe can emit rotons: the excitations with minimal ω/k.
- Scaling form of F_{drag} above v_L depends on the roton emission process.

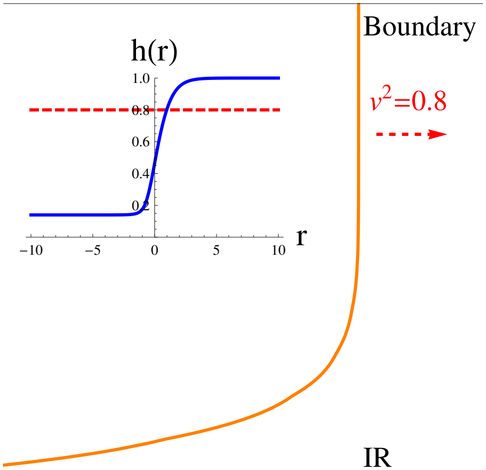


We've got a nice example of a strongly coupled superfluid, and we can trail a string through it [Gubser and Yarom 2009]... so what happens?

For $v < v_{\rm IR} \approx 0.373$, string hangs straight down: NO DRAG.



For $v > v_{\rm IR} \equiv \sqrt{h_{\rm IR}}$, get trailing string.



As before, worldsheet horizon is located by solving $h(r_*) = v^2$. If $v < v_{\rm IR}$, there are no solutions!

Calculating drag, worldsheet temperature, and stochastic forces is complicated slightly by having to pass from 5-d Einstein frame to 10-d string frame: lagrangian is

$$\mathcal{L}_{\rm string} = -\frac{1}{2\pi\alpha'} Q(\eta) \sqrt{-\det \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}} \qquad Q(\eta) = \cosh \frac{\eta}{2} \,. \qquad (33)$$

Two last analytic results:

1. Starting from differential first law,

$$dP = s\,dT + \rho_n\,d\mu - \frac{\rho_s}{2\mu} d\xi^2\,, \tag{34}$$

where ρ_n and ρ_s are normal and superfluid densities, and $\xi_m = \partial_m \varphi$ is proportional to superfluid velocity, one can extract

$$v_{\rm IR}^2 = \lim_{T\to 0} \frac{sT}{sT + \mu\rho_n}\,. \tag{35}$$

2. Using IR asymptotics of the background, one can demonstrate that

$$F_{\rm drag} \propto -(v - v_{\rm IR})^{1/(\Delta_\Phi - 4)} \tag{36}$$

where the exponent Δ_Φ is the dimension of J_0 in the IR AdS_5 region. Also find $\mathrm{Re}\,\sigma(\omega) \propto \omega^{2\Delta_\Phi - 5}$ for small ω. For explicit type IIB example of [Gubser et al. 2009ab], $\Delta_\Phi = 5$.

3. Conclusions

- Heavy-ion application has some striking experimental support. The combination of $F_{\rm drag}$, $S/S_{\rm free}$, and η/s gives a pretty encouraging picture [Noronha et al. 2009].
  - Bulk viscosity estimates have also seen some phenomenological application [Song and Heinz 2009].
  - Failure of Einstein relation suggests that we still have an imperfect understanding of how to treat thermalization via trailing string.
- Condensed matter applications seem to me less closely tied to experiment, but the string theory constructions are rich and interesting.
  - AdS-to-AdS domain walls look like a pretty general construction at finite chemical potential, but other behaviors may be possible [Gubser and Nellore 2009b].
  - Trailing string at $v > v_{\rm IR}$ has $T_{WS} > 0$ even though $T = 0$ for the background.

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