Hydrodynamics of Holographic Superconductors

Outline

- Review of the Model
- Hydrodynamics
- Holographic Hydro by Quasinormal Modes
- Summary and Outlook

The Model

Can we realize spontaneous symmetry breaking as function of temperature in AdS/CFT?

Hartnoll, Herzog, Horowitz: YES, we can! [arXiv: 0803.3295]

based on Gubser [arXiv:0801.2977]

Abelian Higgs model in AdS-Blackhole background

decoupling limit charge q -> Infty

\[ ds^2 = -\left(\frac{r^2}{L^2} - \frac{M}{r}\right)dt^2 + \frac{dr^2}{\frac{r^2}{L^2} - \frac{M}{r}} + \frac{r^2}{L^2}(dx^2 + dy^2) \]

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - m^2 \Psi \bar{\Psi} - (\partial_{\mu} \Psi - i A_{\mu} \Psi)(\partial^{\mu} \bar{\Psi} + i A^{\mu} \bar{\Psi}) \]
The Model

eoms: \( \Psi'' + \left( \frac{f'}{f} + \frac{2}{\rho} \right) \Psi' + \frac{\Phi^2}{f^2} \Psi + \frac{2}{L^2 f} \Psi = 0 \)

\( \Phi'' + \frac{2}{\rho} \Phi' - \frac{2\Psi^2}{f} \Phi = 0 \)

boundary conditions at Horizon: \( \Phi(\rho_H) = 0 \) , \( \Psi(\rho_H) \)

why? bulk current \( J_\mu = \psi^2 A_\mu \) finite norm at the Horizon

values at boundary

\( \Phi = \bar{\mu} - \frac{\bar{n}}{\rho} + O\left(\frac{1}{\rho^2}\right) \)

\( \Psi = \frac{\psi_1}{\rho} + \frac{\psi_2}{\rho^2} + O\left(\frac{1}{\rho^2}\right) \)

\( \bar{\mu} = \frac{3L}{4\pi T^4} \mu, \)

\( \bar{n} = \frac{9L}{16\pi^2 T^2} n, \)

\( \psi_1 = \frac{3}{4\pi TL^2} \langle O_1 \rangle, \)

\( \psi_2 = \frac{9}{16\pi^2 T^2 L^4} \langle O_2 \rangle, \)
The Model

solve eom with either $\psi_2 = 0$ or $\psi_1 = 0$

$$\frac{\langle O_2 \rangle^2}{T_c^4}$$

$$\frac{\langle O_1 \rangle^2}{T_c^2}$$

$$\langle O_i \rangle^2 \propto \left(1 - \frac{T}{T_c}\right)$$
Hydrodynamics

- Hydrodynamics = slow modes \[ \lim_{k \to 0} \omega(k) = 0 \]
- Conservation law \[ \frac{\partial n}{\partial t} + \vec{\nabla} \vec{j} = 0 \]
- Constitutive relation with external source \[ \vec{j} = -D \vec{\nabla} n + \sigma \vec{E} \]
- Taking time derivative and using the continuity eqn

\[ \langle j_L \rangle = \frac{i\sigma \omega^2}{\omega + iD \vec{k}^2} A_L \]
\[ \sigma = -i \frac{\langle j_L j_L \rangle}{\omega} \]
Hydrodynamics

broken phase: take Goldstone mode into account (Chaikin, Lubensky)

generic prediction: appearance of sound modes

predicts correlator

\[ \langle j_L j_L \rangle = \frac{\hat{\sigma} \omega^2}{\omega^2 - \hat{D} \vec{k}^2} \]

and conductivity

\[ \sigma(\omega) = \frac{-i}{\omega + i\epsilon} \hat{\sigma} = -i P \left( \frac{1}{\omega} \right) + \pi \hat{\sigma} \delta(\omega) \]

\[ \hat{\sigma} = n_s \]

including dissipation:

\[ \omega = \pm v_s k - i \Gamma_s k^2 \]
Hydro and QNMs

- poles of retarded Green functions = Quasinormal Modes
- “Eigenmodes”
  - Horizon: infalling \( \Psi_H = (\rho - 1)^{-i\omega/3}(1 + O(\rho - 1)) \)
  - Boundary: Pole of holographic GF
  - complex scalar field \( \Psi_B = \frac{A}{\rho} + \frac{B}{\rho^2} + O \left( \frac{1}{\rho^3} \right) \)
  - theory I \( \langle O_1 \bar{O}_1 \rangle = \frac{A}{B} \) theory II \( \langle O_2 \bar{O}_2 \rangle = \frac{B}{A} \)
  - complex frequencies \( \Psi \propto e^{-i\omega R t} e^{-\omega I t} \)
Hydro and QNMs

Unbroken phase: superconducting Instability

Vector channel: Diffusion mode \( D = \frac{3}{4\pi T} \)
Hydro and QNMs

Broken phase: Second sound and Pseudodiffusion

\[ 0 = f \eta'' + \left( f' + \frac{2f}{\rho} \right) \eta' + \left( \frac{\phi^2}{f} + \frac{2}{L^2} + \frac{\omega^2}{f} - \frac{k^2}{\rho^2} \right) \eta - \frac{2i\omega\phi}{f} \sigma - \frac{i\omega\psi}{f} a_t - \frac{ik\psi}{r^2} a_x, \]

\[ 0 = f \sigma'' + \left( f' + \frac{2f}{\rho} \right) \sigma' + \left( \frac{\phi^2}{f} + \frac{2}{L^2} + \frac{\omega^2}{f} - \frac{k^2}{\rho^2} \right) \sigma + \frac{2\phi\psi}{f} a_t + \frac{2i\omega\phi}{f} \eta, \]

\[ 0 = f a_t'' + \frac{2f}{\rho} a_t' - \left( \frac{k^2}{\rho^2} + 2\psi^2 \right) a_t - \frac{\omega k}{\rho^2} a_x - 2i\omega\psi \eta - 4\phi \sigma, \]

\[ 0 = f a_x'' + f' a_x' + \left( \frac{\omega^2}{f} - 2\psi^2 \right) a_x + \frac{\omega k}{f} a_t + 2ik\psi \eta. \]

\[ \text{constraint:} \quad \frac{\omega}{f} a_t' + \frac{k}{\rho^2} a_x' = 2i (\psi' \eta - \psi \eta'). \]

\[ \text{local ward identity:} \quad \partial \mu \langle j^\mu \rangle = 2 \langle O_i \rangle \eta_0^i. \]
Hydro and QNMs

How to compute QNMs of coupled system

four l.i. solutions (one is pure gauge)

$$\eta^{IV} = i\lambda \psi, \quad \sigma^{IV} = 0, \quad a_t^{IV} = \lambda \omega, \quad a_x^{IV} = -\lambda k.$$ 

rescale scalar fields

$$\tilde{\eta}(\rho) = \rho \eta(\rho), \quad \tilde{\sigma}(\rho) = \rho \sigma(\rho)$$

general solution is now

$$\varphi_i = \alpha_1 \varphi_i^I + \alpha_2 \varphi_i^{II} + \alpha_3 \varphi_i^{III} + \alpha_4 \varphi_i^{IV}$$

QNMs = no-source term $\rightarrow$ zero determinant

\[
0 = \begin{vmatrix}
\varphi_{\eta}^I & \varphi_{\eta}^{II} & \varphi_{\eta}^{III} & \varphi_{\eta}^{IV} \\
\varphi_{\sigma}^I & \varphi_{\sigma}^{II} & \varphi_{\sigma}^{III} & \varphi_{\sigma}^{IV} \\
\varphi_t^I & \varphi_t^{II} & \varphi_t^{III} & \varphi_t^{IV} \\
\varphi_x^I & \varphi_x^{II} & \varphi_x^{III} & \varphi_x^{IV} \\
\end{vmatrix}_{\rho = \Lambda}
\]
Hydro and QNMs

Dispersion relation: \( \omega = v_s k - i\Gamma_s k^2 \)
Hydro and QNMs

Pseudo Diffusion

\[ \omega = -iDk^2 - i\gamma \]
Hydro and QNMs

Higher Quasinormal modes

![Graph showing real and imaginary parts of omega versus omega](image_url)
Summary and Outlook

Relevant modes of the phase transition

- unbroken phase: 1 Diffusion mode
- critical point: 2 massless scalar modes + Diffusion
- broken phase: 2 modes of sound, Pseudo Diffusion, dynamical scaling $z=2$

Outlook:

- study hydro QNMs in the backreacted model
- (much) more complicated 11 coupled diff eqns
- two different mechanism of spontaneous symmetry breaking? (2 different QNMs cross the real axes for large and small charges)
- include fermionic operator