Insightful D-branes

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Outline

I. Introduction

II. A singularity with a gauge theory dual

III. Gauge theory vs. spacetime coordinate transformations

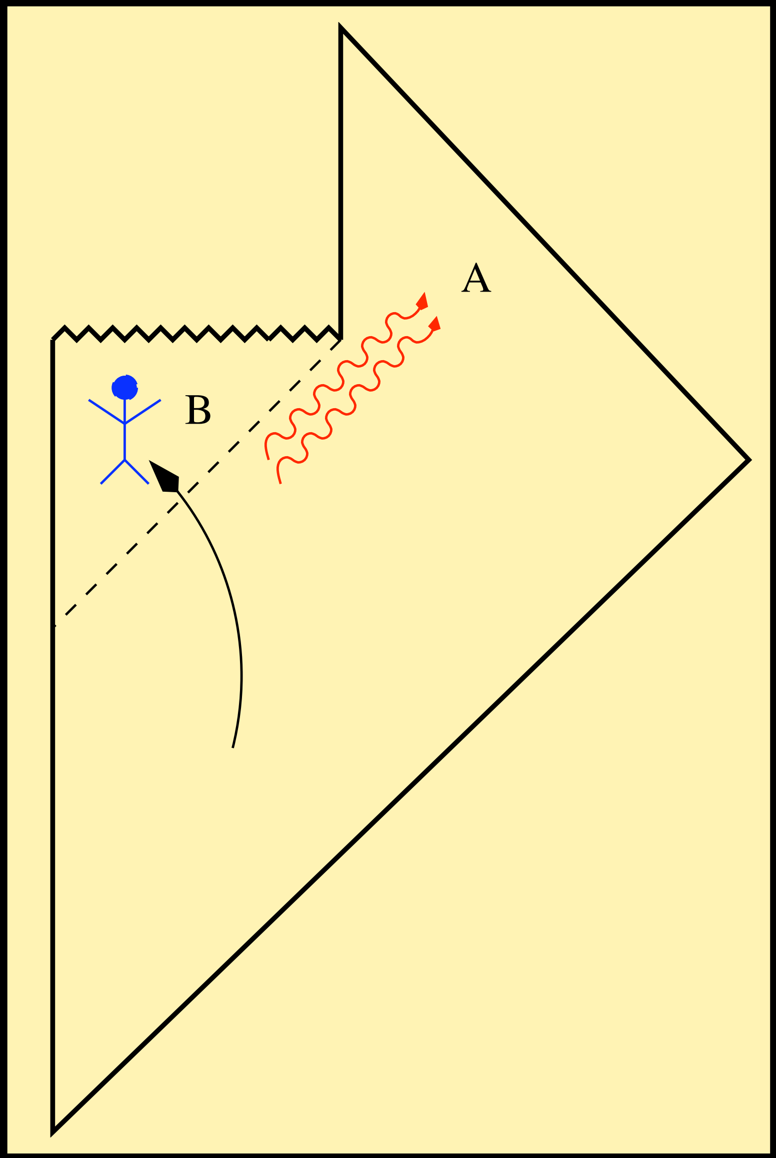
IV. Gauge theory dynamics and hyperbolic black holes

V. Conclusions

Based on work with G. Horowitz and E. Silverstein

arxiv:0904.3922

I. Introduction



Large black hole: curvature remains weak well inside the horizon.

1. Infalling observer (B) remains semiclassical until it reaches the singularity.

2. External observer (A) sees black hole evaporate via long-wavelength, thermal, Hawking radiation. Infalling observer is "cooked" near the horizon and re-emitted as Hawking radiation.

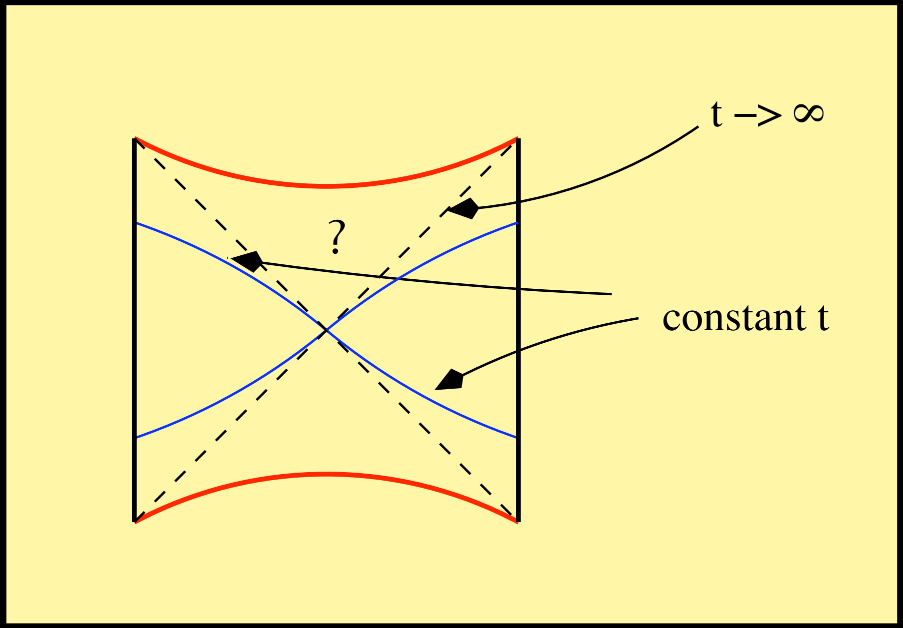
"Black hole complementarity":

't Hooft, Susskind

Unitarity of BH evaporation implies that these two pictures are equivalent (dual).

If so, what is the map?

Black holes in AdS/CFT



AdS_5 black hole $\sim$ 4d gauge theory at temperature $T = T(M)$

Gauge theory time $\sim$ Schwarzschild time

(Time experienced by observer at fixed distance from BH).

Bulk: infalling objects approach horizon as $t \longrightarrow \infty$

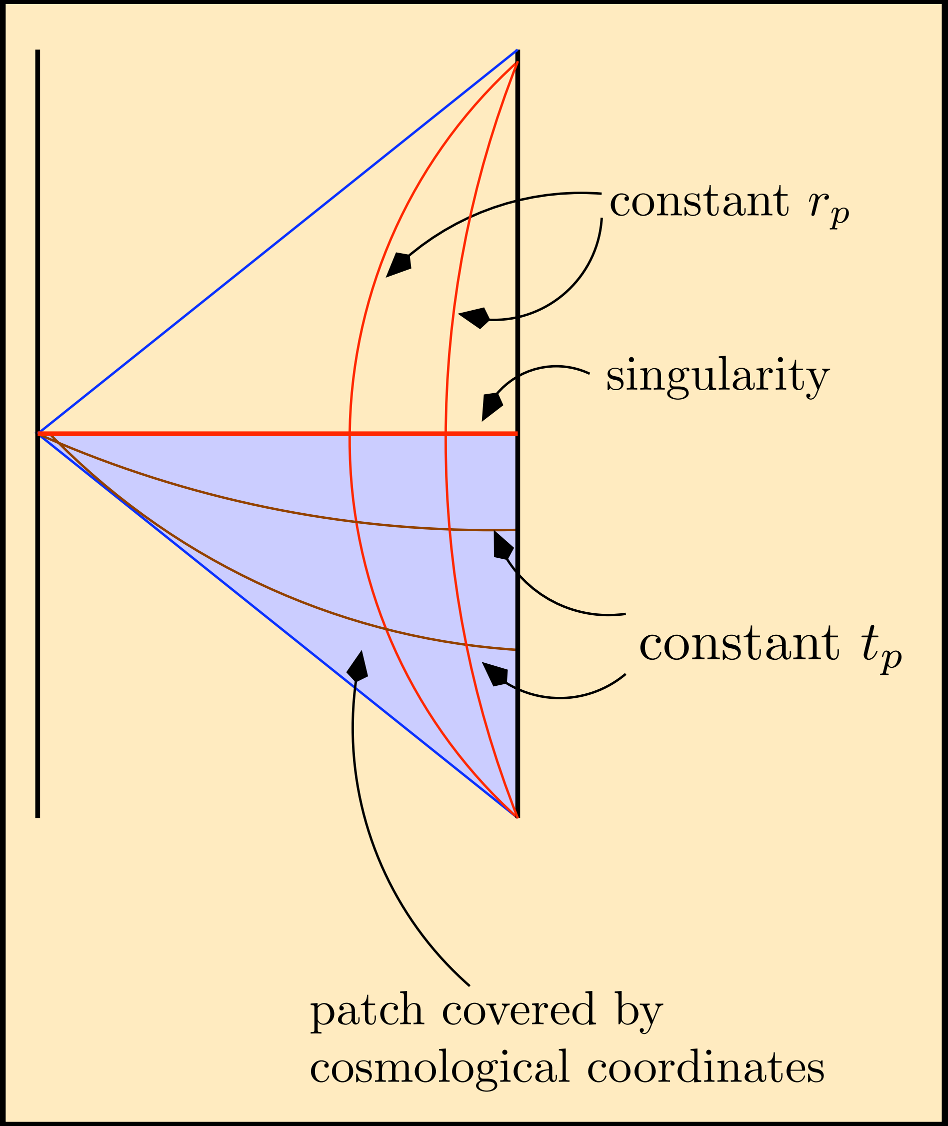
Gauge theory: excitations spread and thermalize

Gauge theory description of semiclassical physics behind the horizon?

Gauge theory description of singularity?

II. A singularity with a gauge theory dual

Green, Lawrence, McGreevy, Morrison, and Silverstein
HLS



Poincare coordinates:

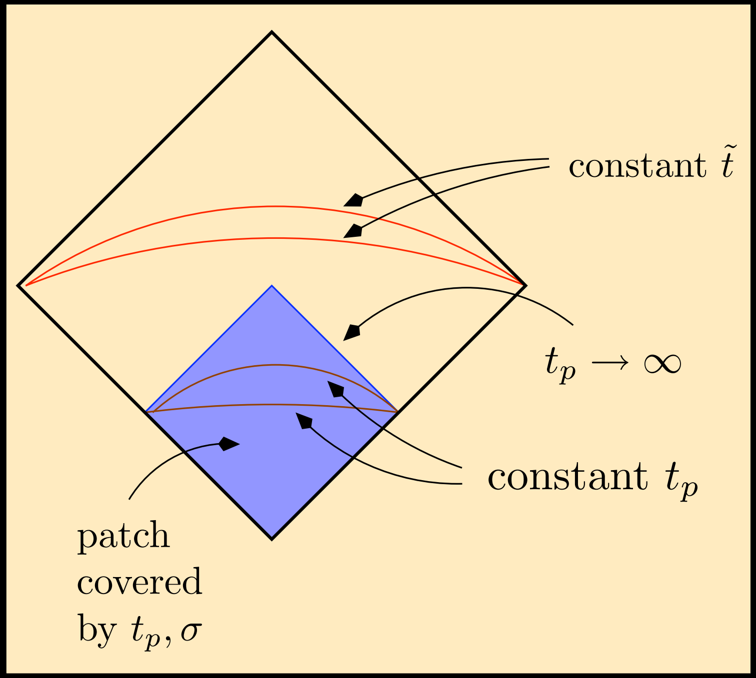
$$ds_5^2 = \frac{r_p^2}{\ell^2}\left(-d\tilde{t}^2 + d\vec{x}_3^2\right) + \ell^2 \frac{dr_p^2}{r_p^2}$$ (ℓ is AdS radius)

Patch of $\mathbb{R}^{3,1}$:

$$ds_4^2 = -dt_p^2 + t_p^2 d\sigma_{\mathbb{H}_3}^2$$

Orbifold: $\Sigma = \mathbb{H}_3/\Gamma$

$t_p \to 0^-$ becomes singular



Final bulk metric:

$$ds_5^2 = \frac{r_p^2}{\ell^2}\left(-dt_p^2 + t_p^2 d\sigma_\Sigma^2\right) + \ell^2 \frac{dr_p^2}{r_p^2}$$

Dual gauge theory lives on "collapsing cone" metric: $$ds_4^2 = -dt_p^2 + t_p^2 d\sigma_\Sigma^2$$

1. Similar in spirit to other time-dependent QFTs

Das *et. al.*
Awad *et. al.*
Craps, Sethi, *et. al.*
Martinec *et. al.*

2. Distinct from example of unstable QFTs

Horowitz and Hertog
Craps, Hertog, and Turok
Bernamonti and Craps

- $D3$ branes at constant r_p are solutions of e.o.m.
- Stretched W bosons have mass m_W constant in time.
- Momentum m_{KK} along Σ grows with $t_p \to 0^-$.
- Dimensionless ratio $m_W/m_{KK} \to 0$.

Singularity associated with IR of QFT

3. Unclear if QFT is well defined as $t_p \to 0^-$

Static coordinate system

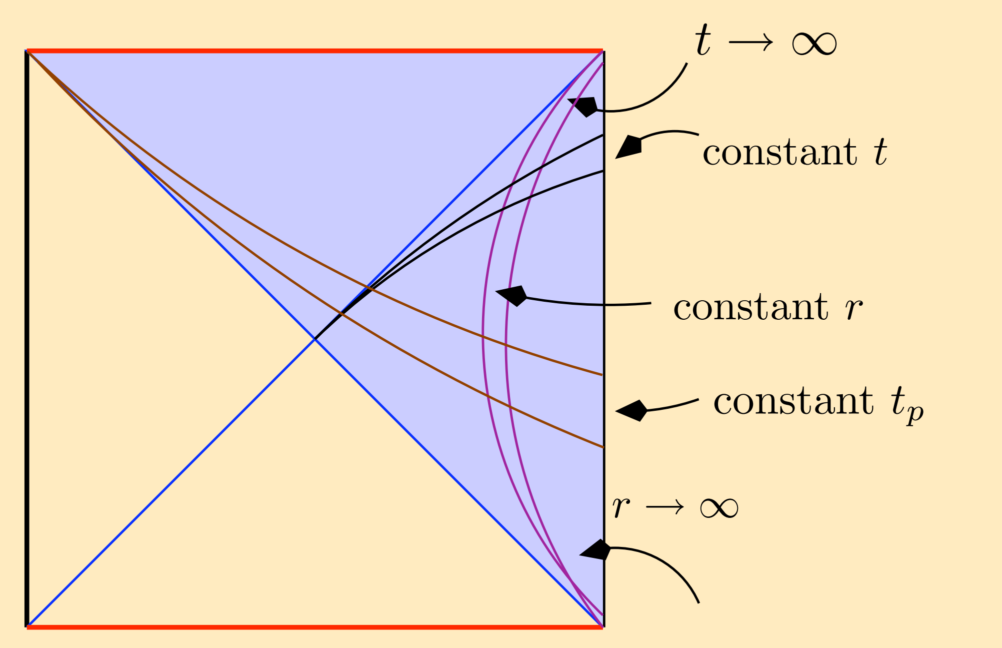
$$r = -\frac{r_p t_p}{\ell} \qquad t = -\frac{\ell}{2} \ln\left(\frac{t_p^2 r_p^2 - \ell^2}{\ell^2 r_p^2}\right)$$

$$ds_5^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma_\Sigma^2$$

$$f(r) = \frac{r^2}{\ell^2} - 1$$

"M=0 topological black hole":
5d version of BTZ black hole

Emparan



Shaded patch covered by t_p, r_p

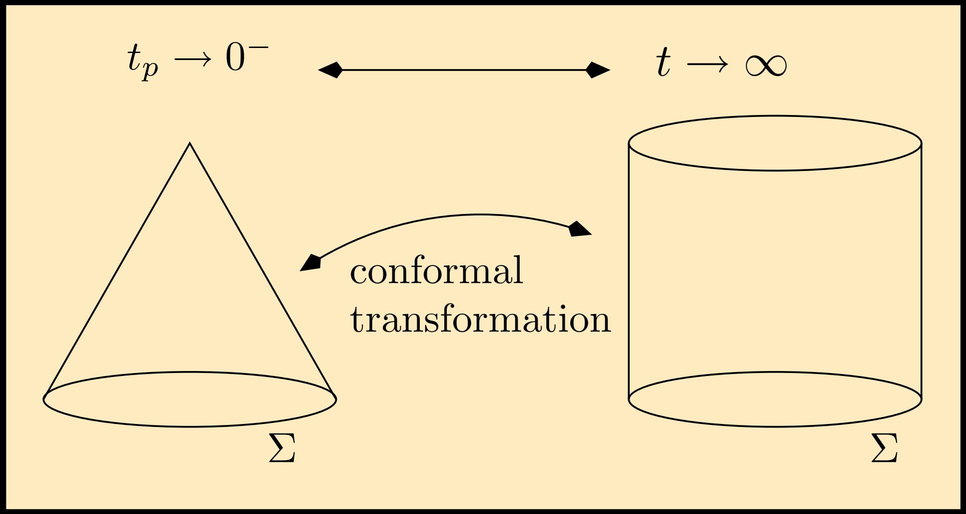
- Negatively curved horizon at $r = \ell$.
- Temperature $T \sim 1/\ell$.
- Horizon area $\sim \ell^3$; entropy $\sim N^2$.

Dual: gauge theory on $\Sigma \times \mathbb{R}$ at $T = 1/R_\Sigma$

Conformal transformation:

$$t_p = -\ell e^{-t/\ell}$$

$$ds^2_{cone} \to ds^2_{cyl} = e^{2t/\ell} ds^2_{cone}$$



Seems to map QFT variables describing Scwharzschild observers to QFT variables describing infalling observers

- How does the map act on bulk probes?
- Can this be generalized to other BHs?

III. Gauge theory vs. spacetime coordinate transformations

A. $\mathcal{N} = 4$ SYM on $\Sigma \times \mathbb{R}$, $T = 1/R_\Sigma$

Consider D3-brane wrapping Σ and moving in r, t

String theory: D3-brane probe dynamics described by DBI action

$$S_{DBI} = \frac{1}{g_s(\alpha')^2} \int d\tau d^3\sigma \left(\sqrt{\det \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)} - A^{(4)}_{RR} \right)$$

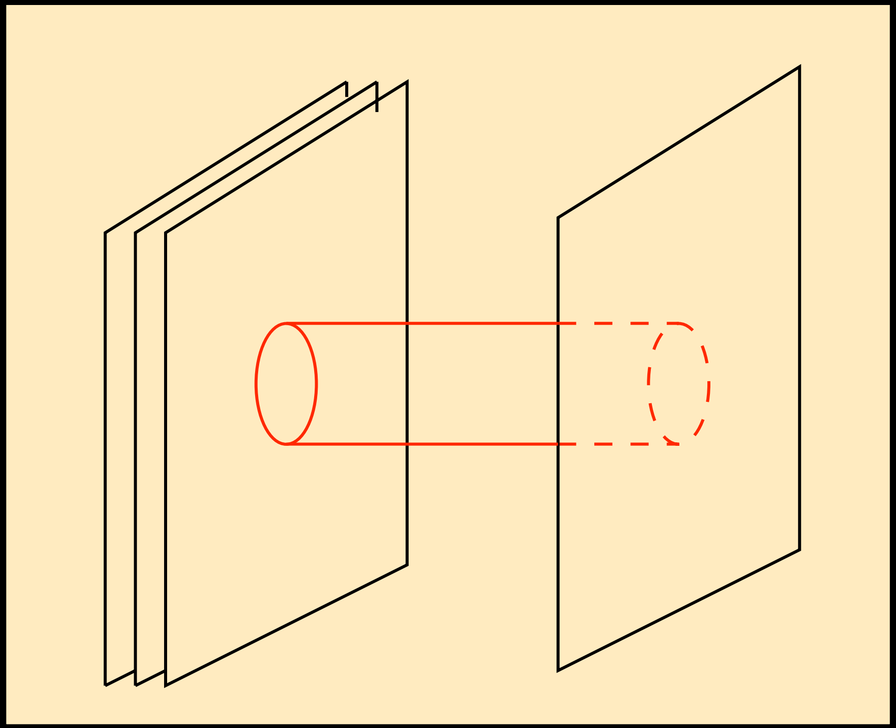
$$\Big\downarrow \quad \begin{array}{rcl} t & = & \tau \\ r & = & r(t) \\ \sigma^i & : & \Sigma \to \Sigma \quad \text{one} - \text{to} - \text{one} \end{array}$$

$$S_{static} = -\frac{\hat{V}}{g_s(\alpha')^2} \int dt \left(r^3 \sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - \frac{r^4 - \ell^4}{\ell} \right)$$

$$f(r) = \frac{r^2}{\ell^2} - 1$$

Gauge theory:

- $t =$ gauge theory time.
- Take adjoint scalar out on Coulomb branch, $\phi = \alpha' r$.
- Integrate out W-bosons charged under $U(1) \times U(N-1)$.



- S_{static} is resulting effective action for ϕ

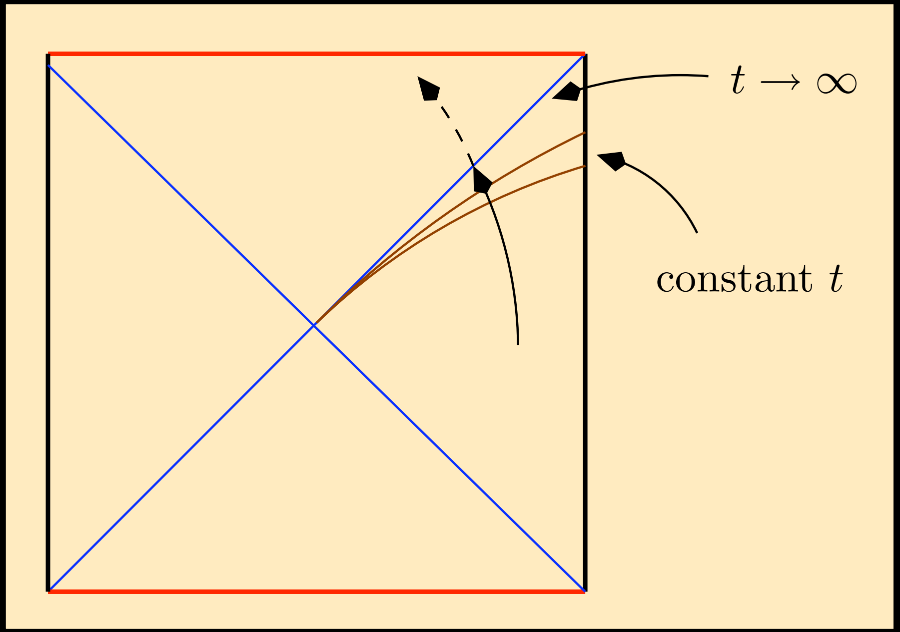
$$S_{static} = -\frac{\hat{V}}{g_s(\alpha')^2}\int dt \left(r^3\sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - \frac{r^4-\ell^4}{\ell} \right)$$

$$f(r) = \frac{r^2}{\ell^2} - 1$$

1. $\dot{r}^2 < f(r)^2$: "scalar speed limit".

Silverstein and Tong

$r \to \ell$ as $t \to \infty$



2. Let $r = r_0(t) + \delta r(t)$:
   - $r_0(t)$ solves classical e.o.m.
   - Expand S_{static} in δr

Expansion in δr breaks down as $f \to 0$.

Semiclassical physics in (t, r) breaks down at horizon

B. $\mathcal{N} = 4$ SYM on collapsing cone

Consider D3-brane wrapping Σ and moving in r_p, t_p

String theory:

$$S_{DBI} = \frac{1}{g_s(\alpha')^2}\int d\tau d^3\sigma \left(\sqrt{\det \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)} - A^{(4)}_{RR}\right)$$

$$\Big\downarrow \qquad \begin{aligned} t_p &= \tau \\ r_p &= r_p(t_p) \\ \sigma^i &: \Sigma \to \Sigma \quad \mathrm{one-to-one} \end{aligned}$$

$$S_{cosmo} = -\frac{\hat{V}}{g_s(\alpha')^2}\int dt_p \left(t_p^3 r_p^3 \sqrt{\frac{r_p^2}{\ell^2} - \ell^2 \frac{\dot{r_p}^2}{r_p^2}} - \frac{r_p^4 t_p^3}{\ell^4}\right)$$

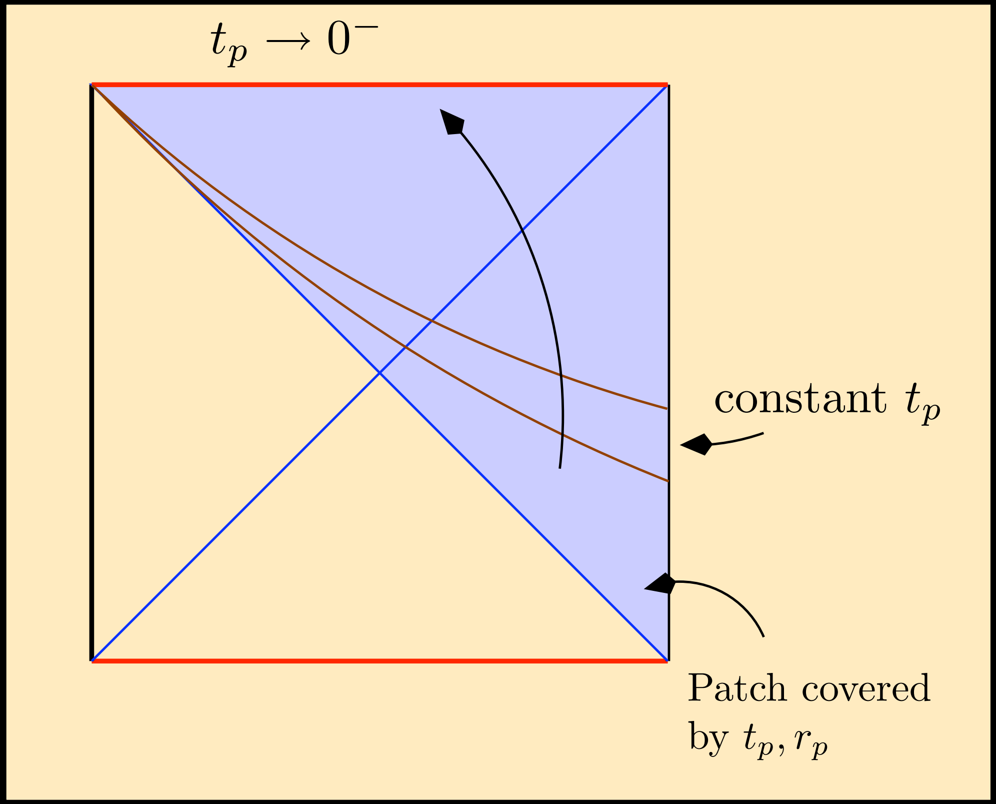
Gauge theory:

- t_p = gauge theory time.
- Take adjoint scalar out on Coulomb branch, $\phi = \alpha' r_p$.
- Integrate out W-bosons charged under $U(1) \times U(N-1)$.

$$S_{cosmo} = -\frac{\hat{V}}{g_s(\alpha')^2}\int dt_p \left(t_p^3 r_p^3 \sqrt{\frac{r_p^2}{\ell^2} - \ell^2 \frac{\dot{r_p}^2}{r_p^2}} - \frac{r_p^4 t_p^3}{\ell^4} \right)$$

1. Horizon at $r_p t_p = \ell^2$, singularity as $t_p \to 0^-$.

   Horizon reached in finite time



2. Let $r_p = r_{p,0}(t_p) + \delta r_p(t_p)$:

   - $r_{p,0}(t_p)$ solves classical e.o.m.
   - Expand S_{static} in δr_p

Expansion in δr_p

- regular at horizon
- breaks down at singularity

C. Transformation of QM variables of probes

1. Conformal transformation of QFT

$$t_p = -\ell e^{-\frac{\tilde{t}}{\ell}}$$

$$ds^2_{cone} \to ds^2_{cyl} = e^{2\tilde{t}/\ell} ds^2_{cone}$$

maps collapsing cone to $\Sigma \times \mathbb{R}$

$\phi = r/\alpha'$ dimension-1 field:

$$r_p = e^{\tilde{t}/\ell}\tilde{r}$$

$$\tilde{t}, \tilde{r} \neq t, r$$

Conformal transformation: $S_{cosmo} \not\to S_{static}$

$$S_{cosmo} = \tilde{S} = -\frac{\hat{V}}{g_s(\alpha')^2}\int d\tilde{t}\left(\tilde{r}^3\sqrt{\frac{\tilde{r}^2}{\ell^2} - \ell^2\frac{(\dot{\tilde{r}}+\tilde{r}/\ell)^2}{\tilde{r}^2}} - \frac{\tilde{r}^4}{\ell}\right)$$

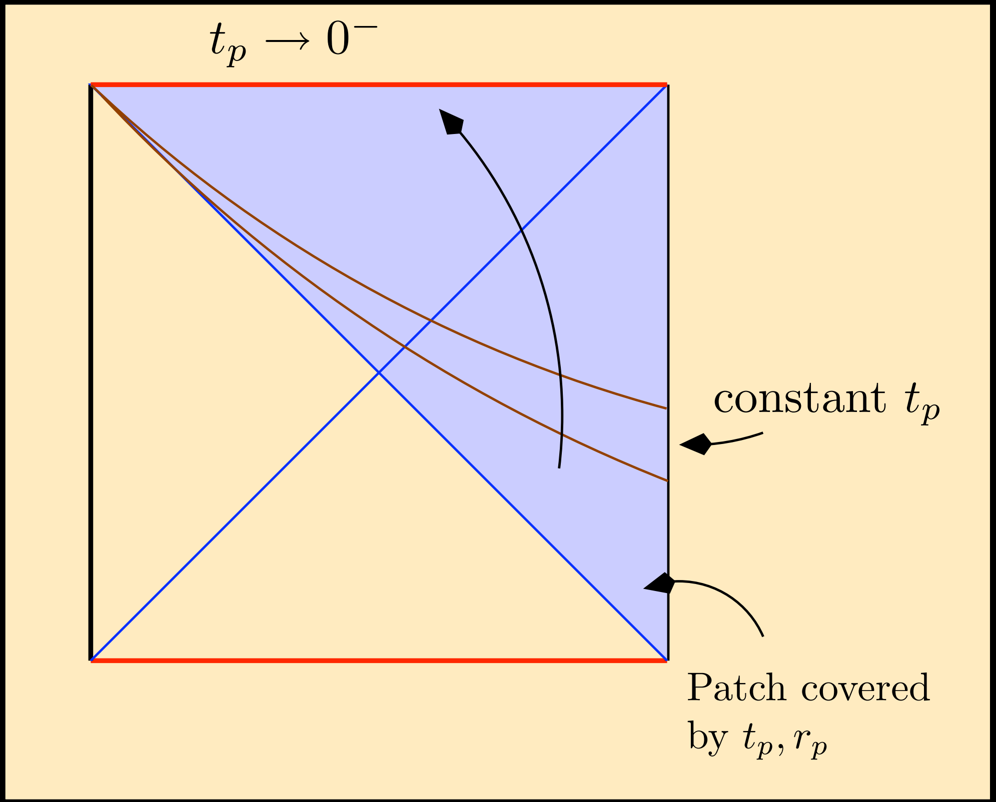
This is not surprising:

(a) Coordinate transformations

$$t_p = -\ell e^{-\tilde{t}/\ell}$$

$$r_p = \tilde{r} e^{\tilde{t}/\ell}$$

do not change equal-time slices in bulk



(b) Bulk metric:

$$ds_5^2 = -f(\tilde{r})d\tilde{t}^2 + \tilde{r}^2 d\sigma_\Sigma^2 + \frac{2\ell}{\tilde{r}} d\tilde{t} d\tilde{r} + \frac{\ell^2}{\tilde{r}^2} d\tilde{r}^2 \qquad f(\tilde{r}) = \frac{\tilde{r}^2}{\ell^2} - 1$$

$$S_{DBI} = \frac{1}{g_s(\alpha')^2} \int d\tau d^3\sigma \left(\sqrt{\det \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X)} - A_{RR}^{(4)} \right)$$

$$\downarrow \qquad \begin{aligned} \tilde{t} &= \tau \\ \tilde{r} &= \tilde{r}(\tilde{t}) \\ \sigma^i &: \Sigma \to \Sigma \end{aligned}$$

$$\tilde{S} = -\frac{\hat{V}}{g_s(\alpha')^2} \int d\tilde{t} \left(\tilde{r}^3 \sqrt{\frac{\tilde{r}^2}{\ell^2} - \ell^2 \frac{(\dot{\tilde{r}} + \tilde{r}/\ell)^2}{\tilde{r}^2}} - \frac{\tilde{r}^4}{\ell} \right)$$

2. Bulk coordinate transformations in dual QFT

$$r = -\frac{r_p t_p}{\ell}$$
$$t = -\frac{\ell}{2} \ln\left(\frac{t_p^2 r_p^2 - \ell^2}{\ell^2 r_p^2}\right)$$

map "cosmological" to "static" coordinates

In QFT:

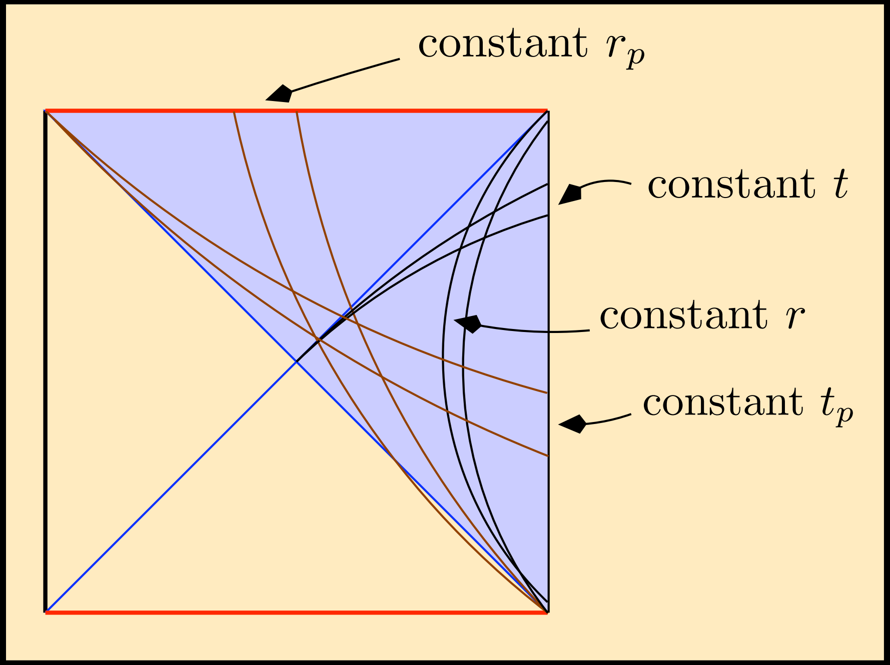
- t, t_p are QFT times
- r, r_p are quantum fields

Map $(r_p, t_p) \to (r, t)$ includes a field-dependent time reparametrization

Generic to any nontrivial change of bulk equal-time slicings

Seems exotic in QFT

(but remember gauge transformations for quantum inflaton fluctuations)



Solution: add gauge invariance

Let $t_p = t_p(\tau), r_p = r_p(\tau)$.

$$S_{cosmo} \to S_{DBI} = -\frac{\hat{V}}{g_s(\alpha')^2\ell^3}\int d\tau\left(t_p^3 r_p^3\sqrt{\frac{r_p^2\dot{t}_p^2}{\ell^2} - \ell^2\frac{\dot{r}_p^2}{r_p^2}} - \frac{\dot{t}_p t_p^3 r_p^4}{\ell}\right)$$

Invariant under reparametrizations of τ

Reduces to S_{cosmo} if we gauge fix $t_p = \tau$

$$r = -\frac{r_p t_p}{\ell}$$

$$t = \frac{\ell}{2}\ln\left(\frac{r_p^2 t_p^2 - \ell^4}{\ell^2 r_p^2}\right)$$

is now a simple field redefinition

Fix $t = \tau$:

$$S_{DBI} \to S_{static} = -\frac{\hat{V}}{g_s(\alpha')^2}\int dt\left(r^3\sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - \frac{r^4 - \ell^4}{\ell}\right)$$

(Up to boundary term/RR gauge transformation)

Conformal transformation to $\Sigma \times \mathbb{R}$

$$ds_5^2 = -f(\tilde{r})d\tilde{t}^2 + \tilde{r}^2 d\sigma_\Sigma^2 + \frac{2\ell}{\tilde{r}} d\tilde{t} d\tilde{r} + \frac{\ell^2}{\tilde{r}^2} d\tilde{r}^2$$

$\downarrow \quad \tilde{r} \to \infty$

$$ds_5^2 \sim \frac{\tilde{r}^2}{\ell^2}\left(-d\tilde{t}^2 + d\sigma_\Sigma^2\right) + \ell^2 \frac{d\tilde{r}^2}{\tilde{r}^2}$$

Field theory dual seems to live on $\Sigma \times \mathbb{R}$

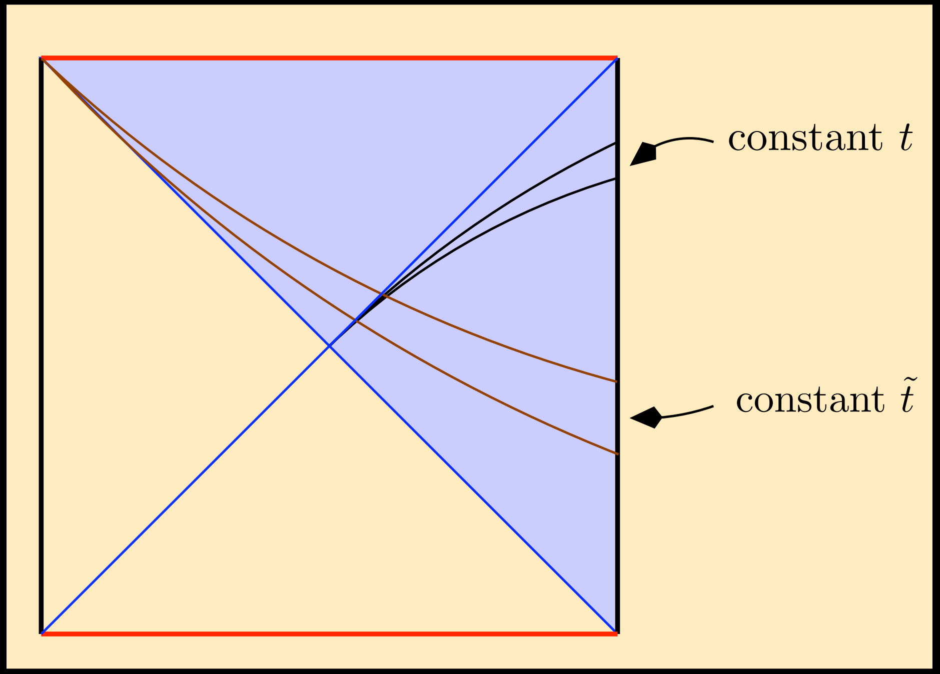
Nonsingular field theory on nonsingular space: $t_p \to 0^-$ mapped to $\tilde{t} \to \infty$.

$\tilde{t}, \tilde{r}$ extend behind horizon.

$\tilde{S}[\tilde{r}]$ well behaved at horizon

Better variables to see behind horizon

Which action arises as
QFT effective action?



Transformation of quantum observables

Conjugate momenta:

$$p_{\tilde{r}} = p_r - \frac{\ell}{r f(r)} p_t + \frac{r^4 - \ell^4}{\ell^4 r f(r)} \hat{V} N$$

$$p_{\tilde{t}} = p_t + \frac{\hat{V} N}{\ell}$$

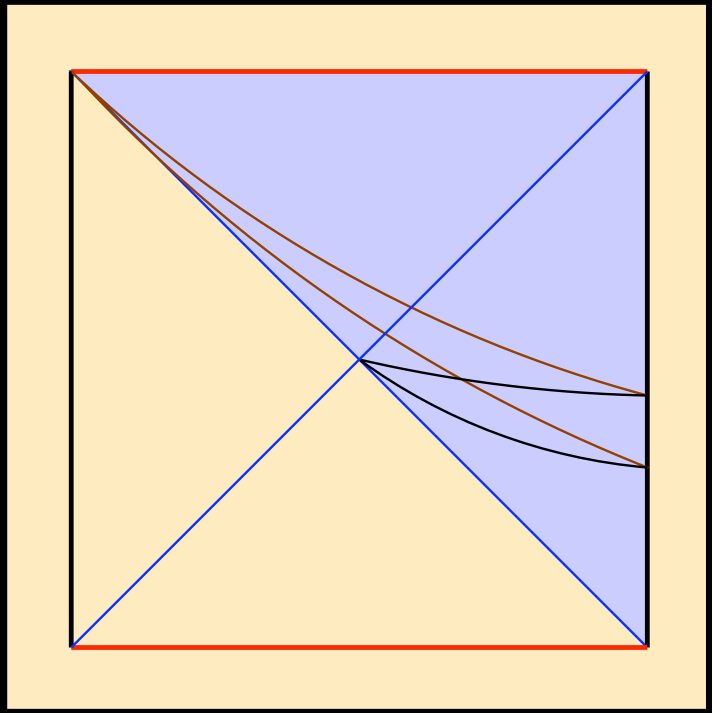
Hamiltonians:

$$H_t = -p_t \; ; \; H_{\tilde{t}} = -p_{\tilde{t}}$$

Equal up to a constant

$r \to \infty$: $\tilde{r} \to \infty$

Equal-t slices asymptotically identical to equal-$\tilde{t}$ slices



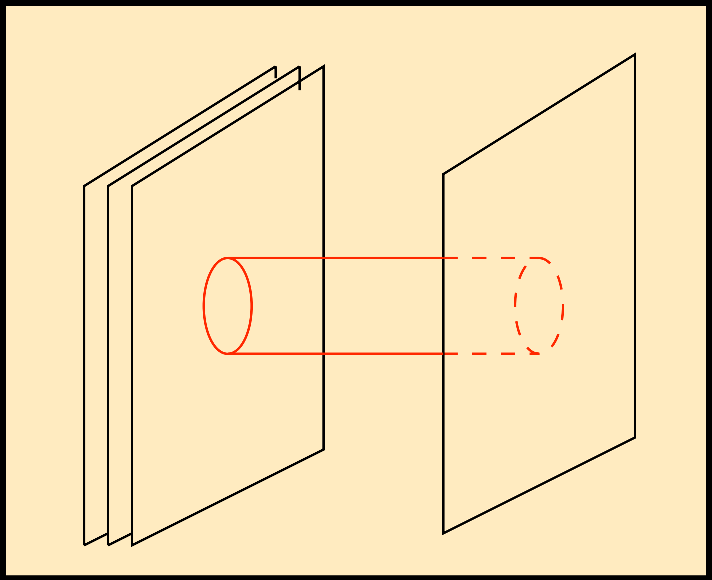
D. Transformation of full QFT

Puzzles:

1. Yang-Mills action invariant under conformal transformation
2. $S_{cosmo} \not\to S_{static}$ under conformal transformation
3. S_{cosmo}, S_{static} effective actions for SYM?
4. Is S_{static} or $\tilde{S}$ effective action for SYM on $\Sigma \times \mathbb{R}$?

Gauge theory:

- t = gauge theory time.
- Take adjoint scalar out on Coulomb branch, $\phi = \alpha' r$.
- Integrate out W-bosons charged under $U(1) \times U(N-1)$.



- S_{static} is resulting effective action for ϕ

Hidden step: *must fix gauge* before integrating out W bosons.

Functional form of effective action depends on gauge choice.

Standard gauge for computing DBI actions:

Background field gauge: expand around

$$A_\mu = \bar{A}_{cl,\mu} + \delta A_\mu, \; \Phi = \bar{\phi}_{cl} + \delta\phi$$

$$G = D_\mu^{\bar{A}} \delta A_\mu + i[\bar{\phi}, \delta\phi]$$

(This is the gauge implicit in string theory computations)

Under conformal transformation:

$$G \to \tilde{G} = \tilde{D}_{\tilde{\mu}}^{\bar{A}} \delta A_{\tilde{\mu}} + i[\bar{\phi}, \delta\phi] + \frac{2}{\ell} A_{\tilde{t}}$$

- Background field gauge not conformally invariant
- $\tilde{G}$ breaks $\tilde{t}$-reversal invariance
- $\tilde{S} \propto \int d\tilde{t}\tilde{r}^3 \sqrt{\frac{\tilde{r}^2}{\ell^2} - \frac{\ell^2}{\tilde{r}^2}(\dot{r} + \frac{\tilde{r}}{\ell})^2} + S_{RR}$ is not $\tilde{t}$-reversal invariant

Proposal:

1. S_{static} is effective action for SYM on $\Sigma \times \mathbb{R}$ in background field gauge.
2. $\tilde{S}$ is effective action for SYM on $\Sigma \times \mathbb{R}$ in gauge $\tilde{G} = 0$.
3. Full 5d coordinate transformations $\leftrightarrow$ Yang-Mills gauge transformations.

Related story: special conformal transformations in SYM vs. DBI

Jevicki, Kazama, and Yoneya

IV. Gauge theory dynamics and black hole formation

A. Phases of gauge theory dynamics

Lagrangian for adjoint scalars:

$$\mathcal{L} \sim \mathrm{Tr}\left[|D\Phi^I|^2 - ([\Phi^I, \Phi^J])^2 - \mathcal{R}^{(4)}(\Phi^I)^2 + \ldots\right]$$

Consider N D3-branes smeared over transverse S^5

Coincident radial position: dual to scalar zero mode $\phi(t)$

$\Sigma \times \mathbb{R}$ has negative curvature

- Zero modes of Φ are unstable
- Small number of momentum modes unstable
- $\Sigma = \mathbb{H}/\Gamma$ exist such that only zero modes unstable

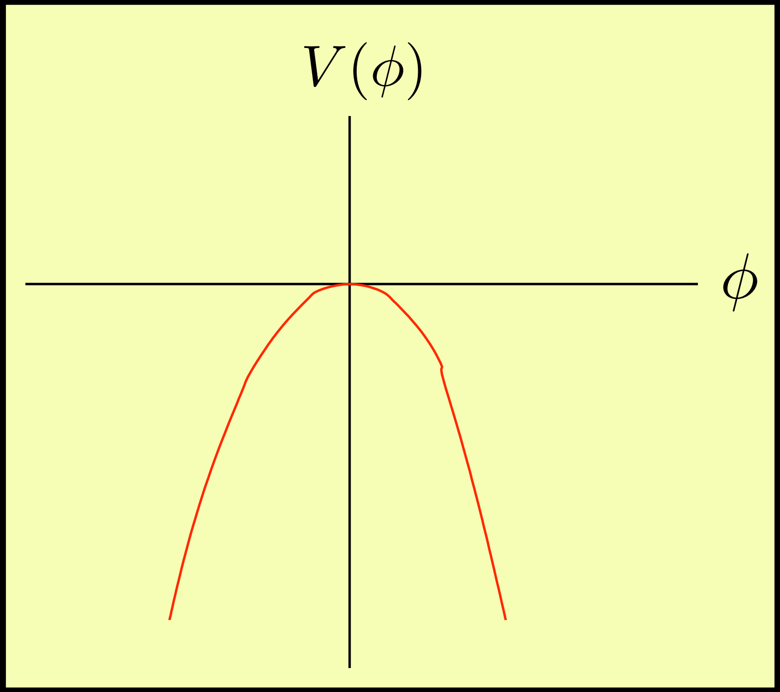
Cornish, Spergel, and Starkman

Quantum mechanics of $\phi(t)$

Lagrangian for large ϕ (no corrections from W loops):

$$L \sim (\partial_t \phi)^2 + \frac{1}{\ell^2}\phi^2$$

upside-down SHO



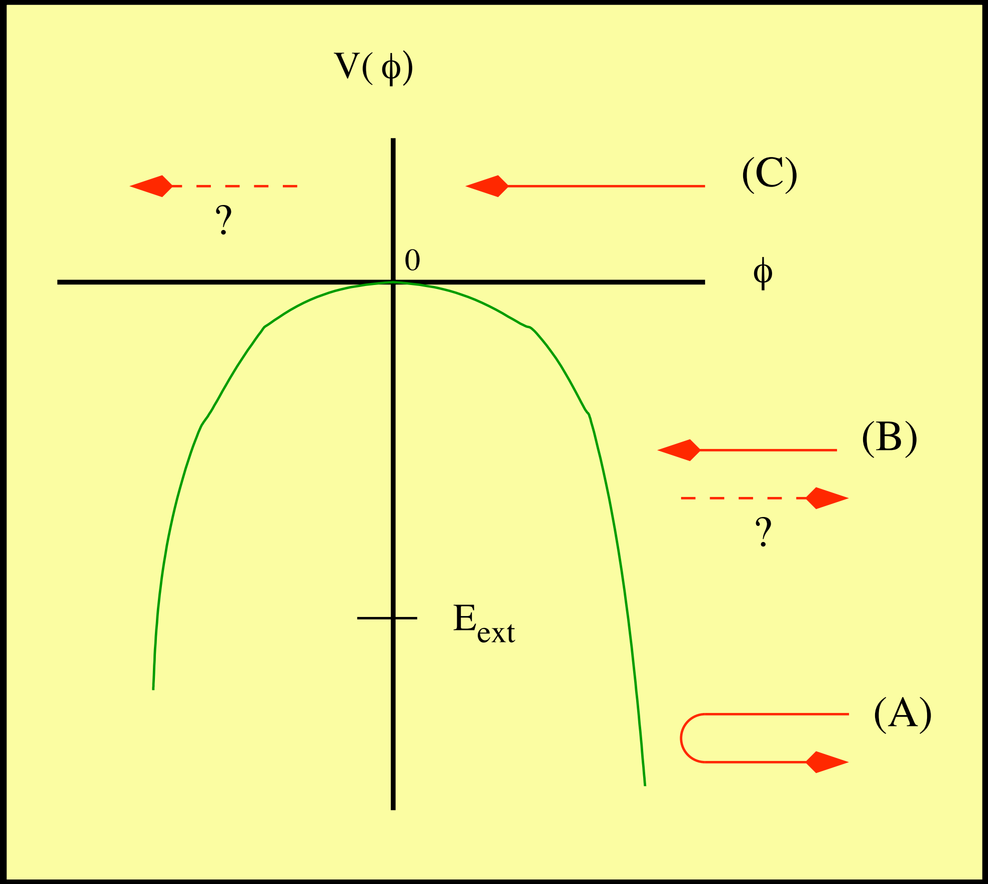
- Classically $\phi \to \infty$ in *infinite* time.
- Continuous spectrum, no ground state.
- Quantum mechanics nonsingular.

Quantum corrections to SHO action

1. Loops of W bosons when $\lambda \frac{\dot{\phi}}{\phi^2} \sim 1$.
2. W bosons produced when $\frac{\dot{\phi}}{\phi^2} \sim 1$.
3. Loops and production of
   - KK modes
   - Wilson lines on Σ
   - Flux tubes
   - ...

As ϕ evolves inwards, (1) becomes important first

Quantum corrections



$E_{ext} = \frac{\mu_{ext}}{G_N}$

(A) Scalar bounces before $\lambda\dot{\phi}/\phi^2 \sim 1$

(B) $\lambda\dot{\phi}/\phi^2 \sim 1$ before bounce

- $r = \alpha'\phi \sim r_{horizon}$ for $M = E$ black hole
- Expect W loops to slow down evolution (as with probe)

(C) $\lambda\dot{\phi}/\phi^2 \sim 1$ before $r \to 0$ reached

- Uncorrected motion describes a "bounce"
- Expect W loops to slow evolution near $r_{horizon} \sim \alpha'\phi$
- As $\phi \to 0$, production of QFT modes thermalizes system, traps branes

B. Topological black holes

Emparan

Solutions to 5d SUGRA with negative c.c.:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\sigma^2_{\Sigma_k}$$

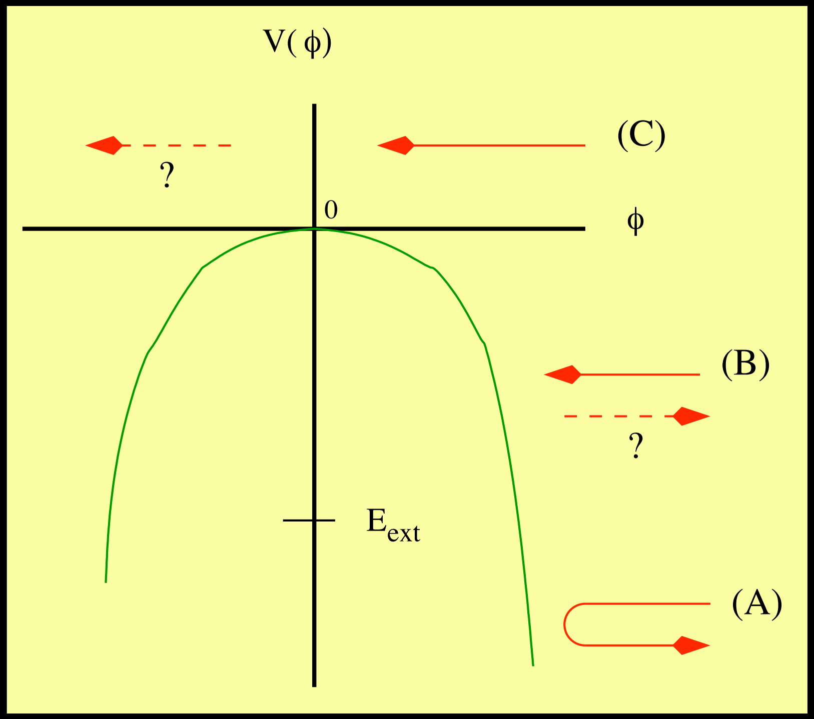
$$f(r) = \frac{r^2}{\ell^2} + k - \frac{\mu}{r^2}\ ;\ k = 0, \pm 1 \qquad \mu = G_N M$$

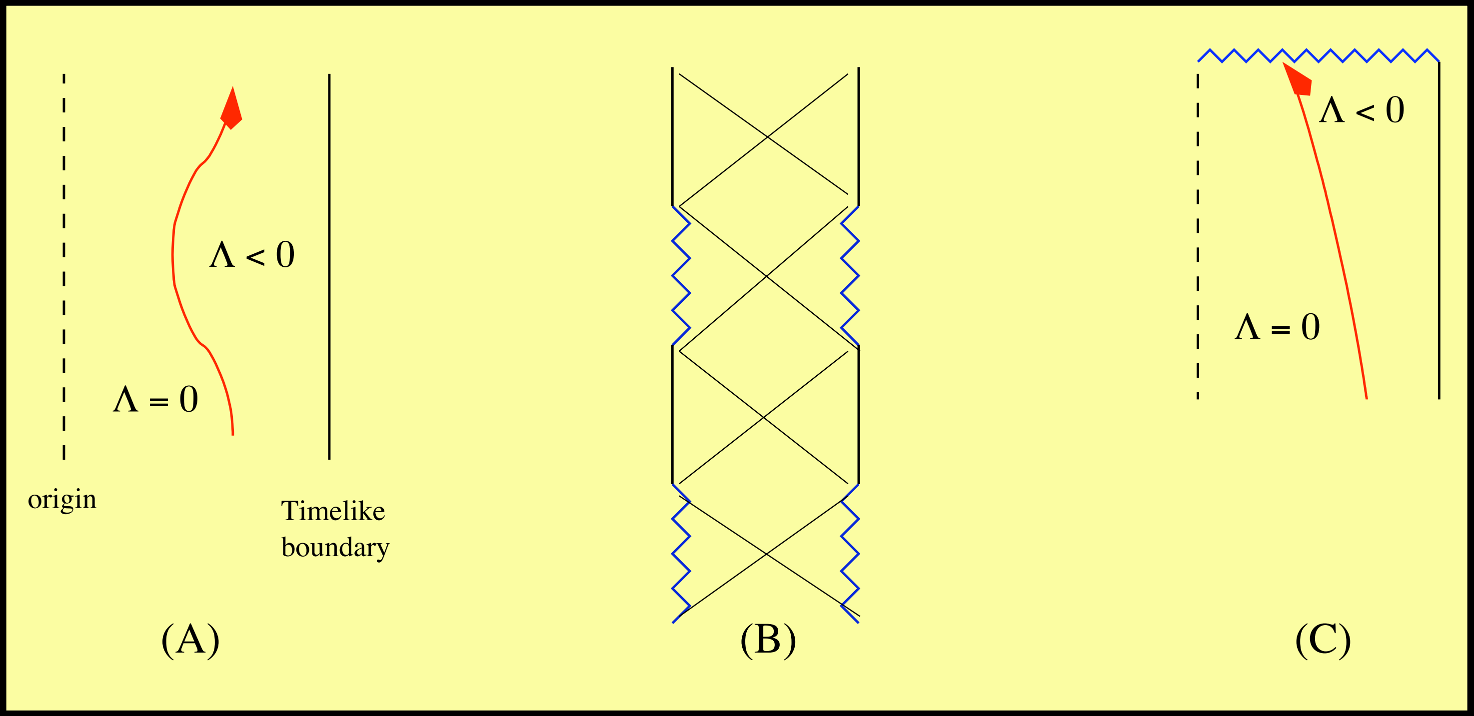
Σ_k 3-manifold of constant curvature:

- $\Sigma_1 = S^3$: AdS-Schwarzschild
- $\Sigma_0 = \mathbb{R}^3$: near horizon limit of black D3-brane
- $\Sigma_{-1} = \mathbb{H}_3/\Gamma$: "topological" black hole

$$\mu \propto E_{gauge}$$

C. Gauge theory phases and spacetime causal structure





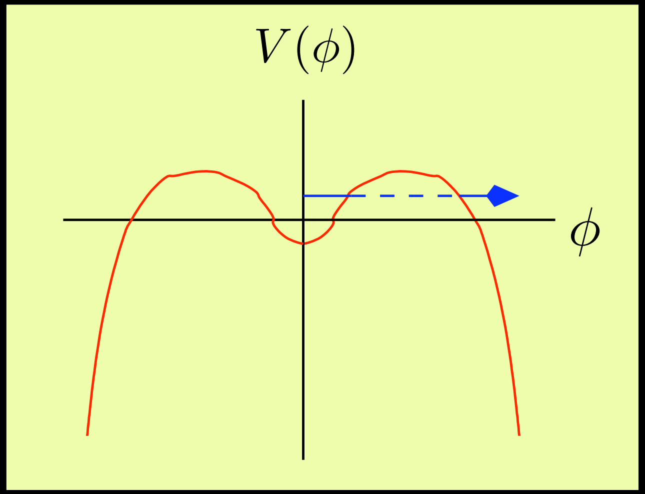
- Shell of D3-branes screens Λ.
- Outside of shell with energy E, spacetime is $M = E$ black hole.
- Trajectory (A) removes singularity a la enhancon mechanism.
- Trajectory (B) unknown: recall instability of inner horizon.
- Trajectory (C) stalls near origin.
  Thermal effective potential traps D3-branes.

D. Late time behavior

Shell with $E \geq 0$ thermalizes gauge theory as $\phi \to 0$

Thermal effects modify effective potential for Φ^I:

- Eigenvalues trapped near origin by W-bosons
- W effects small for large ϕ: instability dominates



Nonperturbative instability to brane emission

DBI action $S \sim cN$ for single brane

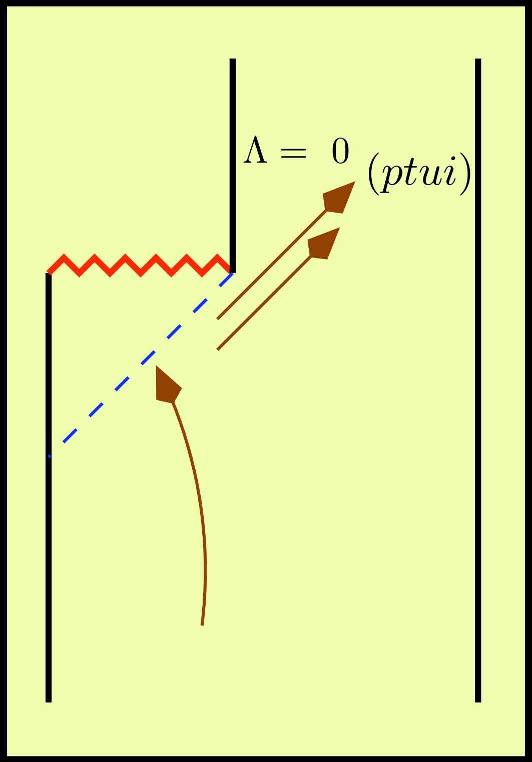
$$t_{emission} \sim e^{cN}$$

- Shorter than recurrence time for AdS-Schwarzschild $\sim e^{N^2}$
- Longer than lifetime of "small" BHs in AdS: $t_{evap} \sim M^{\alpha}$.

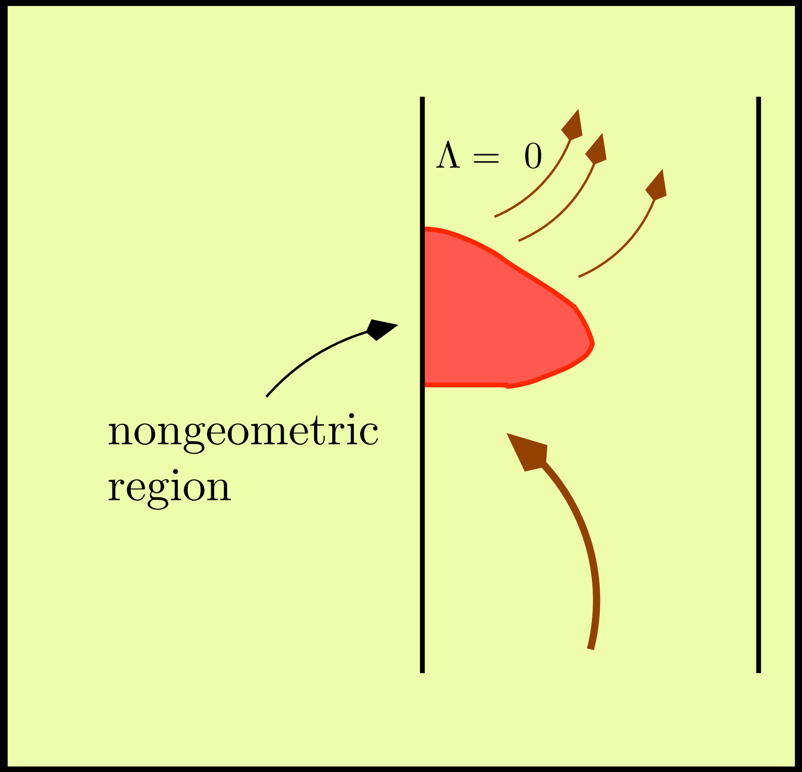
Branes emitted incoherently over time scale $\sim N e^{cN}$.

Candidate spacetimes

(a)



(b)



(a) Unitarity: should not continue past singularity

(b) Not a simple bounce: branes re-emitted one by one quantum-mechanically

(Are (a) and (b) physically distinct?)

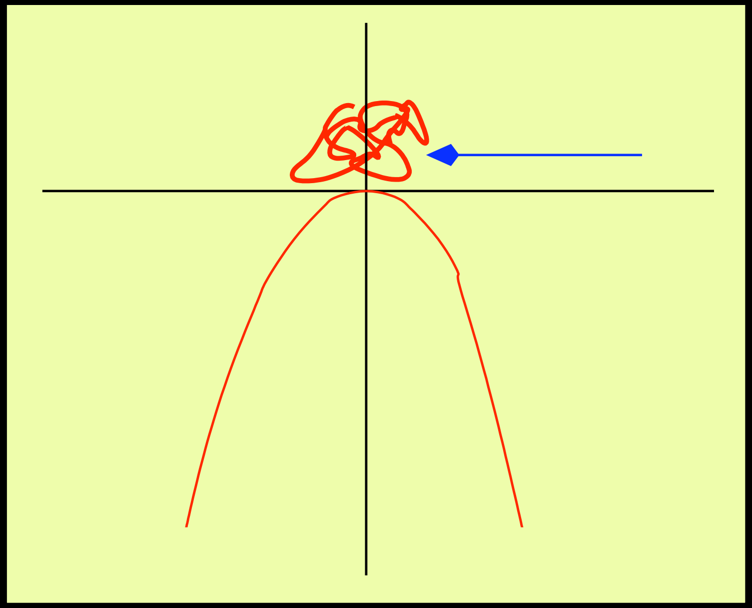
V. Conclusions

A. Lessons

1. Bulk coordinate transformations ~ boundary gauge transformations

2. Schwarzschild and infalling observers described by same
   Hamiltonian in different variables: dual descriptions

3. Singularity associated with origin of field space; physics well-defined



vs. Horowitz and Hertog; Craps, Hertog, and Turok

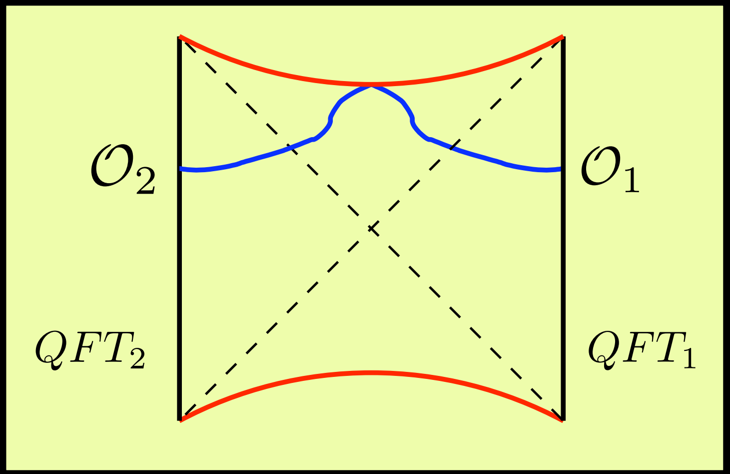
4. Singularity accessible in static QFT

vs. Das *et. al.*
Awad *et. al.*
Craps, Sethi, *et. al.*
Martinec *et. al.*

5. No sign of cosmological bounce

B. Future work

1. Relation to work using TFD correlators to probe singularity



Kraus, Ooguri, and Shenker;
Fidkowski, Hubeny, Kleban, and Shenker
Liu and Festuccia

2. Distinction between horizon in (t, r) and singularity in $(\tilde{t}, \tilde{r})$
3. Understand transformation of full gauge theory (study other probes?)

Horowitz, AL, Shenker, Silverstein

4. Better understand $M < 0$ black holes
5. Source of $\mathcal{O}(N^2)$ ground state entropy?
6. Coordinate transformation for other black holes:

- $\mu \neq 0$
- $k = 0, 1$

Use ingoing Eddington-Finkelstein coordinates?