Holographic Flavor Transport

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Credits

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- 0708.1994 A. O'B.
- 0808.1115 A. O'B.
- 0812.3629 A. Karch, A. O'B., E. Thompson
- 0908.2625 M. Ammon, H.T. Ngo, A. O'B.

Outline:

- I. Motivation
- II. The System
- III. The Conductivity
- IV. Summary and Outlook

I. Motivation

REAL Strongly-coupled Systems

Quantum Chromodynamics (QCD)

Relativistic Heavy-Ion Collider (RHIC)

QCD at $T \leq 2 \times T_c$

$T_c \approx 170$ MeV

Strongly-coupled, Nearly-ideal FLUID

Question:

Can we compute

TRANSPORT COEFFICIENTS

for QCD

at RHIC temperatures?

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Can we compute

TRANSPORT COEFFICIENTS

for QCD

at RHIC temperatures?

Answer:

NO.

Philosophy:

CHANGE the Question

Question:

Can we find ANY STRONGLY-COUPLED system for which we CAN compute TRANSPORT COEFFICIENTS?

Answer:
YES!

$\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills (SYM)

$$N_c \to \infty$$

$$\lambda \to \infty$$

$$\lambda = g_{YM}^2 N_c$$

Use Gauge-gravity Duality

Shear Viscosity

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

GOAL

Compute a
CONDUCTIVITY
associated with
“Quarks” or “Electrons”
using
Gauge-gravity Duality

RESULT

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

Current $\langle J^x \rangle = \sigma E$ nonlinear in E

Pair Production

Drude Conductivity

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II. The System

$\mathcal{N}$ = 4 supersymmetric SU(N_c) Yang-Mills (SYM)

$$N_c \to \infty$$

$$\lambda \to \infty$$

$$\beta = 0$$

II. The System

$\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills (SYM)

$$N_c \to \infty$$

$$\lambda \to \infty$$

No Quarks!

ADD

N_f $\mathcal{N} = 2$ hypermultiplets

$$\beta = +\mathrm{O}\left(\frac{N_f}{N_c}\right)$$

"Probe Limit"

N_f fixed << $N_c \to \infty$

Scales

Temperature T

Mass m

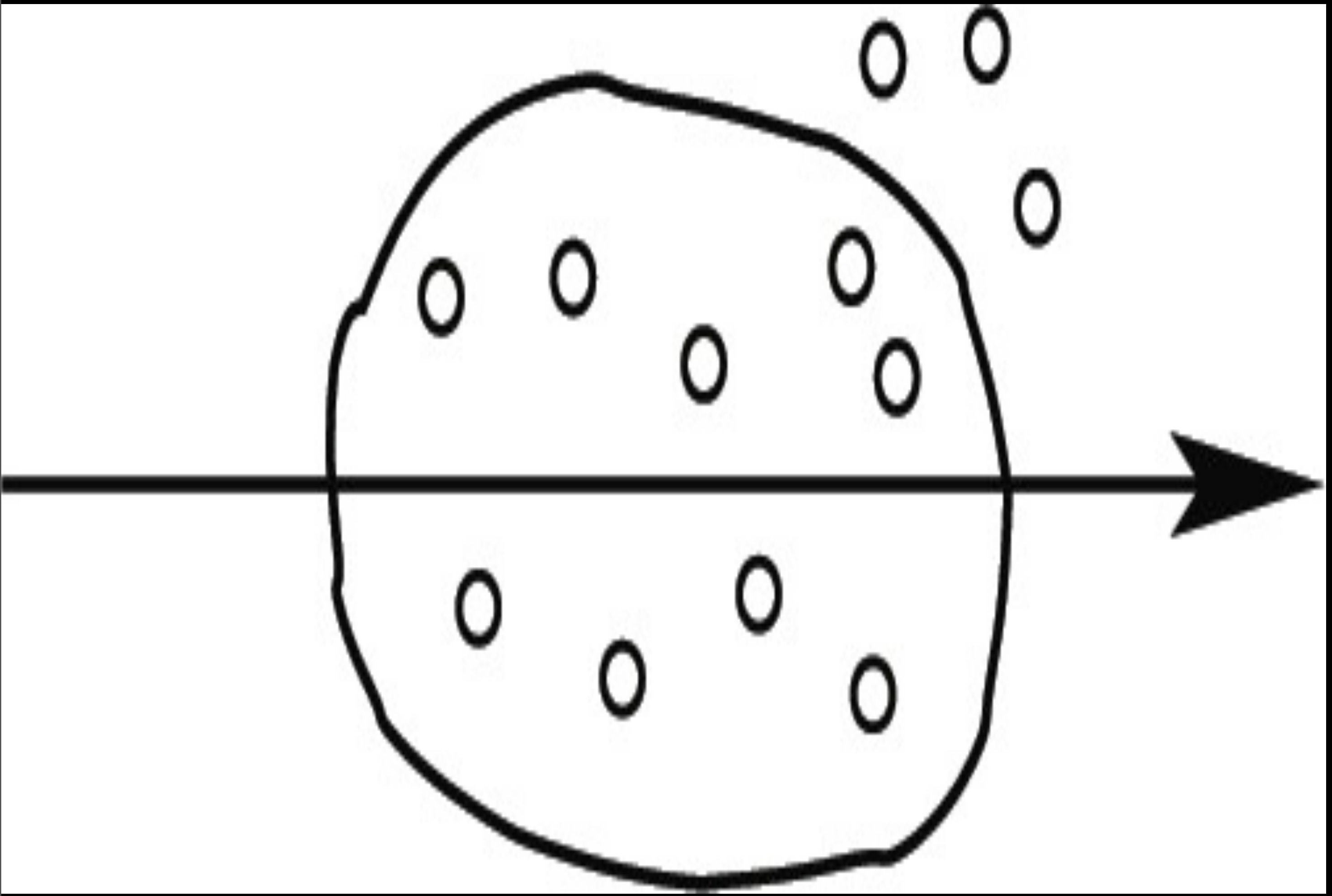
$U(1)_B$ symmetry current J^μ

Baryon Number Density $\langle J^t \rangle$

Electric Field E

"Two-fluid" picture

Lorentz force = Drag Force



$$\left\langle J^{x} \right\rangle = \sigma E$$

Steady-state???

Translation Invariance $\Rightarrow$ Momentum Conservation

Net Charge + Constant Electric Field $\Rightarrow$ Net Work

ENTIRE SYSTEM ACCELERATES FOREVER

NO

DISSIPATION!

Probe Limit MIMICS Dissipation

$$\langle T_{\mu\nu} \rangle = \mathrm{O}(N_c^2)_{\mu\nu} + \mathrm{O}(N_f N_c)_{\mu\nu}$$

$$\partial_\mu \langle T^{\mu\nu} \rangle = F^{\nu\sigma} \langle J_\sigma \rangle$$

$$\partial_t \langle T^{tt} \rangle = E \langle J_x \rangle \qquad \partial_t \langle T^{tx} \rangle = -E \langle J_t \rangle$$

$$\langle J_\mu \rangle = \mathrm{O}(N_f N_c) \qquad E = \mathrm{O}(1)$$

Holographic Dual

$\mathcal{N} = 4$ SYM $N_c, \lambda \to \infty$	=	Supergravity $AdS_5 \times S^5$
Finite temperature	=	AdS-Schwarzschild
N_f $\mathcal{N} = 2$ hypers.	=	N_f probe D7-branes $AdS_5 \times S^3$
m	=	Embedding
J^μ	=	A_μ

The Method

$$S_{D7} = -N_f T_{D7} \int d^8x \sqrt{-\det(g_{ab} + (2\pi\alpha')F_{ab})}$$

$$J^t \leftrightarrow A_t \qquad J^x \leftrightarrow A_x$$

- NOT Kubo formula!
- Compute 1-pt. function DIRECTLY
- Exploit Born-Infeld dynamics with E field
- Valid for any Dp/Dq system

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III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$e = \frac{E}{\frac{\pi}{2}\sqrt{\lambda}\, T^2}$$

$$d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda}\, T^2}$$

III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

Depends on E!

$$\langle J^x \rangle = \sigma(E) E$$

Linearize in E

$$\langle J^x \rangle = \sigma(0) E$$

III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$\langle J^t \rangle = 0 \quad \text{BUT} \quad \sigma \neq 0$$

Pair Production

III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$m \to \infty \quad \Rightarrow \quad f(m) \to 0$$

$$m \to 0 \quad \Rightarrow \quad f(m) \to 1$$

III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1}\, f(m) + \frac{d^2}{e^2+1}}$$

$$\langle T^{tx} \rangle \propto \langle J^t \rangle$$

NO momentum flow at zero density

III. The Conductivity

$$\sigma = \sqrt{\frac{N_f^2 N_c^2}{16\pi^2} T^2 \sqrt{e^2+1} f(m) + \frac{d^2}{e^2+1}}$$

Drude Conductivity

Linearize in E $\sigma(0)$ Take $m \to \infty$

$$\sigma \to d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda} T^2}$$

Why $m \to \infty$?

Charges behave as semi-classical quasi-particles:

$$\frac{dp}{dt} = -\mu p + E$$

Separate calculation

$$\boxed{\mu m = \frac{\pi}{2}\sqrt{\lambda}T^2}$$

$$\mu m = \frac{\pi}{2}\sqrt{\lambda T^2}$$

$$\sigma \to d = \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{\lambda T^2}} = \frac{\langle J^t \rangle}{\mu m}$$

Drude Conductivity

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IV. Summary + Outlook

Probe Branes are Great!

Computed CONDUCTIVITY

for a "DISSIPATIVE"

STRONGLY-COUPLED

Non-Abelian Gauge Theory

FUTURE DIRECTIONS

MORE TRANSPORT COEFFICIENTS:

Magnetic Fields

Anomalous currents

Condensed Matter Applications:

Thermo-electric Transport

Quantum Hall Effect

Superfluidity

Non-relativistic Theories

Thank You.