Domain wall flow equations and flux compactifications

Paul Smyth @ Zurich Strings Workshop `09

based on: P.S. & S. Vaula 0905.1334 [hep-th]

Motivation

- Flux compactifications and their 4D low energy effective actions have seen much attention recently.

  see also the talks of Martucci, Quevedo, Roest, Wrase...

- It has long been appreciated that the resulting 4D theories are gauged supergravities with can have electric and magnetic charges Polchinski & Strominger, Michleson `95

  recall De Rydt's talk

Fluxes ⟶ *Curvature/Torsion* ⟶ *Scalar Potential in 4d*

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- Domain walls are ubiquitous vacua in gauged supergravity, yet rarely considered, so why not look at them too? Mayer & Mohaupt, Louis et al
- Our aim is to find the general set equations describing these vacua, which might allow us to find new solutions.
- We test the proposal for flux compactifications given by Graña et al using the language of *SU(3)* x *SU(3)* structures ~ a handy way to deal with manifolds with torsion.

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Domain wall
in 4D *N*=2 theory
$\mathbb{R}^{1,3}$

Lifted ansatz
in 10D *Type II* theory
$\mathbb{R}^{1,3} \times Y$

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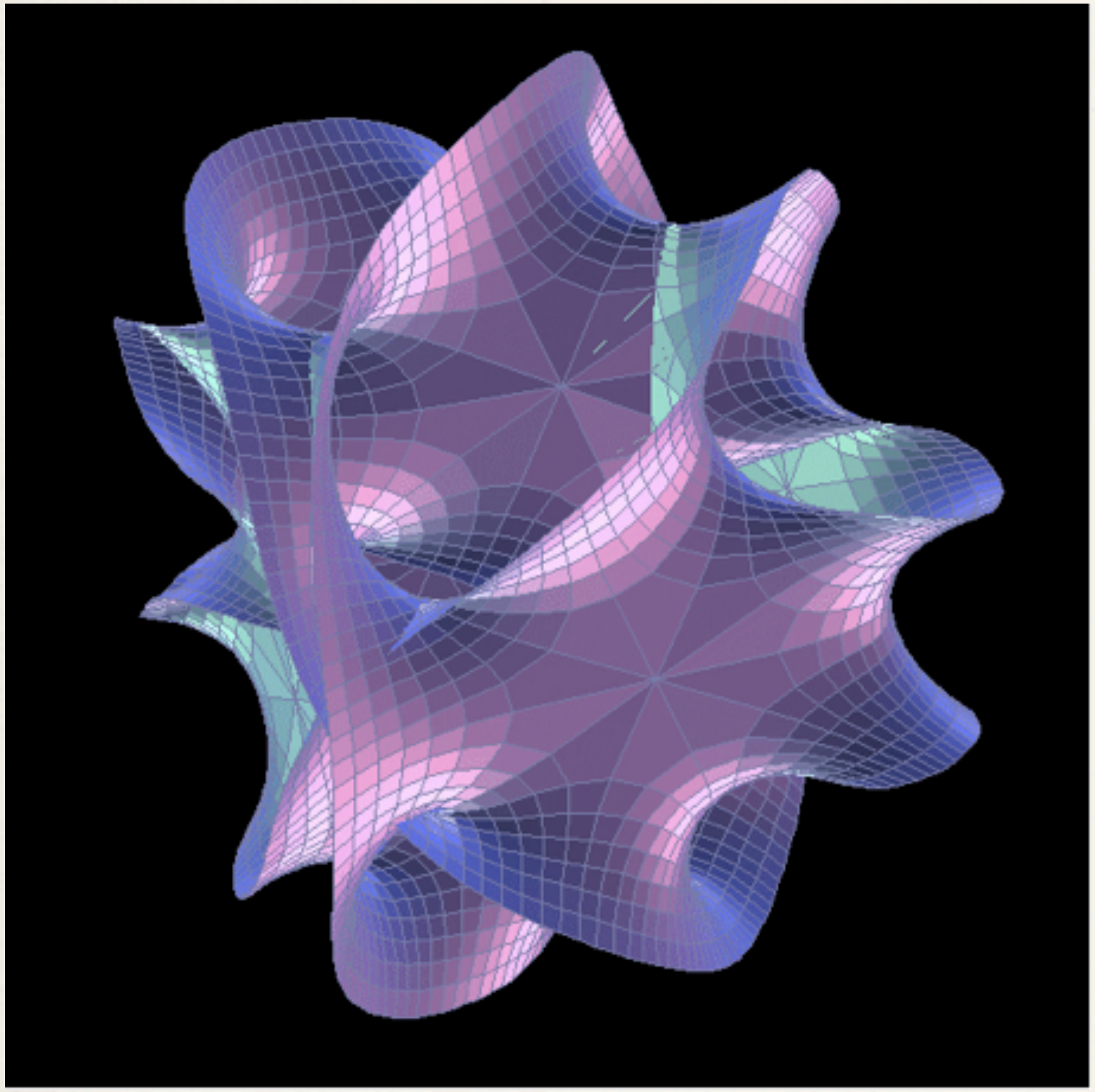
Test the dictionary - do the results for vacua agree?

From SU(3) holonomy to SU(3) structure



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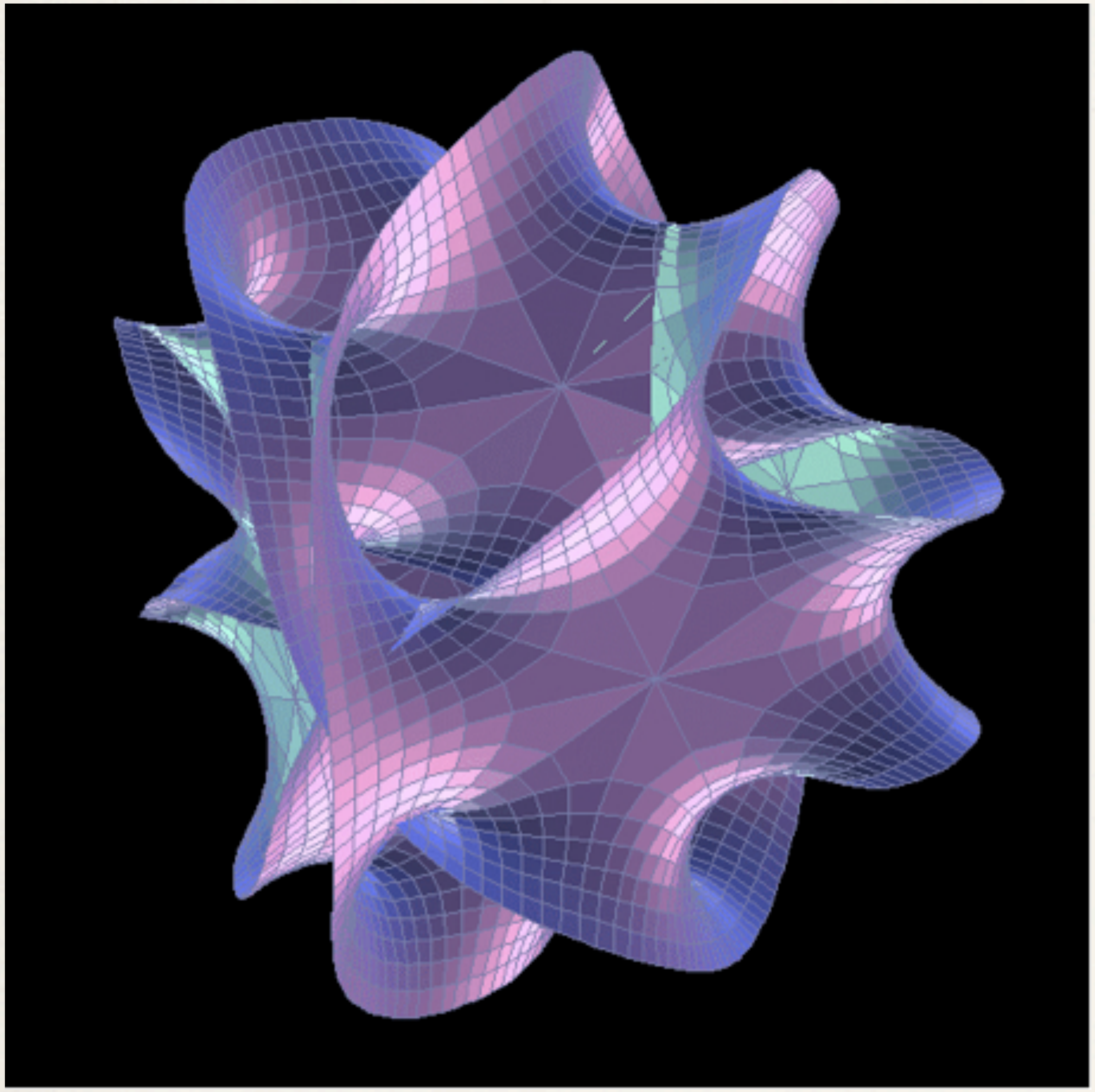

Thanks to Jeff Bryant @ Wolfram Research and calabi yau

Domain Walls from 10D

We consider an ansatz for a D=4 supersymmetric domain wall in a 10D bosonic warped background of Type II supergravity:

$$ds^2 = e^{2A(y,r)}\left(e^{2V(r)}\eta_{\alpha\beta}dx^\alpha dx^\beta + e^{2G(r)}dr^2\right) + g_{mn}(r,y)dy^m dy^n \ ,$$

$$F_n^{(10)} = \hat{F}(y)_n + \text{vol}_4 \wedge \tilde{F}(y)_{n-4} \quad , \quad H^{(10)} = H_3(y) \quad , \quad \phi = \phi(r,y)$$

where g_{mn} is the metric on the compact manifold Y and we use the democratic formalism.

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Use the SUSY variations to find the conditions for domain wall vacua

A first look at the SUSY variations

$$\delta\psi_M = (D_M \pm \frac{1}{4} H_M \mathcal{P})\epsilon + \frac{e^{\phi}}{16} \sum_n \hat{\not{F}}_{2n} \Gamma_M \mathcal{P}_n \epsilon$$

For such a symmetric configuration the SUSY variations are considerably simplified. To proceed, we still need some reasonable assumption - no worldvolume dependence:

$$\epsilon(x^{\alpha}, r, y^m) = \epsilon(r, y^m)$$

Radial dependence plays an important role, so we can initial focus on this component of the gravitino variation $\delta\psi_r$ and $\delta\psi_\alpha$:

$$\delta\psi_r \Rightarrow \epsilon(r, y^m) = e^{\frac{1}{2}(A(r)+V(r))}\epsilon_0(y^m)$$

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10D Killing Spinors $\epsilon_0(y^m)$

We take the standard ansatz for the 10D Killing spinors:

$$\begin{aligned}\epsilon_0^1(y^m) &= \varepsilon_+ \otimes \eta_+^{(1)}(y^m) + \varepsilon_- \otimes \eta_-^{(1)}(y^m) \\ \epsilon_0^2(y^m) &= \varepsilon_+ \otimes \eta_-^{(2)}(y^m) + \varepsilon_- \otimes \eta_+^{(2)}(y^m)\end{aligned}$$

where the subscripts label chirality, ε is a 4D spinor and $\eta^{(1)},\ \eta^{(2)}$ are 6D spinors on Y.

Also, we will need an ansatz for the DW projection condition:

$$\gamma_{\underline{r}}\varepsilon_+ = \mathrm{i}\alpha\varepsilon_-$$

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Now follow the story in Green, Schwarz and Witten 2:

$$\eta \to \eta^{(1)} \ \& \ \eta^{(2)}$$

The spinors $\eta^{(1)} \ \& \ \eta^{(2)}$ each locally define an SU(3) structure on *Y*.

Globally, the structure can be SU(3) $\eta^{(1)} = \eta^{(2)}$ or SU(2) $\eta^{(1)} \neq \eta^{(2)}$.

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$\delta\psi_m$ & $\delta\lambda$ then give conditions on the spinors on *Y*, or on the associated structures ~ dJ=0 and dΩ=0 in the Calabi-Yau case.

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Internal fluxes ⇒ $R_Y \neq 0$ and *Y* is a manifold with torsion: $\nabla_M \varepsilon \neq 0$

Generalised Geometry

Hitchin, Cavalcanti, Gualtieri, Witt, Graña et al 04-09

Generalised geometry provides a compact description of manifolds with torsion by considering structures on $T_Y \oplus T^*{}_Y$:

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Use the tensor product to define *pure spinors* on $T_Y \oplus T^*_Y$:

$$\Phi_\pm = \eta^{(1)}_+ \otimes \eta^{(2)\dagger}_\pm \qquad \leftrightarrow \qquad \textit{Fierz id.} \equiv \textit{Sums of even/odd forms}$$

purity means $\Phi_\pm$ is annihilated by a 6D subspace of Clifford(6,6).

c.f. the definition in Boels talk.

Pure spinor supersymmetry conditions

Using the tools of AdS4 compactifications, we can take the conditions on the η 's for supersymmetric domain walls found from $\delta\psi_m$ & $\delta\lambda$ and rewrite them in terms of pure spinors $\Phi_\pm$:

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\begin{aligned}
d_H\left[e^{2A-\phi}\mathrm{Im}\Phi_-\right] &= 0\ , \\
d_H\left[e^{4A-\phi}\mathrm{Re}\Phi_-\right] &= e^{4A}\tilde{\not{F}} - e^{-3V-G}\mathrm{Im}\left(\alpha^*\partial_r\left[e^{3A+3V-\phi}\Phi_+\right]\right), \\
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Radial flow terms reduce to the cosmological constant in the AdS limit

P. Smyth @ Zurich '09

Example: Domain walls and *Half-flat* SU(3) structure

Pure spinors $\Phi_+ = e^{iJ} \; , \qquad \Phi_- = \Omega \; , \qquad \left(\eta^{(1)} = \eta^{(2)}\right)$

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$$d\mathrm{Re}\Phi_+ \equiv -d(J \wedge J) = 0 \; ,$$
$$d\mathrm{Im}\Phi_- \equiv d(\mathrm{Im}\;\Omega) = 0 \; ,$$
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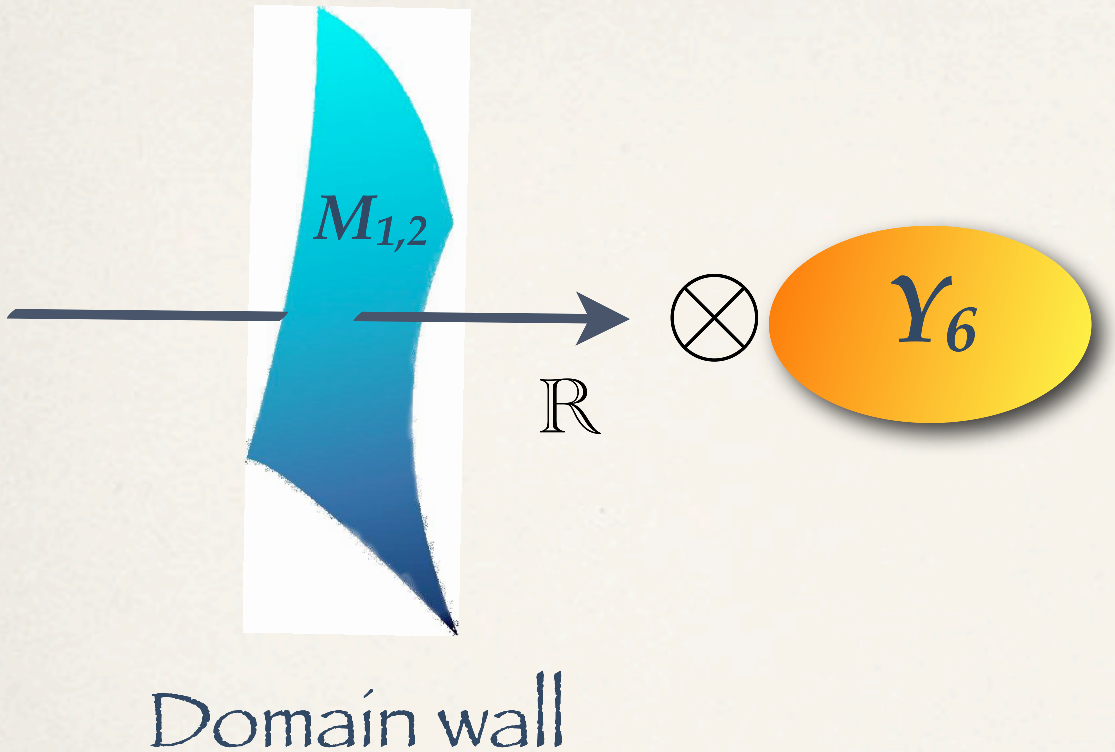
Defining 3 & 4 form in 7 dimensions (i.e. Y + dr) we find,

$$\rho = dr\wedge J + \mathrm{Im}(\Omega) \quad , \quad \star\rho = dr\wedge\mathrm{Re}(\Omega) + J\wedge J$$

$\mathrm{SU}(3)$ structure $\hookrightarrow \mathrm{G}_2$ holonomy : $d_7\rho = 0 = d_7(\star\rho)$

Gurrieri et al `02

From SU(3) structure to G_2 holonomy for domain walls



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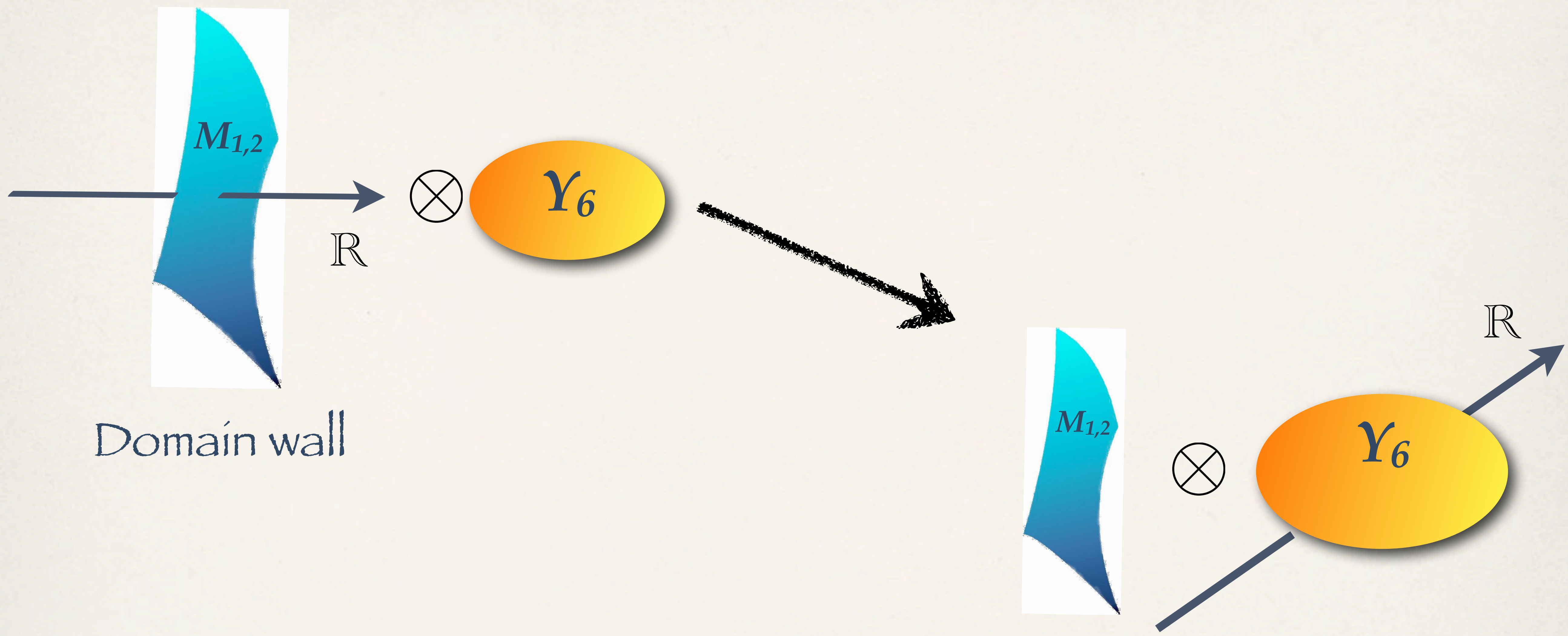

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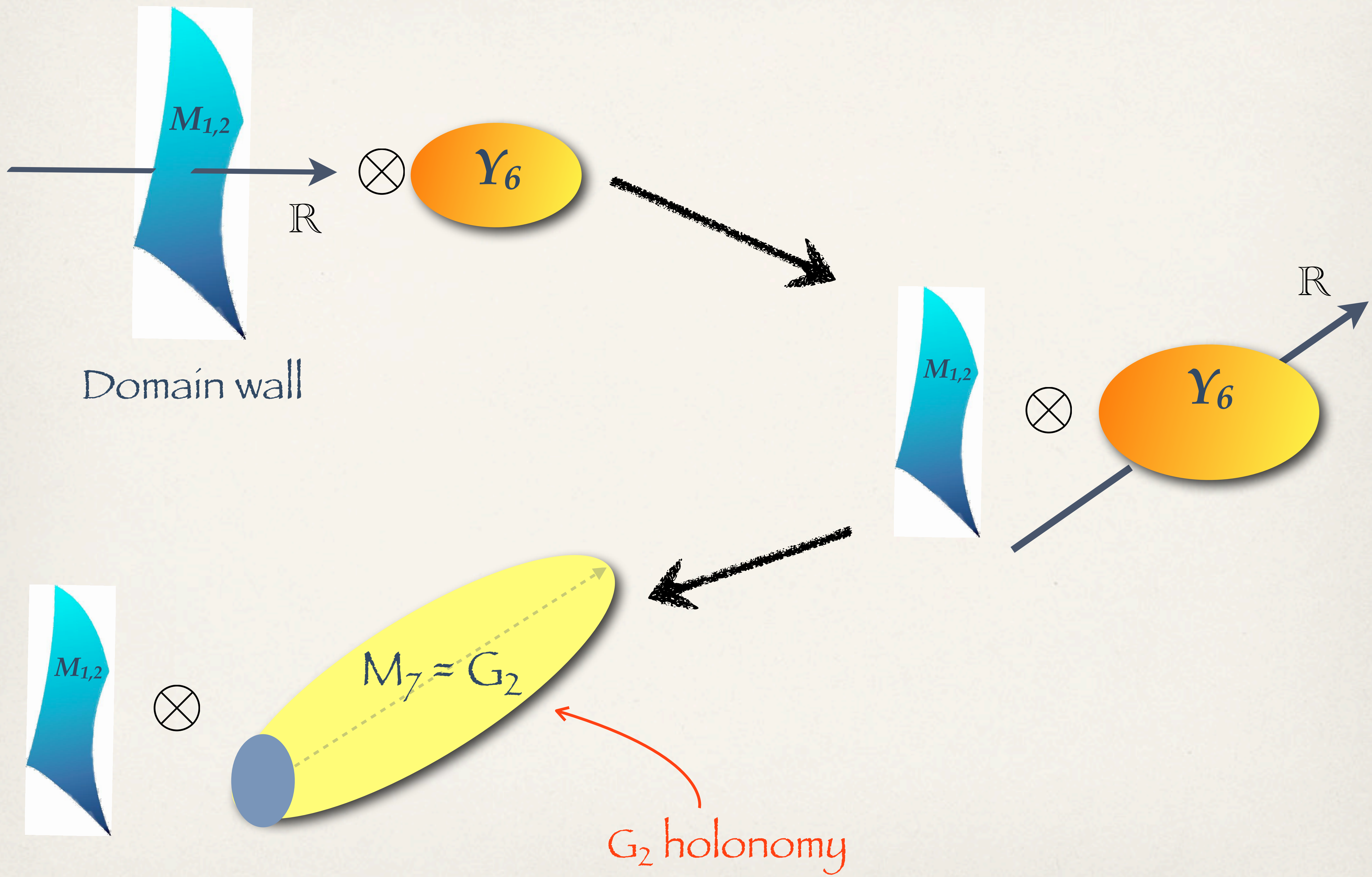

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Our 10D result for the SU(3) x SU(3) structure case:

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They describe the embedding of the SU(3) x SU(3) structure manifold into an *generalised G_2 manifold* i.e. *G_2 x G_2 structure*. Jeschek & Witt `05

A quick recap

- We used the 10D SUSY variations to derive a set of equations describing a domain wall in 4D.
- This approach obviously does not require any truncation of fields, therefore it will give a useful tool to compare with the truncated 4d results.
- The resulting `flow equations' describe the embedding of a SU(3) x SU(3) structure manifold into a generalised G_2 manifold.

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- The resulting `flow equations' describe the embedding of a SU(3) x SU(3) structure manifold into a generalised G_2 manifold.

Do they agree with the 4D gauged supergravity equations?

The 4D story

Now forget everything and start again...

We will consider N=2 supergravity in 4d, which can contain vector- and hyper-multiplets with scalar fields t^i and q^urespectively.

For particular gaugings, the N=2 theory is believed to match the proposed reduction of Type II supergravity on a SU(3) x SU(3) structure manifold. Graña, Louis & Waldram `05-`06

We will use the GLW dictionary between 4D and 10D fields to compare the compare the flow equations.

4D gauged supergravity

D'Auria...Vaula et al `03-`07

The supersymmetry variations are:

Gravitino: $$\delta\psi_{\mu A} = D_\mu \varepsilon_A + i S_{AB}\gamma_\mu \varepsilon^B = 0\ ,$$

Gaugino: $$\delta\lambda^{iA} = i\partial_\mu t^i \gamma^\mu \varepsilon^A + W^{iAB}\varepsilon_B = 0\ ,$$

Hyperino: $$\delta\zeta_{\hat\alpha} = i P_{uA\hat\alpha}\partial_\mu q^u \gamma^\mu \varepsilon^A + N^A_{\hat\alpha}\varepsilon_A = 0\ ,$$

where

$$S_{AB} = \frac{i}{2}\vec\sigma_{AB}\cdot\vec W\ ,$$

$$W^{iAB} = i\vec\sigma^{AB} g^{i\bar\jmath}\nabla_{\bar\jmath}\vec W\ ,$$

$$P^{v\hat\alpha}_{(A} N_{B)\hat\alpha} = i\vec\sigma^{AB} h^{vu}\nabla_u \vec W\ ,$$

$g^{i\bar\jmath}$ & h^{vu} are the vector- and hyperscalar σ-model metrics, $P_{uA\hat\alpha}$ is the quaternionic vielbein and $\vec W$ is a triplet of superpotentials.

Domain wall ansatz again...

Now we play the same game, plugging a domain wall ansatz into the SUSY variations:

$$ds_4^2 = e^{2U(r)}\eta_{\alpha\beta}dx^\alpha dx^\beta + e^{-2pU(r)}dr^2 \ ,$$

$$t^i(x^\alpha, r) = t^i(r) \qquad\qquad q^u(x^\alpha, r) = q^u(r)$$

Using some standards tricks, we find:

$$\partial_r t^i = \mp e^{-pU} g^{i\bar{\jmath}}\,\nabla_{\bar{\jmath}}\bar{W} \ ,$$
$$\partial_r q^u = \mp e^{-pU} h^{uv}\,\partial_v\bar{W} \ ,$$
$$U' = \pm e^{-pU} W \ ,$$

How can do we compare this to the 10D result?

The GLW dictionary (IIA) - $A(y,r) = 0$

Graña, Louis & Waldram `05-`06

We need to express the 4D quantities in terms of $\Phi_\pm$ and $\tilde{\not{F}}$:

hyper: q^u → $z^a \longrightarrow \Phi^0_-$

q^u → $\xi^A_{RR} \longrightarrow \tilde{\not{F}}$

vector: $t^i \longrightarrow \Phi^0_+$

$$\Phi^0_\pm \equiv \Phi_\pm\big|_{\mathrm{U}}$$

→ Truncated, finite-dim special Kähler subspace U of pure spinors

Similarly, there is an expression for the superpotential $\vec{W}$ in terms of $\tilde{\not{F}}$ and $d_H\Phi^0_\pm$ which we can apply on the RHS of our flow equations.

Flow equations for domain walls in 4D

After some work we find more complicated expressions,

$$
\begin{aligned}
d_H \mathrm{Re}(e^{-\phi} h \Phi^0_+) &= e^{-\varphi + pU - 2(U+\varphi)} \partial_r \mathrm{Im}(e^{2(U+\varphi)} e^{-\phi} \Phi^0_-) \ , \\
d_H \mathrm{Re}(e^{-\phi} \Phi^0_-) &= \not{\tilde{F}} - e^{-\varphi + pU - (U+\varphi)} \partial_r \mathrm{Im}(e^{U+\varphi} e^{-\phi} h \Phi^0_+), \\
d_H \mathrm{Im}(e^{-\phi} \Phi^0_-) &= 0 \ , \\
d_H \mathrm{Im}(e^{-\phi} h \Phi^0_+) &= 0 \ .
\end{aligned}
$$

where we have also made use of the relation between the 4D and 10D dilatons,

$$\phi \propto \varphi - \frac{1}{2} K_+$$

Now we can try to match metrics, 4D spinors, etc

Comparing with our result from 10D

- *Warp factor* $A(y,r)=0$ - KK reduction is better understood.

- Then we need to take care of the 4d spinors and match the metric.

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The results agree

- We find a precise agreement between the (truncated) 10D result and the pure 4D supergravity result.

- Examples? From the 4D perspective, we were able to find domain walls with more general NS charges (R.-R.=0).

Conclusions

What is this really good for?

- Our results provide a non-trivial check of the generalised compactification procedure proposed by GLW.

- The SUSY variations for domain wall vacua produce *generalised Hitchin flow equations,* describing the embedding of a SU(3) x SU(3) structure manifold into a generalised G_2 manifold.

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Some further work

- More general examples and the relation to the *N=1* story.

- What about other vacua? (PS, J. Louis and H. Triendl - coming soon)