Refined bound state indices for D-particles ArXiv:0909.0508 with Thomas Wyder

Walter Van Herck

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Outline



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- Elliptic genus and polar states
- (Split) flow trees
- Bound states and DT invariants
- 3 Index refinement scheme
 - Case study 1: the sextic
 - Case study 2: the decantic

4 Summary

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D-particles as toy models for black holes

- Interest in multi-centered BPS black holes: attractor mechanism, split flows, OSV, entropy enigma, scaling solutions,...
- Split attractor flow tree conjecture (Denef, Moore 2007)
- Refinement of indices enumerating BPS microstates in IIA on CY₃
- The chromosomes of D-particles: polar states

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Index refinement scheme

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Elliptic genus and polar states

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1 Motivation

2 Setup

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Summary

Elliptic genus and polar states

D-particles/black holes

D-branes wrapped on cycles of CY in type IIA One modulus: H²*(X) = H⁰(X) ⊕ H²(X) ⊕ H⁴(X) ⊕ H⁶(X) Brane system with charge (p⁰, p, q, q₀): Γ = p⁰ + pH + ^q/_H H² + ^{g₀}/_H H³, with H := ∫_X H³ ⟨Γ₁, Γ₂⟩ := q_{0,1}p⁰₂ - q₁p₂ + p₁q₂ - p⁰₁q_{0,2}

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Summary

Elliptic genus and polar states

D-particles/black holes

- D-branes wrapped on cycles of CY in type IIA
- One modulus: $H^{2*}(X) = H^0(X) \oplus H^2(X) \oplus H^4(X) \oplus H^6(X)$
- Brane system with charge (p^0, p, q, q_0) : $\Gamma = p^0 + pH + \frac{q}{H}H^2 + \frac{q_0}{H}H^3$, with $\mathcal{H} := \int_X H$ $\langle \Gamma_1, \Gamma_2 \rangle := q_{0,1}p_2^0 - q_1p_2 + p_1q_2 - p_1^0q_{0,2}$

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Summary

Elliptic genus and polar states

Elliptic genus

Mixed ensembles of D4–D2–D0 branes with fixed magnetic charge H and variable (q, q₀)

Modified elliptic genus: BPS indices of (0,4) MSW CFT

$$Z(q,\bar{q},y) = \operatorname{Tr}_{R}\left(\frac{1}{2}F^{2}(-1)^{F}q^{L_{0}-\frac{c_{L}}{24}}\bar{q}^{\bar{L}_{0}-\frac{c_{R}}{24}}e^{2\pi i y^{A}q_{A}}\right)$$

$$Z(q, \bar{q}, y) = \sum_{\gamma} Z_{\gamma}(q) \Theta_{\gamma}(q, \bar{q}, y), \quad F = \frac{H}{2} + f + \gamma$$

$$Z_\gamma(q) = q^{-lpha}(\# + \# \, q + \# \, q^2 + \ldots), \quad \hat{q}_0 \equiv q_0 - rac{1}{12} D^{AB} q_A q_B$$

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Summary

Elliptic genus and polar states



Z (q, \bar{q}, y) is a weak Jacobi form with weight $\left(-\frac{3}{2}, \frac{1}{2}\right)$

Farey tail expansion and polar states

In our setup, these are D6 – D6 bound states giving rise to a factorization for the index

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Summary

Elliptic genus and polar states



Z(q, q
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 Index refinement scheme

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(Split) flow trees





2 Setup

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Refined bound state indices for D-particles



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Summary

(Split) flow trees



BPS equations for static, spherically symmetric solutions

Central charge flows to a fixed point $(|Z| \rightarrow minimum)$

This gives rise to a single flow in moduli space

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Refined bound state indices for D-particles

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(Split) flow trees



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Summary

(Split) flow trees

Attractor mechanism

- BPS equations for static, spherically symmetric solutions
- Central charge flows to a fixed point ($|Z| \rightarrow \text{minimum}$)
- This gives rise to a single flow in moduli space

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Motivation	Setup ○○○○ ○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○○	Index refinement scheme ooooooooooooooooooooooooooooooooooo	Summary
(Split) flow trees			
Single flow			

In moduli space this looks like:



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Summary

(Split) flow trees

Attractor points

We distinguish three cases:

- |Z| attains a minimum \neq 0: there is a single flow
- |Z| reaches 0 at a regular point in moduli space (crash point): there is no single flow
- |Z| reaches 0 at a singular or boundary point of moduli space: further information is necessary

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Summary

(Split) flow trees

Multicenter solutions

Take a more general (stationary instead of static) ansatz

- The total charge Γ could also split into two charges $\Gamma = \Gamma_1 + \Gamma_2$
- This splitting is marginally stable when $|Z| = |Z_1| + |Z_2|$ or arg $Z = \arg Z_1 = \arg Z_2$
- Stability condition: $\langle \Gamma_1, \Gamma_2 \rangle (\alpha_1 \alpha_2) > 0$

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Refined bound state indices for D-particles



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Summary

(Split) flow trees

Split attractor flow conjecture

Conjecture by Denef, Moore (2007)

- Weak form: Single or split flows are an existence criterium for BPS states in supergravity and their number is finite.
- Strong form: Single or split flows are an existence criterium for BPS states in the full string theory and their number is finite.

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Bound states and DT invariants





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Summary

Bound states and DT invariants

Bound states of $D6 - \overline{D6}$

■ D-particles with low charge $(0, 1, q, q_0)$ $\rightarrow D6 - \overline{D6}$ bound state

Index splits up into a sum of split flows

$$\Omega(\Gamma) = \sum_{\Gamma \to \Gamma_1 + \Gamma_2} (-1)^{|\langle \Gamma_1, \Gamma_2 \rangle| - 1} |\langle \Gamma_1, \Gamma_2 \rangle| \, \Omega(\Gamma_1) \, \Omega(\Gamma_2)$$

 $\Delta\Omega(\Gamma_{D4}) = (-1)^{|\langle \Gamma_{D6}, \Gamma_{\overline{D6}} \rangle| - 1} |\langle \Gamma_{D6}, \Gamma_{\overline{D6}} \rangle| N_{\rm DT}(\beta_1, n_1) N_{\rm DT}(\beta_2, n_2)$

$$\Gamma_{D6} = e^{F_1} \left(1 - \beta_1 - \left(\frac{1}{2} \chi(C_{\beta_1}) + N_1 \right) \omega \right) \left(1 + \frac{c_2(X)}{24} \right)$$

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 Index refinement scheme

Summary

Bound states and DT invariants

Bound states of $D6 - \overline{D6}$

- **D**-particles with low charge $(0, 1, q, q_0)$
 - $\rightarrow D6 \overline{D6}$ bound state
- Index splits up into a sum of split flows

$$\Omega(\Gamma) = \sum_{\Gamma \to \Gamma_1 + \Gamma_2} (-1)^{|\langle \Gamma_1, \Gamma_2 \rangle| - 1} |\langle \Gamma_1, \Gamma_2 \rangle| \, \Omega(\Gamma_1) \, \Omega(\Gamma_2)$$

 $\Delta\Omega(\Gamma_{D4}) = (-1)^{|\langle \Gamma_{D6}, \Gamma_{\overline{D6}} \rangle| - 1} |\langle \Gamma_{D6}, \Gamma_{\overline{D6}} \rangle| \, \textit{N}_{\rm DT}(\beta_1, n_1) \, \textit{N}_{\rm DT}(\beta_2, n_2)$

$$\Gamma_{D6} = e^{F_1} \left(1 - \beta_1 - \left(\frac{1}{2} \chi(C_{\beta_1}) + N_1 \right) \omega \right) \left(1 + \frac{c_2(X)}{24} \right)$$

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Motivation	Setup ○○○○ ○○○○○○○ ○○●○	Index refinement scheme ooooooooooooooooooooooooooooooooooo	Summary
Bound states and DT invariants			
Refinement (1)			

The tachyon index jumps between different D6 – D6 microstates

- Tachyon field: $T \in \Gamma(F_2^* \otimes F_1)$
- In our case $T \in \Gamma(H)$
- **Riemann–Roch:** $\mathcal{I}(\mathcal{M}_T) = \langle \Gamma_1, \Gamma_2 \rangle$

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Index refinement scheme

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Bound states and DT invariants



• Consider for example CY in \mathbb{WP}^4_{11125}

- Tachyon field: $T = a_1 x_1 + a_2 x_2 + a_3 x_3$
- **T** has to vanish on bound $\overline{D0}$'s
- In general, \mathcal{M}_T reduces from \mathbb{CP}^2 to \mathbb{CP}^1 (for one bound $\overline{D0}$)
- What about a $\overline{D0}$ at $x_1 = x_2 = x_3 = 0$?

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Image: A matrix

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Case study 1: the sextic





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- Elliptic genus and polar states
- (Split) flow trees
- Bound states and DT invariants
- 3 Index refinement scheme
 - Case study 1: the sextic
 - Case study 2: the decantic

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Case study 1: the sextic

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Our Calabi-Yau: The sextic

The sextic is a degree 6 hypersurface in \mathbb{WP}^4_{11112} :

- Homogeneous coördinates x_1, x_2, x_3, x_4, x_5 with weights (1, 1, 1, 1, 2)
- General transversal polynomial of degree 6:

$$P(x) = x_5^3 + x_5 f^{(4)} + f^{(6)}$$

where the $f^{(i)}$ denotes a homogeneous polynomial of degree *i* in x_1, x_2, x_3, x_4

• Euler number of CY: $\chi = -204$

One Kähler modulus: basiselement $H \in H^2(X, \mathbb{Z})$

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Refined bound state indices for D-particles

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Summary

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Case study 1: the sextic

Elliptic genus: factorization

■
$$\int_X H^3 = 3$$
 → there are 2 gluing vectors
= $Z(q, \bar{q}, y) = \sum_{\gamma=0}^2 Z_{\gamma}(q) \Theta_{\gamma}(\bar{q}, y)$

• We will focus on $Z_0(q)$ from here on.

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Case study 1: the sextic

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Summary

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Elliptic genus: most polar state

The most polar state: pure D4-brane

 $q = H \cdot F, \quad q_0 = \frac{F^2}{2} + \frac{c_2(P)}{24} - N, \quad \hat{q}_0 \equiv q_0 - \frac{1}{12} D^{AB} q_A q_B$

■ → pure D4 charge: $(0, 1, \frac{3}{2}, \frac{9}{4})$

• $\hat{q}_0 = \frac{45}{24}$ so 2 polar states

• Only one split flow into $D6_H$ and $\overline{D6}$ with $\langle \Gamma_1, \Gamma_2 \rangle = -4$

This gives

$$\Omega = (-1)^{\langle \Gamma_1, \Gamma_2 \rangle - 1} |\langle \Gamma_1, \Gamma_2 \rangle | N_{DT}(0, 0) \cdot N_{DT}(0, 0)$$

= $-4 \cdot 1 \cdot 1 = -4$

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Motivation	Setup 0000 0000000 0000	Index refinement scheme oooo●oooooo oo	Summary
Case study 1: the sextic			
Elliptic genus:	$Z_0(q)$		

In the same way we find for pure D4 + 1 D0 with \$\hat{q}_0 = \frac{21}{24}\$
 Only one split flow into D6_H and D6 - D0 and

$$\Omega = (-1)^{\langle \Gamma_1, \Gamma_2 \rangle - 1} |\langle \Gamma_1, \Gamma_2 \rangle | N_{DT}(0, 0) \cdot N_{DT}(0, 1)$$

= 3 \cdot 1 \cdot 204 = 612

These polar states completely determine Z_0 : $Z_0(q) =$ $q^{-\frac{45}{24}} (-4 + 612q - 40'392q^2 + 146'464'860q^3 + \cdots)$

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Summary

Case study 1: the sextic

The first non-polar state

For D4 + 2 $\overline{D0}$ with $\hat{q}_0 = \frac{-3}{24}$

Still only one split flow into $D6_H$ and $\overline{D6} - 2\overline{D0}$

The naive index would be

$$\Omega = (-1)^{\langle \Gamma_1, \Gamma_2 \rangle - 1} |\langle \Gamma_1, \Gamma_2 \rangle| N_{DT}(0, 0) \cdot N_{DT}(0, 2)$$

= -2 \cdot 1 \cdot 20'298 = -40'596

$$-40'596 \neq -40'392!$$

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Case study 1: the sextic

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Summary

Refining: a blind tachyon...

• We had: $T = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$

This field does not 'see' x_5 coordinates of the $\overline{D0}$'s

Number of constraints of the two D0's :

$$\operatorname{rank}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{pmatrix}$$

Special locus: $x_i = y_i$, i = 1, ..., 4 and $x_5 \neq y_5$ with $f = x_5^3 + f^{(6)} \rightarrow 2(\chi - \chi_0)\frac{1}{2}$ where χ_0 is the Euler characteristic of the locus $x_5 = 0$

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Refined bound state indices for D-particles

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Special locus: $x_i = y_i$, i = 1, ..., 4 and $x_5 \neq y_5$ with $f = x_5^3 + f^{(6)} \rightarrow 2(\chi - \chi_0)\frac{1}{2}$ where χ_0 is the Euler characteristic of the locus $x_5 = 0$

Walter Van Herck

Refined bound state indices for D-particles

K.U.Leuven

Case study 1: the sextic

Setup 0000 0000000 Index refinement scheme

Summary

K.U.Leuven

Refining: a blind tachyon...

- We had: $T = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$
- This field does not 'see' x_5 coordinates of the $\overline{D0}$'s
- Number of constraints of the two $\overline{D0}$'s :

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Summary

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Refining: a hidden blowup...

- If both $\overline{D0}$'s coincide, a blowup procedure is needed
- We replace the coincidence locus with the tangent directions (CP²)

Number of constraints of the overlapping $\overline{D0}$'s :

$$\operatorname{rank}\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ X^1 & X^2 & X^3 & X^4 \end{pmatrix}$$

Special blowup direction: $X^i = \lambda x_i$, i = 1, ..., 4 $\nabla_X f = 3X^5 x_5^2 + 6\lambda f^{(6)} = 3x_5^2 (X^5 - 2\lambda x_5) = 0$

• Only solution when $x_5 = 0$ and only one direction, so special blowup: $\chi_0 \cdot 1$

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Case study 1: the sextic

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Summary

K.U.Leuven

Refining: the result

The special locus is the sum: $2 \cdot \frac{1}{2}(\chi - \chi_0) + \chi_0 = \chi = -204$

We now have:
$$\mathcal{N}_{DT}^{(s)}(0,2) = -204$$
 and
 $\mathcal{N}_{DT}^{(g)}(0,2) = N_{DT}(0,2) - \mathcal{N}_{DT}^{(s)}(0,2) = 20'298 + 204 = 20'502$

■ And the index becomes -2 · 20′502 - 3 · (-204) = -40′392

Exact confirmation of the modular prediction!

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Case study 1: the sextic

Setup 0000 0000000 0000 Summary

Confirmation of the refinement

Sextic:

$$Z_{0}(q) = q^{-\frac{45}{24}}(-4+612q-40'392q^{2}+146'464'860q^{3}+...)$$
Octic:

$$Z_{0}(q) = q^{-\frac{23}{12}}(-4+888q-86'140q^{2}+131'940'136q^{3}+...)$$
Octic:

$$Z_{1}(q) = q^{-\frac{7}{6}}(-59'008q+8'615'168q^{2}+...)$$

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Case study 1: the sextic

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Summary

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Case study 2: the decantic





2 Setu

- Elliptic genus and polar states
- (Split) flow trees
- Bound states and DT invariants
- 3 Index refinement scheme
 - Case study 1: the sextic
 - Case study 2: the decantic

4 Summary

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Index refinement scheme

Summary

Case study 2: the decantic

New elliptic genus

Decantic: degree 10 hypersurface in WP⁴₁₁₁₂₅, $\int_X H^3 = 1$ Pure D4, $\hat{q}_0 = \frac{35}{24}$: $\Omega = 3$ Plus one $\overline{D0}$, $\hat{q}_0 = \frac{11}{24}$: naively -576
Special locus: $x_1 = x_2 = x_3 = 0$, $\chi_0 = 1 = -\mathcal{N}_{DT}^{(s)}(0, 1)$ $\Omega = -2 \cdot (N_{DT}(0, 1) - (-1)) - 3 \cdot (-1) = -575$ $Z_0(q) = q^{-\frac{35}{24}}(3 - 575q + 271'955q^2 + ...)$

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Index refinement scheme

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Index refinement scheme

Summary

Case study 2: the decantic

New elliptic genus

Decantic: degree 10 hypersurface in \mathbb{WP}_{11125}^4 , $\int_X H^3 = 1$ Pure D4, $\hat{q}_0 = \frac{35}{24}$: $\Omega = 3$ Plus one $\overline{D0}$, $\hat{q}_0 = \frac{11}{24}$: naively -576
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Index refinement scheme

Summary

Case study 2: the decantic

New elliptic genus

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Index refinement scheme

Summary

Case study 2: the decantic

New elliptic genus

- Decantic: degree 10 hypersurface in \mathbb{WP}^4_{11125} , $\int_X H^3 = 1$ Pure D4, $\hat{q}_0 = \frac{35}{24}$: $\Omega = 3$
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Refined bound state indices for D-particles



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Confirmation of the refined computational scheme

- New predictions for elliptic genera
- Strong evidence for split attractor flow conjecture
- New invariants: Donaldson–Thomas partitions

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Find confirmation of the new elliptic genus for the decantic

- Elliptic genera for higher class divisors, orientifolds, $\ldots \rightarrow$ generalization
- Implication for OSV conjecture
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Summary



Thank you!

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