(Supersymmetric) Godel Space from Wrapped M2 Branes

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in collaboration with:

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Overview

- Introduction
- Our system
  - Reduction of 11d sugra to 3d effective action
- Solutions:
  - Anti-de Sitter space
  - Godel space
  - (Super)symmetries
- Superglue
  - Gluing Godel space to anti de Sitter space
- Conclusions and outlook

Introduction

Introduction/Motivation

- **Problem**:
  - $AdS_3 \times S^2 \times CY_3$ background + probe M2 wrapping S^2 Backreaction?
- **Original motivation**: Black Hole entropy/microstates
  - Constituent counting: D-branes/CFT states
  - Supergravity regime?
    - Fuzzball proposal
    - Gaiotto-Strominger-Yin (2005), GSY + Denef-Van den Bleeken (2007):

(1) (2)

Type IIA on CY gives N=2 supergravity in 4d

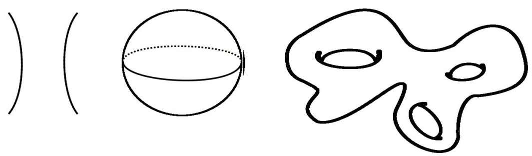
D0-D4 black hole split as D0-D4 background and probe D0, counting agrees with entropy

Introduction/Motivation

1) Gaiotto-Strominger-Yin:

- D0-D4 BH: (background core D0-D4) + (probe D0s)
- Background near-horizon:

$$AdS_2 \times S^2 \times CY_3$$



- AdS superconformal quantum mechanics
- Large # (non-abelian) D0s: puff up to D2s (conjecture)

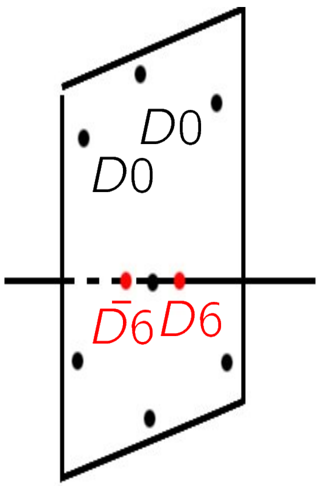
} Counting ground states agrees with entropy

Introduction/Motivation

- GSY+Denef-Van den Bleeken:
  - D0-D4 BH: background core D0-D4 + probe D0s

    $D6 - \bar{D}6$ purely fluxed
  - Two interesting regions in scaling limit

    Consider 5d/11d interpretation:
    - FAR region: quotient of $AdS_3 \times S^2$ (not surprising, D0-D4 BH near horizon)
    - NEAR region: global $AdS_3 \times S^2$ (surprising)



  - Entropy?
    - Start in near region (take D0 charge to infinity)
    - Bring in D0 charge as D2/M2 branes (non-abelian degrees of freedom)
    - M2's wrap near horizon S^2: spinning particles in AdS_3
    - Counting ok in leading order à la GSY

- ISSUES:
  - Myers effect?
  - Backreaction of probe branes on S^2

    Either solves black hole problems or is interesting new solution

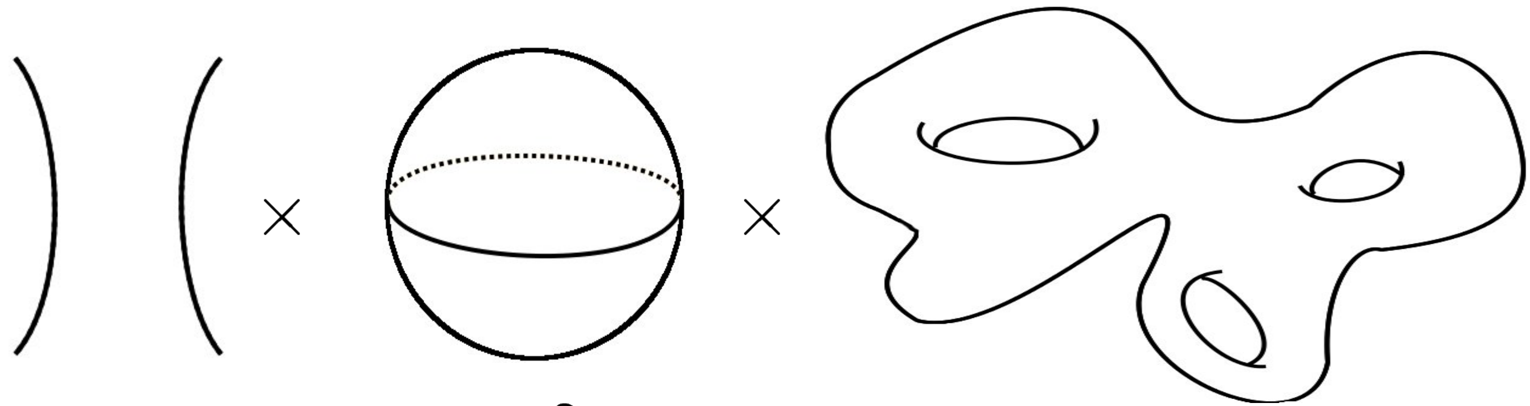
Our System

Our System

From M2 probes to an ansatz for the backreaction

- Backreaction of M2 branes on S^2 ? 11 dimensional picture

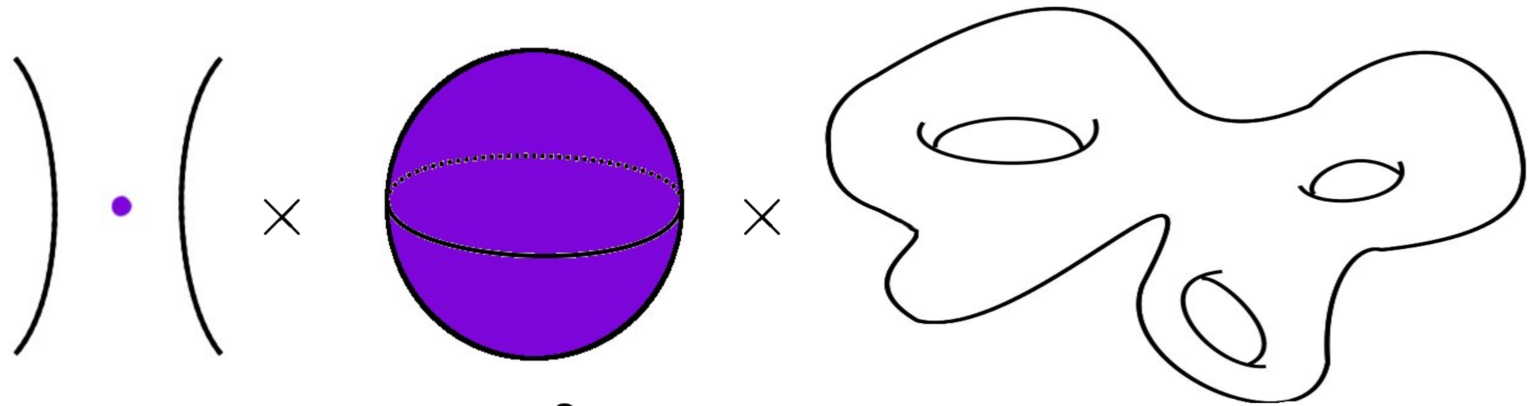
D6-anti D6
near horizon:



$$AdS_3 \times S^2 \times CY_3$$

- Backreaction of M2 branes on S^2 ? 11 dimensional picture

M2 brane probes
(Susy when rotating with constant angular velocity in global AdS_3, see later)



$AdS_3 \times S^2 \times CY_3$

Our System

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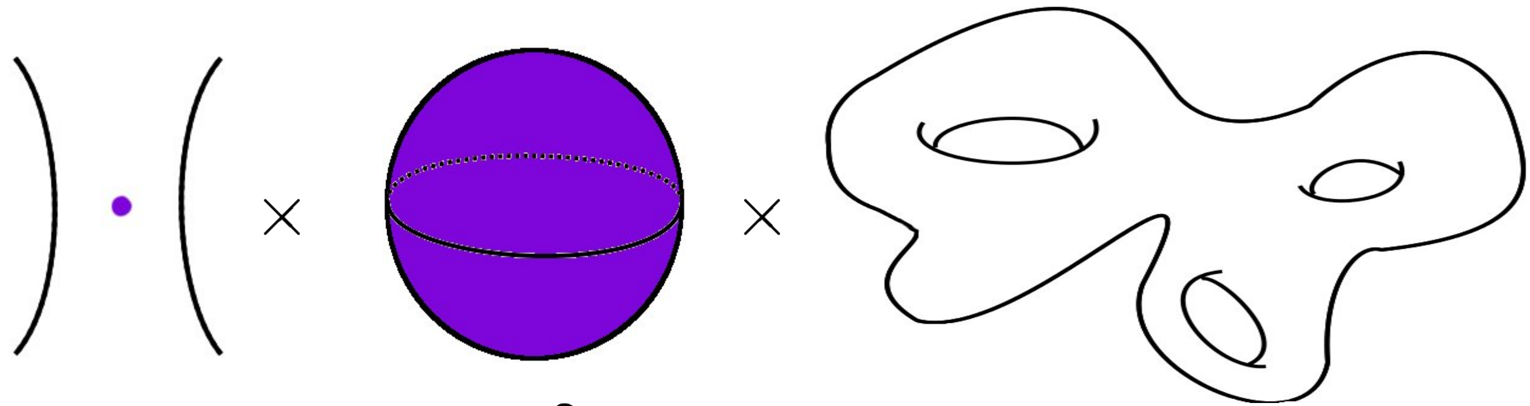
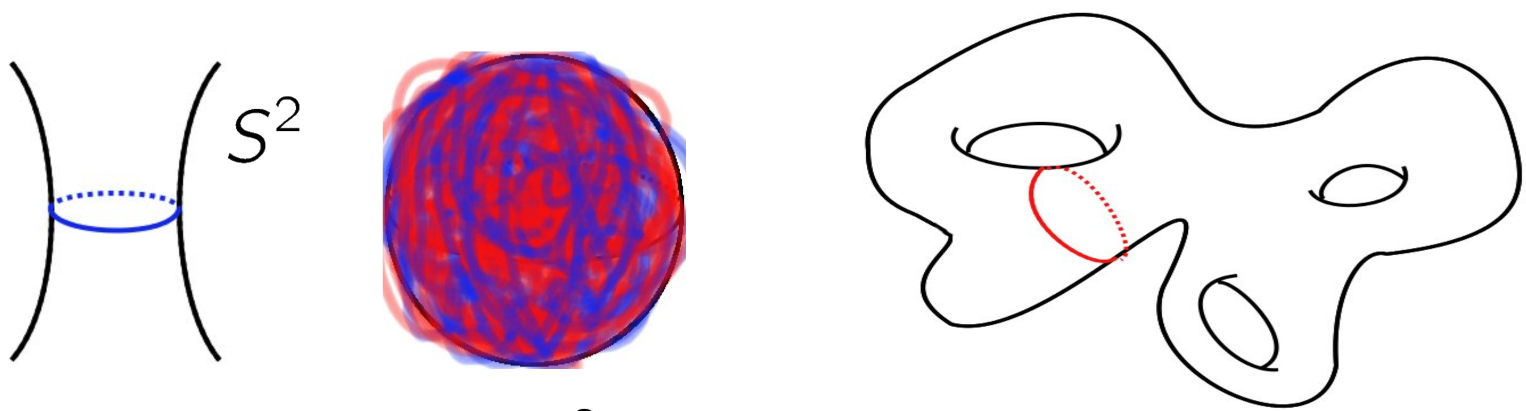


$$AdS_3 \times S^2 \times CY_3$$

Backreacted system:
(ASSUME...)



g_{11} → M_3 $(\Lambda < 0)$

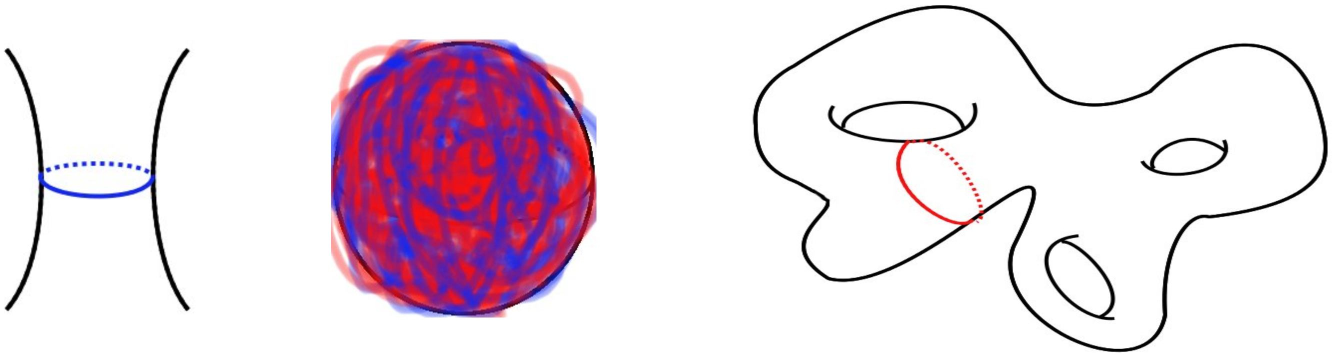
$F^{(4)}$ Red and blue cycles (on S^2)

$$M_3 \times S^2 \times CY_3$$

Our System

Specifying our ansatz in 11 dimensions

- Backreaction of M2 branes on S^2?



$M_3 \times S^2 \times CY_3$

  - Details:

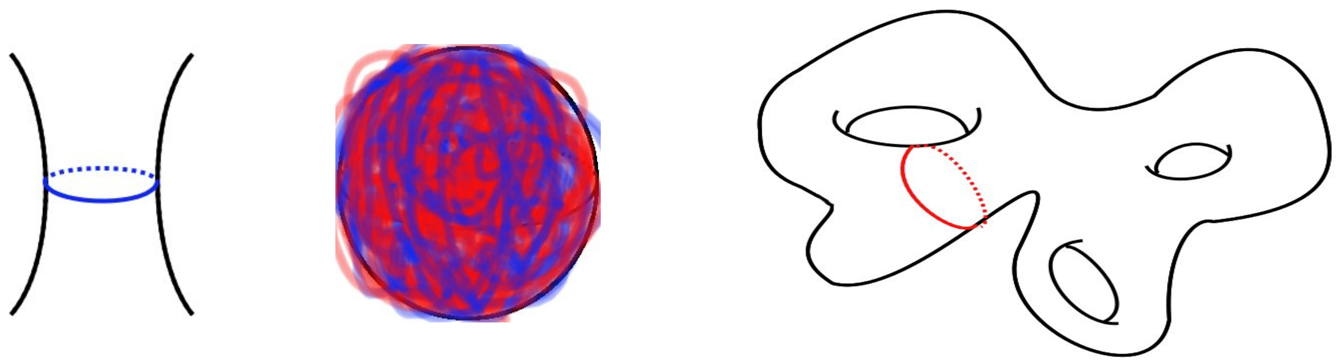
$$ds_{11}^2 = \frac{1}{\tau_2^{2/3}}(ds_3^2 + \frac{l^2}{4} d\Omega_2^2)) + ds^2(CY_3)$$

$$F^{(4)} = p^A D_A \wedge dvol_{S^2} + \frac{1}{\tau_2^2} *_3 d\tau_1 \wedge dvol_{S^2}$$

$\tau_2 = CY_3$ volume

$$d\tau_1 = \frac{1}{\tau_2^2} *_5 dA^{(3)} = \frac{1}{\tau_2^2} \star_3 dA$$

- Backreaction of M2 branes on S^2?



$$M_3 \times S^2 \times CY_3$$

$$\tau_1, \tau_2$$

  - Details:

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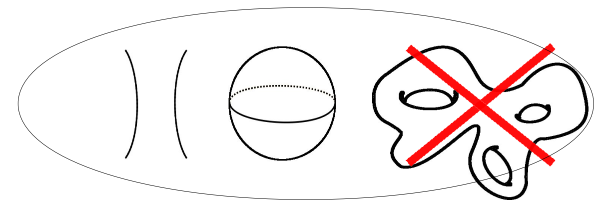
g_{11}, $F^{(4)}$ → (CY_3 Redux) → 5d → (S^2 Redux) → 3d with $\Lambda < 0$ + two scalars τ_1, τ_2

- Reduced to 5 dimensions over CY_3
  - Gives five dimensional N=1 supergravity with vector multiplets and one hypermultiplet:



$$S_5 = \int dx^5 \sqrt{-g}\left(R - \frac{1}{2}\frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2}\right) - \frac{1}{2}\int G_{AB} F^A \wedge \star F^B + \frac{D_{ABC}}{6}\int A^A \wedge F^B \wedge F^C$$

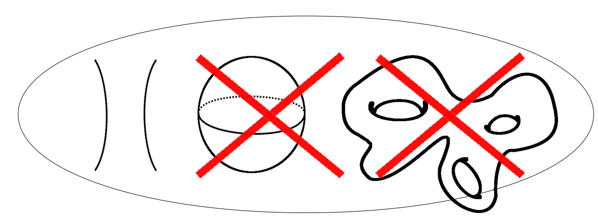
$$F^A = p^A dvol_{S^2}$$

$\tau = \tau_1 + i\tau_2$ complex scalar in universal hypermultiplet

$\sim \star_5 dA^3$

CY volume

- Reduced to 3 dimensions over S^2



$$S_3 = \int dx^5 \sqrt{-g}\left(R + \frac{2}{\ell^2} - (\mu - 1)\frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2}\right)$$

  - $\mu = 1$: no backreaction, AdS $\quad \mu = \frac{3}{2}$: backreaction = ???
  - Constant sphere radius? None of the other fields couple to this modulus! General for codimension 2 branes

- Action and equations of motion in 3d:

$$S_3 = \int dx^3 \sqrt{-g}\left(R + \frac{2}{\ell^2} - (\mu - 1)\frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2}\right)$$

> $\mu = 1$: no backreaction, AdS
>
> $\mu = \frac{3}{2}$: backreaction = ???

  - Einstein eqns. $R_{\alpha\beta} + \frac{2}{\ell^2} g_{\alpha\beta} = (\mu - 1)\frac{\partial_{(\alpha}\tau\partial_{\beta)}\bar{\tau}}{\tau_2^2}$
  - Scalar field eqn. $\partial_\alpha\left(\sqrt{-g}g^{\alpha\beta}\partial_\beta\tau\right) + i\sqrt{-g}g^{\alpha\beta}\frac{\partial_\alpha\tau\partial_\beta\tau}{\tau_2} = 0$
  - We search for stationary solutions

- Inspiration: codimension 2 branes are special!
  - Greene, Shapere, Vafa, Yau (1990) (Stringy cosmic strings)
  - Gibbons, Green, Perry (1995) (D7 branes)

  } $\ell = \infty$ FLAT SPACE!

  - 3D part of metric:

$$-dt^2 + \underbrace{dx^2 + dy^2}_{dz d\bar{z}} \rightarrow \tau(z)$$

$\Rightarrow$ Remains valid when spatial part of $\sqrt{-g}g^{\alpha\beta}$ is "flat"

Flat space: $-dt^2 + e^{2\phi(z,\bar{z})}dz d\bar{z}$

Our case: $-(dt + \chi)^2 + e^{2\phi(z,\bar{z})}dz d\bar{z}$

Our System

What does our ansatz give us?

- Let's put everything together:
  - Wrapped branes on the sphere in $M_3 \times S^2 \times CY_3$
  - Leads to three-dimensional action:

$$S_3 = \int dx^3 \sqrt{-g} \left(R + \frac{2}{\ell^2} - (\mu - 1) \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{\tau_2^2} \right)$$

  - Ansatz:

$$ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z} \qquad \tau(z)$$

$$\chi = \chi(z,\bar{z}) dz + \chi_{\bar{z}}(z,\bar{z}) d\bar{z}$$

Gauge freedom:

$$z \to f(z)$$

$$t \to t + f(z,\bar{z})$$

  - Einstein eqns.:

$$d\chi = \frac{i e^{2\phi} dz \wedge d\bar{z}}{2}$$

$$\partial\bar{\partial}\phi - \frac{e^{2\phi}}{4} = -(\mu - 1) \frac{\partial\tau\bar{\partial}\bar{\tau}}{4\tau_2^2}$$

  - We will focus on eqn. for ϕ: sourced Liouville equation

$\mu = \frac{3}{2}$ Our case, we will show this is timelike stretched AdS_3

$\mu = 1$ AdS_3

Our System

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$\mu = 1$ AdS_3

Take μ arbitrary

Solutions

- Solving for general value of μ
  - Ansatz and Einstein equations:

$$\begin{cases} ds_3^2 = -(dt+\chi)^2 + e^{2\phi(z,\bar{z})}dzd\bar{z} \\ \tau(z) \end{cases} \qquad \begin{cases} d\chi = \dfrac{ie^{2\phi}\, dz\wedge d\bar{z}}{2} \\ \partial\bar{\partial}\phi - \dfrac{e^{2\phi}}{4} = -(\mu-1)\dfrac{\partial\tau\bar{\partial}\bar{\tau}}{4\tau_2^2} \end{cases}$$

  - Specifics:
    - Spatial base has topology of a disk (UHP) (cf. *AdS*)
    - Imaginary part of τ positive: lives on UHP
    - Possible poles of τ only on the boundary of space

      meromorphic function from disk/UHP to UHP

- Solving for general value of μ
  - Ansatz and Einstein equations:

$$\begin{cases} ds_3^2 = -(dt+\chi)^2 + e^{2\phi(z,\bar z)} dz d\bar z \\ \tau(z) \end{cases} \qquad \begin{cases} d\chi = \dfrac{i e^{2\phi}\, dz\wedge d\bar z}{2} \\ \partial\bar\partial\phi - \dfrac{e^{2\phi}}{4} = -(\mu-1)\dfrac{\partial\tau\bar\partial\bar\tau}{4\tau_2^2} \end{cases}$$

  - Specifics:
    - Spatial base has topology of a disk (UHP) (cf. *AdS*)
    - Imaginary part of τ positive: lives on UHP
    - Possible poles of τ only on the boundary of space

      meromorphic function from disk/UHP to UHP
  - Build solutions:

$$\chi = 2\mathrm{Im}\,(\partial\phi + (1-\mu)\partial\ln\tau_2) + df$$

→ Notice earlier gauge freedom!

$$\mathcal{D}e^{2\phi} = -(\mu-1)\frac{\partial\tau\bar\partial\bar\tau}{2\tau_2^2} = \mathcal{D}\left(\mu\frac{\partial\tau\bar\partial\bar\tau}{\tau_2^2}\right)$$

  - We can take:

$$e^{2\phi} = \mu\frac{\partial\tau\bar\partial\bar\tau}{\tau_2^2}$$

- Literature (sourced Liouville eqn.)
- Uniqueness? Locally: 3D gravity

- We express the Godel metric in coordinates where the spatial base is either the Poincare disk or UHP.

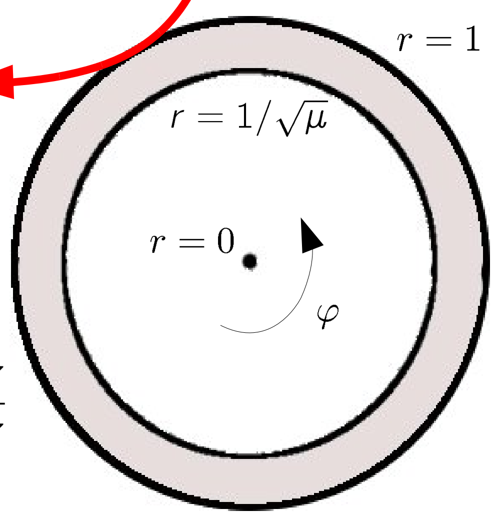
$$ds^2 = -(dt + d\chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z}$$

$$dw d\bar{w}$$

$$e^{2\phi} = \mu \frac{\partial\tau\bar{\partial}\bar{\tau}}{\tau_2^2}$$

- Poincare Disk

$z = re^{i\varphi}$



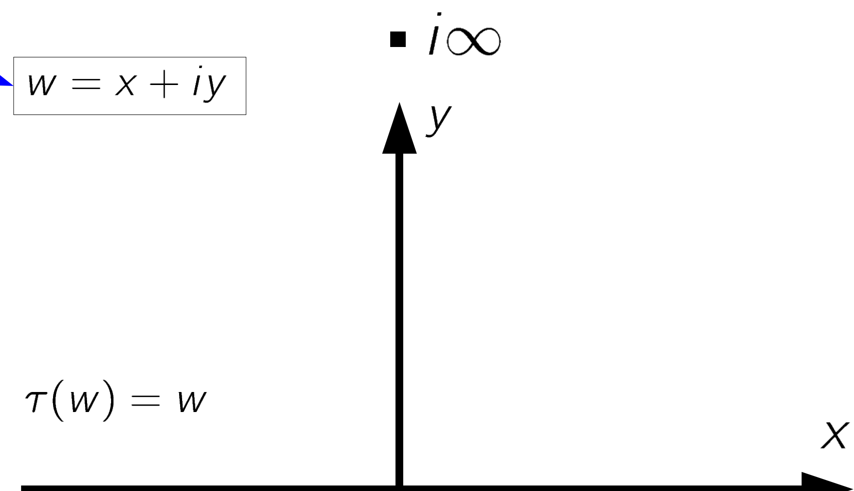
$$\tau(z) = i\frac{1+z}{1-z}$$

$$ds^2 = \frac{\mu\ell^2}{4}\left[-\mu(d\tilde{t} + \frac{2r^2}{1-r^2}d\varphi)^2 + 4\frac{dr^2 + r^2 d\varphi^2}{(1-r^2)^2}\right]$$

- $\mu < 1$ Timelike Squashed AdS (TMG): unphysical!
- $\mu = 1$ AdS
- $\mu > 1$ Timelike Stretched AdS
  $r > 1/\sqrt{\mu}$ → φ circle timelike, CTCs

- Upper Half Plane (UHP)

$w = x + iy$



$$\tau(w) = w$$

$$ds^2 = \frac{\mu\ell^2}{4}\left[-\mu(dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2}\right]$$

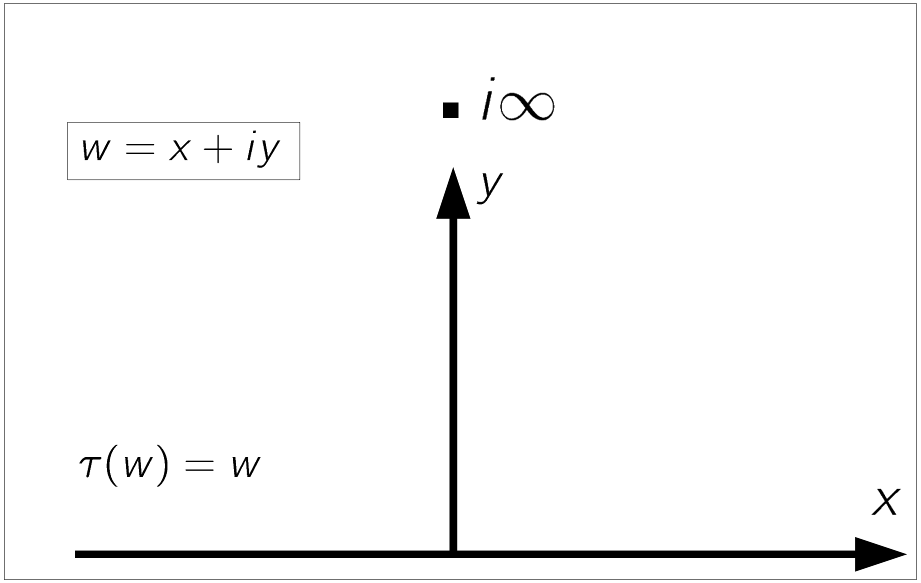
- Original Godel space
  - Metric in four dimensions:
    3d solution above ($\mu = 2$) plus 1 extra dimension
  - Solution to Einstein eqns. with a pressureless fluid source:

$$T_{\mu\nu} = \rho u_\mu u_\nu, \qquad u^\mu = \frac{2}{l}\delta_0^\mu \qquad \rho \text{ constant energy density}$$

  - Closed Timelike Curves (CTCs)
  - Godel rotates around every point: $\star_3(u \wedge du) \neq 0$

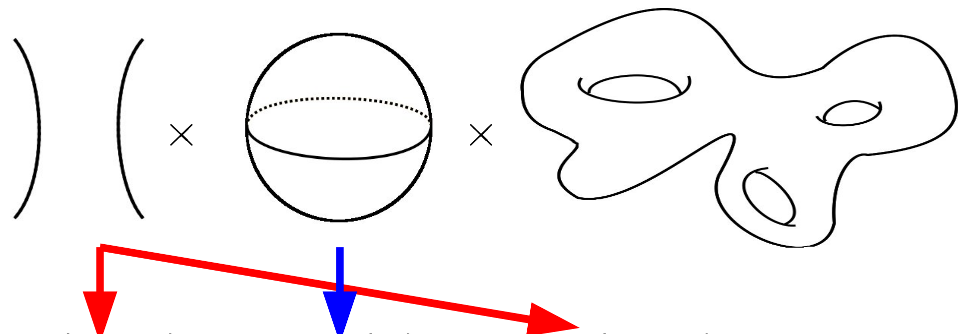
- Our Godel space
  - 3d solution ($\mu = 3/2$)
  - Non-trivial complex scalar $\tau(w)$ with EM tensor of the "Godel" form
  - CTCs
  - Pole of τ on the boundary ($w = i\infty$)
    Infinite U(1)- charge of gauge field:

$$d\tau_1 \sim \star_3 dA$$



- Background geometry

  $$AdS_3 \times S^2 \times CY_3$$



  - Bosonic symmetry group: $SL(2,R)_L \times SU(2)_L \times SL(2,R)_R$
  - Supergroup (8 supersymmetries): $SU(1,1|2)$

- Probe (wrapped) M2 Branes

  - Static w.r.t. l_0 generator of $SL(2,R)_L$ (i.e. t in UHP coords, physically rotating M2!)
  - Minimal energy: $L_0 = Z\,,\qquad Z =$ mass of brane
  - $\frac{1}{2}$ BPS state: (4 supersymmetries)

- Is 3d Godel Space = Backreacted (wrapped) M2 branes ?

  - Bosonic symmetry group $U(1)_L \times SU(2)_L \times SL(2,R)_R$
  - Same (4 supersymmetries) of probe
  - Check: M2 branes in Godel background do not break any susy

Superglue

- Motivation: why glue to Anti de Sitter space?
  - Black hole motivation
  - AdS/CFT
    - Embedding of Godel in AdS:

  ⇒ SIMPLEST REALIZATION: domain wall canceling the M2 charge

    - Outside wall: locally AdS (3d gravity)
  - Analogy with enhancon etc.
    - Resolving CTCs of Godel-type spacetime
- Setting up the domain wall
  - Cancel energy-momentum sourcing Godel space
  - Need M2 brane charge

⇒ Domain wall built up out of M2 branes wrapping internal ,
smeared in AdS on a dimension 1 domain "wall"

- Action

$$S = S_3 + S_{probe}$$

  - Which wall? M2 branes couple to CY volume (τ_2) ⇒ try constant τ_2

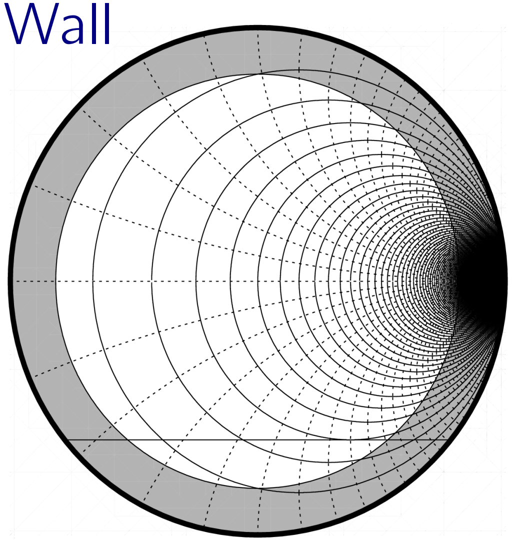
- Action

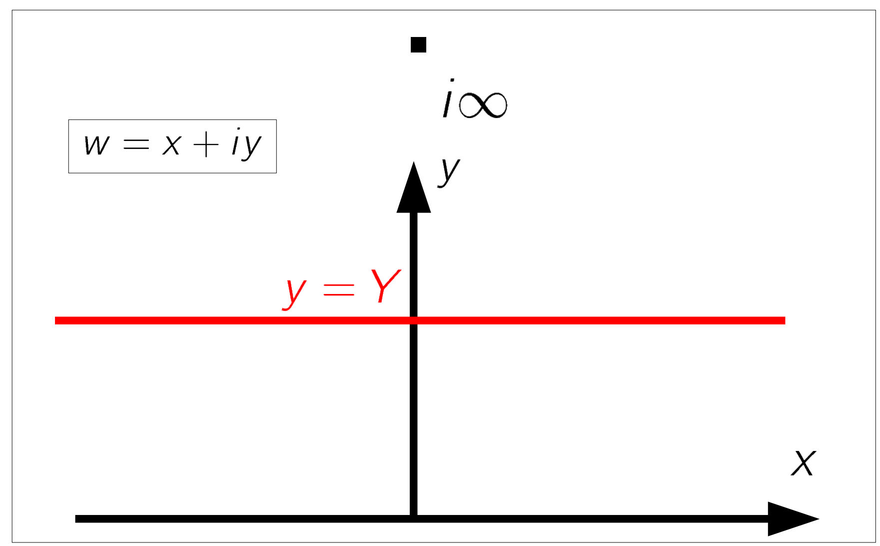
$$S = \int d^3x(L_3 + L_{probe})$$

with $dA = \frac{1}{\tau_2^2} \star_3 d\tau_1$ $\quad X^0(\sigma,\lambda) = \sigma \quad X^1(\sigma,\lambda) = \lambda \quad X^2(\sigma,\lambda) = Y$

Solution supersymmetric (remember static probe branes in UHP are susy!)

- Domain Wall





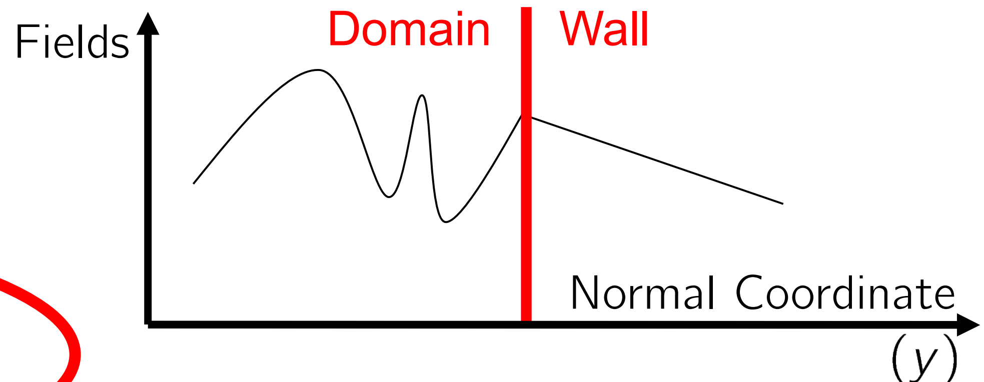
- Ansatz:

Complex Scalar				Metric
$\tau = x + iy$	$\Longleftrightarrow$	$\epsilon(y - Y) \leq 0$	$\Longleftrightarrow$	$ds_3^2 = ds^2_{Godel}$
$\tau = \tau_0$ (cst)	$\Longleftrightarrow$	$\epsilon(y - Y) \geq 0$	$\Longleftrightarrow$	$ds_3^2 = N^2 dy^2 + h_{ab}dx^a dx^b$

- Junction conditions

$\tau = x + iy$	$\Longleftrightarrow$	$\epsilon(y - Y) \leq 0$	$\Longleftrightarrow$	$ds_3^2 = ds^2_{Godel}$
$\tau = \tau_0$ (cst)	$\Longleftrightarrow$	$\epsilon(y - Y) \geq 0$	$\Longleftrightarrow$	$ds_3^2 = N^2 dy^2 + h_{ab} dx^a dx^b$

  - For metric, complex scalar (= CY volume and gauge field)



- Need to match:
  - Continuity across domain wall
  - EOM scalar/metric across wall:
  - Einstein eqn. AdS part: no source

ϵ = the sign of the M2 brane tension!
Godel in/out determined by brane tension neg/pos

Metric:

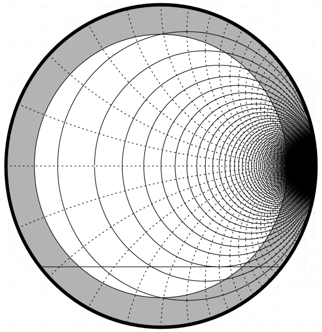
$$\epsilon(y - Y) \leq 0 \quad ds^2 = \frac{\mu\ell^2}{4}\left[-\mu(dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2}\right]$$

$$\epsilon(y - Y) \geq 0 \quad ds^2 = \frac{\mu l^2}{4}(-(dt + \frac{dx}{y})^2 + \frac{(f(y)dx^2 + f^{-1}(y)dy^2))}{y^2}$$

$$f(y) = \mu + (1 - \mu)\frac{y^2}{Y^2}$$

Global AdS!

- Problems:
  - CTCs:
    - Remember poincare disk:
      Domain walls are circles tangent
      to the boundary in $z = 1$



    - Even worse:
      - Out(in)side AdS space can be brought to global coordinates
      - Identification of AdS-angle requires extra timelike identification in Godel
  - Black hole charges? Hoped for, but not realized!

Conclusions & Outlook

Conclusions & Discussion

- Summary?
  - Backreaction of M2 branes wrapped on S^2 in $AdS_3 \times S^2 \times CY_3$ background?
  - Godel space + complex scalar in 3 dimensions:
    - Infinte M2 charge on one boundary point
    - Supersymmetric ($\frac{1}{2}$ BPS)
  - Connect to to asymptotically AdS spacetime: domain wall of M2 branes
    - PROBLEMS: CTCs, no BH equivalent

- Questions?
  - What corresponds to backreacted setup of Gaiotto-Denef-Strominger-Van den Bleeken-Yin? We would expect asymptotics = quotient of $AdS_3 \times S^2$
  - Other alternatives
    - For Godel solutions? Complex scalar solution?
    - of making Domain Wall?
      - 11d picture, need codimension 1 object – other brane sources:
        - M2?
        - M5?
  - Use this technology to resolve problematic solutions (3d conical defect of AdS. . .)

End

Extra Slides

- As an appetizer, let's solve the case

$$S_3 = \int dx^5 \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) \qquad ds_3^2 = -(dt + \chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z}$$

  - We know this should be AdS
  - Equations of motion become

$$\left.\begin{aligned} \partial\bar{\partial}\phi - \frac{e^{2\phi}}{4} &= 0 \\ d\chi &= \frac{i e^{2\phi}\, dz \wedge d\bar{z}}{2} \end{aligned}\right\} \Longrightarrow \begin{aligned} e^{2\phi} &= \frac{4\partial g \bar{\partial}\bar{g}}{(1 - g\bar{g})^2} \\ \chi &= 2\mathrm{Im}\partial\phi + df \end{aligned}$$

in terms of an arbitrary holomorphic function $g(z)$

  - We can show this is AdS with the coordinate transformation:

$$\left.\begin{aligned} g &= \tanh(\rho) e^{i(\psi - \sigma)} \\ \sigma &= \frac{t + f}{2} \end{aligned}\right\} \quad ds^2 = l^2 \left(-\cosh^2\rho\, d\sigma^2 + d\rho^2 + \sinh^2\rho\, d\psi^2 \right)$$

- For later use, two main coordinate systems:
  - The Poincare Disk
  - The Upper Half Plane (UHP)

Solutions: Godel Space

Godel Space in Disk and UHP coordinates

- We express the AdS metric in coordinates where the spatial base is either the Poincare disk or UHP.

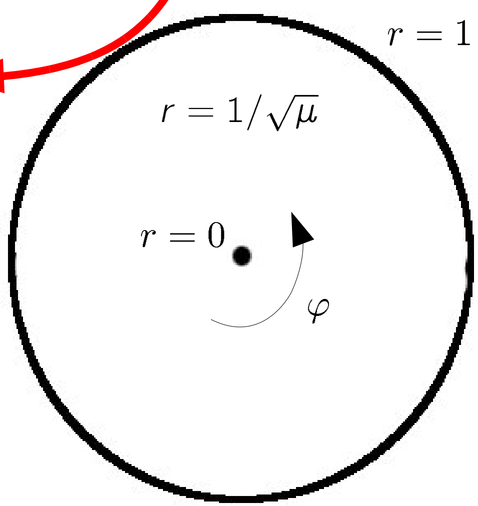
$$ds^2 = -(dt + d\chi)^2 + e^{2\phi(z,\bar{z})} dz d\bar{z} \quad / \quad dw d\bar{w}$$

$$e^{2\phi} = \frac{4\partial g \bar{\partial} \bar{g}}{(1 - g\bar{g})^2}$$

$$\chi = 2\mathrm{Im}\partial\phi + df$$

- Poincare Disk

$z = re^{i\varphi}$

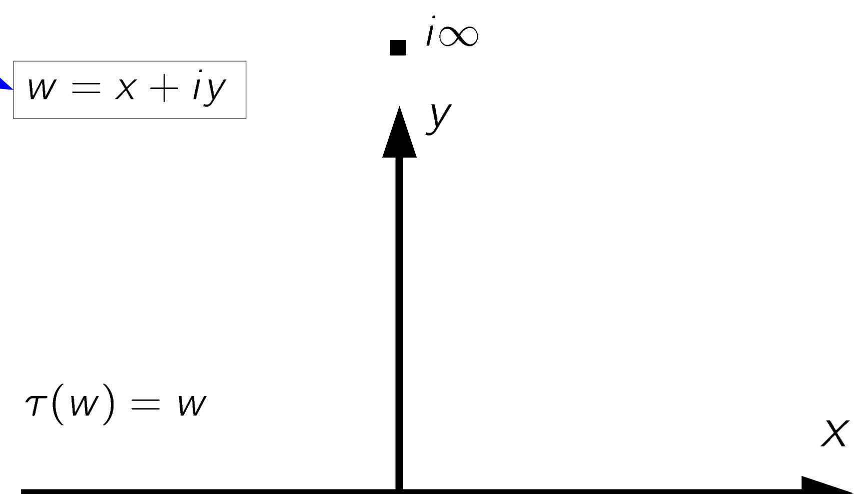


$$\tau(z) = i\frac{1+z}{1-z}$$

$$ds^2 = \frac{\ell^2}{4}\left[-\left(d\tilde{t} + \frac{2r^2}{1-r^2}d\varphi\right)^2 + \frac{4}{(1-r^2)^2}(dr^2 + r^2 d\varphi^2)\right]$$

- Upper Half Plane (UHP)

$w = x + iy$



$$\tau(w) = w$$

$$ds^2 = \frac{\ell^2}{4}\left[-\mu(dt + \frac{dx}{y})^2 + \frac{dx^2 + dy^2}{y^2}\right]$$

- Timelike Squashed AdS (TMG)
- AdS
- Timelike Stretched AdS

circle timelike, CTCs