Effective field theory for axion monodromy inflation

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Based on work in progress with Nemanja Kaloper and L.orenzo Sorbo

Outline

I. Introduction and motivation

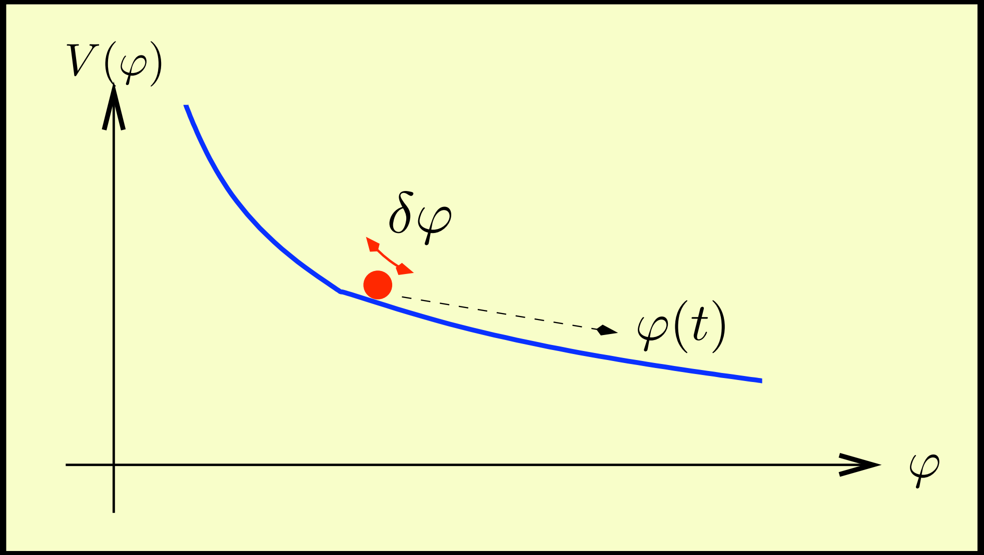
II. Scalar + 4-form dynamics

III. Corrections from UV completions

IV. Conclusions

I. Introduction and motivation

Single field inflation



Slow roll and vacuum dominance:

- $\epsilon = m_{pl}^2 \left(\frac{V'}{V}\right)^2 \ll 1$
- $\eta = m_{pl}^2 \frac{V''}{V} \ll 1$

Spacetime approximately de Sitter:

$$ds^2 \sim -dt^2 + e^{2\int H dt} d\vec{x}^2,\ H^2 = \frac{V}{m_{pl}^2}.$$

Quantum fluctuations of ϕ generate observed density fluctuations:

$$\frac{\delta\rho}{\rho} \propto \frac{V^{3/2}}{m_{pl}^3 V'}$$

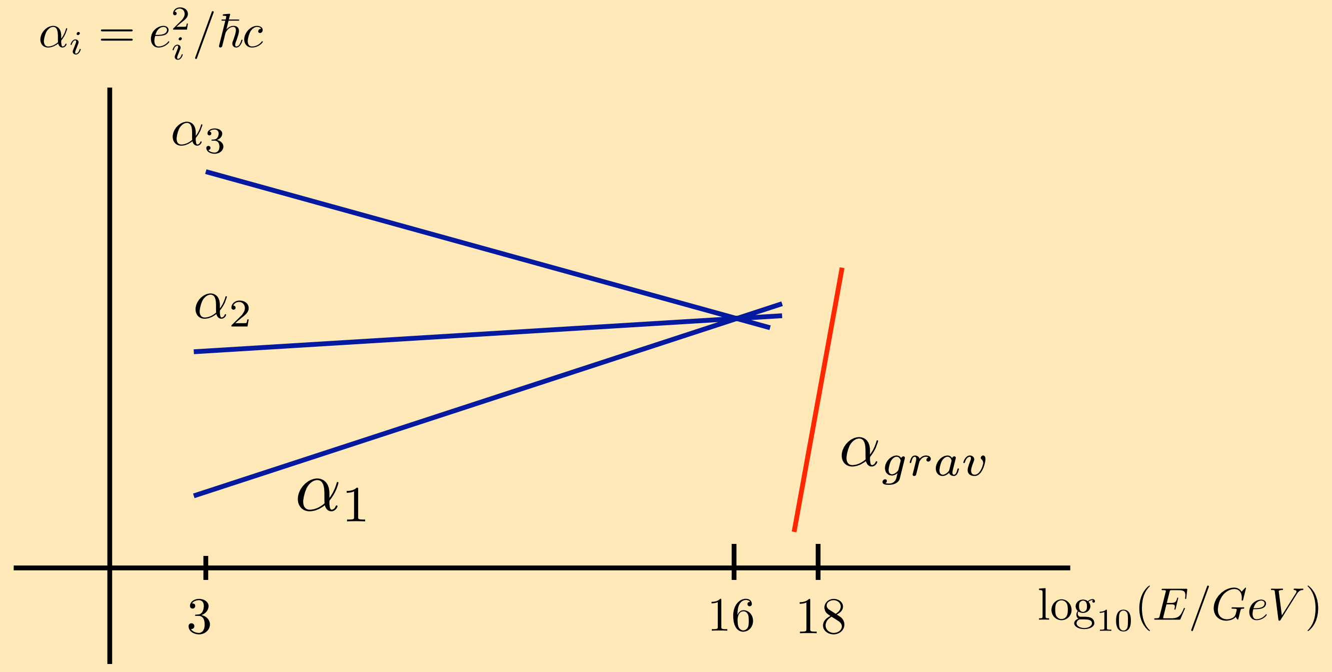
Quantum fluctuations of metric produce gravity waves detectable via CMB polarization

$$\mathcal{P}_g \propto \frac{V}{m_{pl}^4}$$

Large field inflation

Observational upper bound on primordial gravity waves: $V \leq 10^{16}\ GeV \sim M_{gut}$

Close to "unification scale"



Couplings unify (assuming MSSM above 1 TeV) at approximately 10^{16} GeV. Graph not to scale.

Consistent with
- Proton decay
- Neutrino masses

Could be detectable by PLANCK, ground-based experiments

Detectable gravitational waves require large fields

Lyth, hep-ph/9606387

$\left(\frac{\delta\rho}{\rho}\right)^2 \propto \frac{V^3}{(V')^2}$ measured by CMB temperature fluctuations

Primordial gravitational waves $\mathcal{P}_g \propto \frac{V}{m_{pl}^4}$

Upper bound on $V \Rightarrow$ upper bound on $\frac{V'}{V}$

$$N_e = \int dt H = \int \frac{d\phi}{\dot{\phi}} H = \frac{3}{m_{pl}^2} \int d\phi \frac{V}{V'} \sim 60$$

$\Rightarrow$ Upper bound on $\frac{d\phi}{dN}$, $\Delta\phi$ during inflation

$$\Delta\phi \gg m_{pl}$$

Effective field theory and large ϕ

Effective field theory:

Allow all terms in action consistent with symmetries

$V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$ $M \leq m_{pl}$ dynamical scale of UV physics

Generic theory: $g_n \sim 1$

Expansion breaks down for $\phi > M$

- New degrees of freedom become light.
- Relevant degrees of freedom could be very different.

Inflation is a highly nongeneric theory

Consider $V \sim m^2\phi^2$ or $V \sim \lambda\phi^4$ Give observable GW

$$N_e \sim 60,\ \frac{\delta\rho}{\rho} \sim 10^{-5} \Rightarrow m \sim 10^{-6} m_{pl},\ \lambda \sim 10^{-14}$$

Very finely tuned! But in fact the situation is worse:

All coefficients g_n in $V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$ must be exquisitely small

For example such corrections give $\eta \gg 1$

NB: quantum loops of inflatons and gravitons are not dangerous

m, λ give small breaking of shift symmetry:
Guarantees loops of ϕ, gravitons do not make g_n too large

$$V_{loop} = V_{class}(\phi) F\left(\frac{V}{m_{pl}^4}, \frac{V'}{m_{pl}^3}, \dots\right)$$

Coleman and Weinberg; Smolin; Linde

This approximate shift symmetry is the key to building chaotic inflation models

Fine tuning hard to justify

Coupling to other degrees of freedom is the problem!

Tends to give unacceptable breaking of shift symmetry

- Gravity breaks global symmetries: wormholes, virtual black holes,...

Holman et al; Kamionkowski and March-Russell; Barr and Seckel; Lusignoli and Roncadelli; Kallosh,Linde,Linde,Susskind

- String theory: global symmetries tend to be gauged or anomalous.
- Anomalous shift symmetry broken by instantons (eg axion):

Arkani-Hamed, Cheng, Creminelli, Randall

$$V \sim \Lambda^4 \sum_n c_n \cos(n\phi/f_\phi) \sim \Lambda^4 \cos(\phi/f_\phi) + \ldots$$

$f_\phi \geq m_{pl}$ is hard to realize (eg c_n tends to be large).

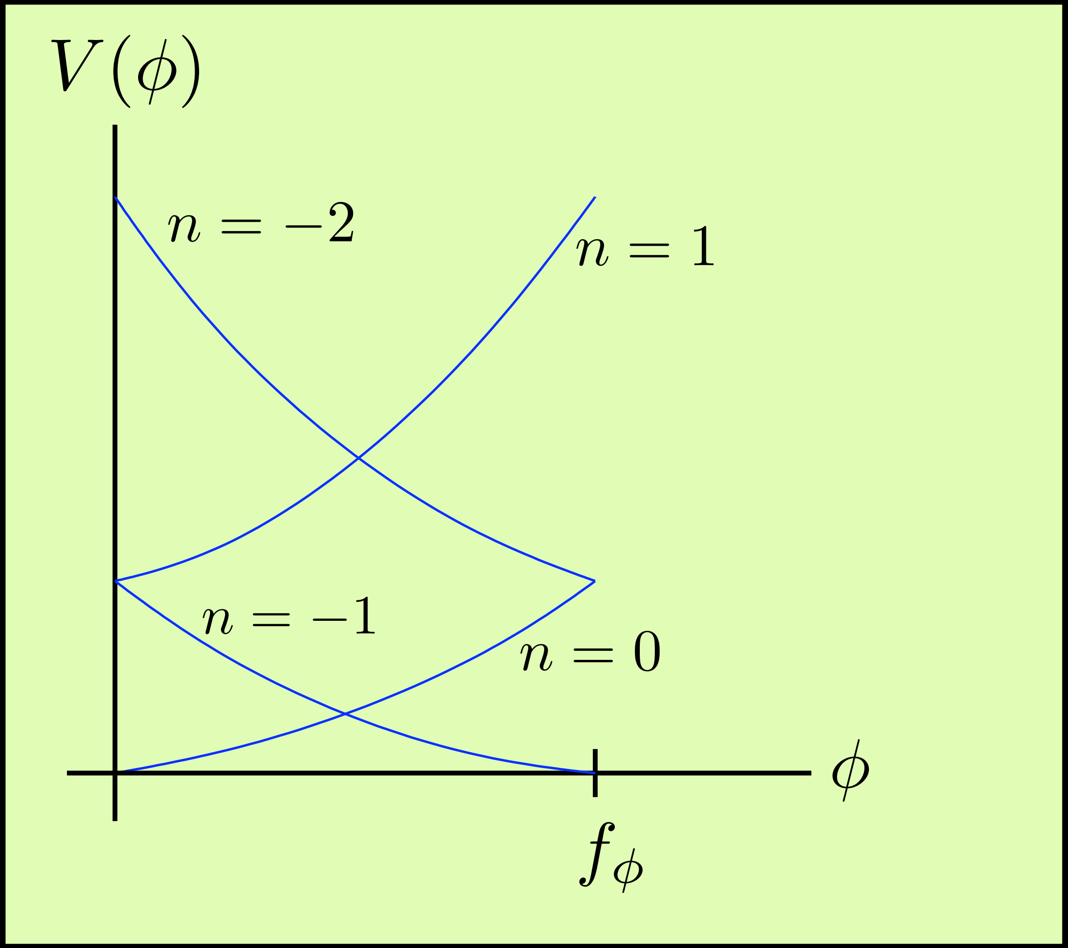
Banks, Dine, Fox, Gorbatov;
Arkani-Hamed, Motl, Nicolis, Vafa

Solution: monodromy inflation

Silverstein and Westphal; McAllister, Silverstein and Westphal; Kaloper and Sorbo; Berg, Pajer, and Sjors; KLS

Consider axion with period (decay constant) f_ϕ

In this scenario, physics invariant under $\phi \to \phi + f_\phi$, but states are not periodic under continuous shift



Spectral flow

$$V(\phi, n) = \mu^2(\phi - n f_\phi)^2$$

$n \in \mathbb{Z}$ discrete variable

$\tau_{\Delta n} \gg \tau_{inflation}$

Inflation: ϕ ranges over many periods

Compact field space (nb: must include n) may keep EFT under control

String theory example

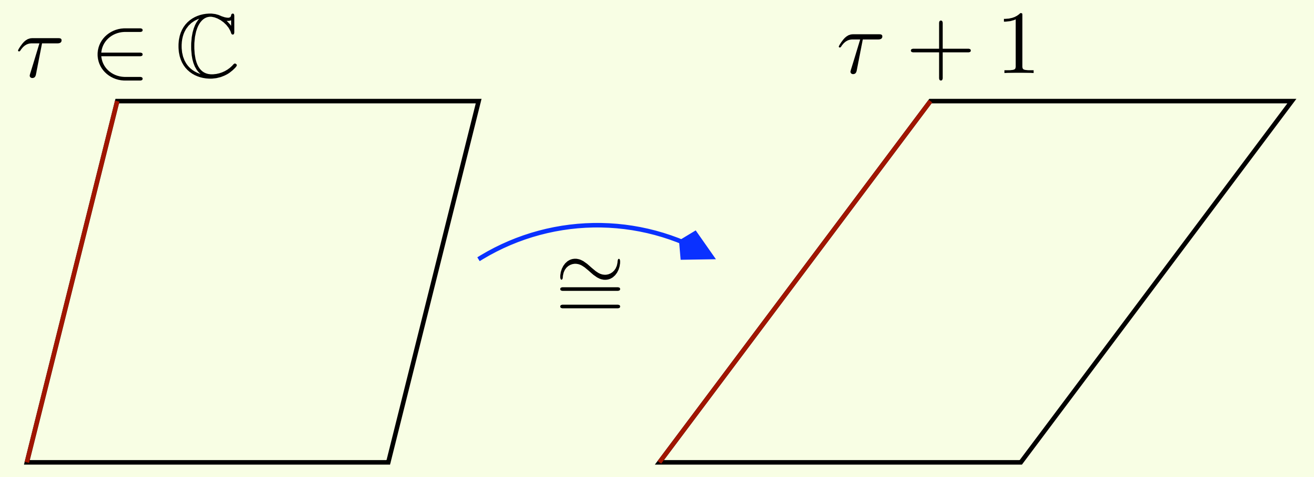
Silverstein and Westphal; McAllister, Silverstein and Westphal

Type II on torus; unit volume and complex structure τ in string units

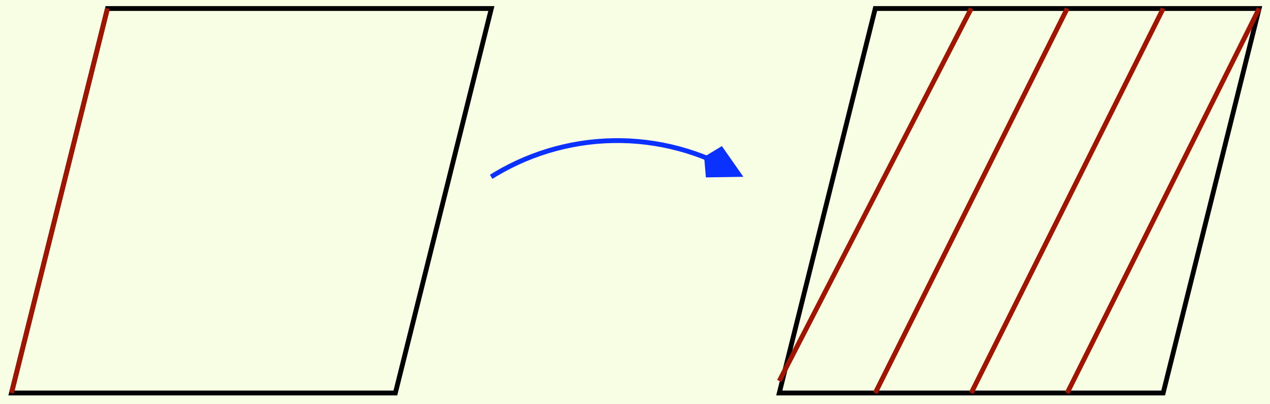
τ has period 1.

Canonically normalized scalar $\phi = m_{pl}\tau$

$$V \sim \frac{m_s^4}{g_s}\sqrt{1+\tau^2}$$



Shift $\tau \to \tau + 1$ is symmetry of torus, but stretches D-brane.


Shift τ n times; D-brane becomes n times as long.

Goal:

These scenarios receive quantum corrections from integrating out UV degrees of freedom:

- moduli
- Kaluza-Klein modes
- light string modes
- ...

Additional effects:

- instantons
- semiclassical gravity

These were analyzed model by model in the string constructions. These models are of necessity complicated.

Silverstein and Westphal; McAllister, Silverstein and Westphal

We study 4d effective field theory to :

- Better understand the physics behind suppressing corrections
- Better understand the degree of fine tuning still needed
- Provide a framework for building and comparing models.

II. Inflaton-4 form dynamics

Kaloper and Sorbo, 0810.5346 and 0811.1989

4d mechanism for generating a potential via spectral flow.

$F_{\mu\nu\lambda\rho}$ totally antisymmetric 4-form field strength

$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]}$

$$S = \int d^4x \sqrt{g} \left(m_p^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\phi)^2 + \frac{\mu}{24} \phi^* F \right) + \text{boundary terms}$$

$U(1)$ gauge invariance $A_{\mu\nu\rho} \to \partial_{[\mu} \Lambda_{\nu\rho]}$

(Will assume $U(1)$ compact as in string theory)

Bousso and Polchinski

F sourced by membranes:

$$S_{membrane} = \frac{e}{6} \int_{\Sigma_3} d^3\sigma \epsilon^{ijk} \partial_i x^\mu \partial_j x^\nu \partial_k x^\rho A_{\mu\nu\rho}$$

Compact $U(1) \Rightarrow$ quantized membrane charge

Theory has 1 scalar degree of freedom with mass μ

Dvali; Kaloper and Sorbo

Hamiltonian dynamics

$$H = \frac{1}{2}\left(p + \mu\phi\right)^2 + \frac{1}{2}\pi_\phi^2 + \text{grav.}$$

π_ϕ: conjugate momentum for ϕ

p: conjugate momentum for A_{123}

- Compact $U(1) \Rightarrow p$ is quantized in units of e^2.
- p is conserved by H: jumps via membrane nucleation.
- ϕ periodicity: $f_\phi = e^2/\mu$.

Realizes monodromy inflation:

$\phi \to f_\phi$, $p \to p + e^2$ leaves H invariant.

III. Corrections from UV completions

$$S = \int d^4x \sqrt{g} \left(m_p^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\phi)^2 + \frac{\mu}{24} \phi^* F \right) + \text{bndry terms} + \text{UV corrections}$$

$\mu \sim 10^{-6} m_{pl}$ for slow roll inflation:

Effective field theory:

Allow all terms in action consistent with symmetries, topology of field space

Corrections controlled by:

- Compactness of ϕ (and of $U(1)$).
- Small coupling $\mu \ll m_{pl}, M_{GUT}$.

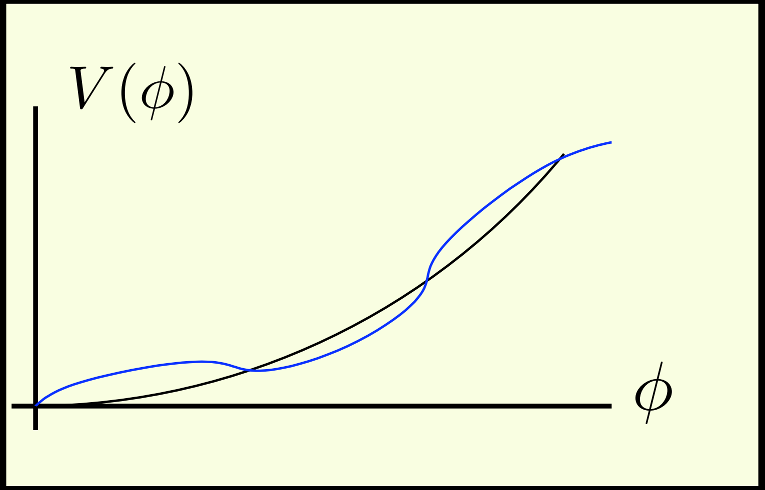
Direct corrections

Direct corrections to V must be periodic in ϕ

$$\sum_n c_n \frac{\phi^n}{M^{n-4}} = \Lambda^4 \cos(\phi/f_\phi) + \ldots$$

Generally $f < m_{pl}$; $\mu^2\phi^2$ potential modulated by oscillations

- Gauge instantons: $\Lambda \sim \Lambda_{QCD}$
- Gravitational effects: $\Lambda^4 \sim f^{n+4}/m_{pl}^n$



$$V_{corr} \ll V_{class} \Rightarrow \Lambda^4 \ll M_{gut}^4$$

$$\eta \ll 1 \Rightarrow \frac{\Lambda^4}{f^2} \ll \frac{V}{m_{pl}^2} = H^2$$

Example: feasible if $f \sim M_{gut}$, $\Lambda \ll 10^{15}\ GeV$

Must be careful with moduli stabilization

Coefficients of $V(\phi)$ typically depend on moduli ψ

$$V = V_0(\psi) + c_1(\psi)\Lambda^4 \cos(\phi/f) + \ldots$$

We must have $|V_0(\psi)| \gg | c_1 \; \Lambda^4|$

Otherwise modulus destabilized whenever $\cos(\phi/f) \sim -1$

(which will happen many times during inflation)

Indirect corrections

Additional corrections must respect periodicity of ϕ.

Instead we can correct dynamics of 4-form sector.

$$\delta\mathcal{L} = \sum_n c_n \frac{F^{2n}}{\Lambda^{4n-4}} \qquad (\Lambda \text{ some UV scale})$$

$$S_{classical} = \int d^4x \sqrt{g} \left(m_p^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\phi)^2 + \frac{\mu}{24} \phi^* F \right) + \ldots$$

We can guess effects of $\delta\mathcal{L}$ by integrating out F classically:

$$F \sim \mu\phi \Rightarrow \delta\mathcal{L} \sim V(\phi) \sum_n c_n \frac{V^{n-1}}{\Lambda^{4n-4}}$$

Multiplicative correction to V: safe if $\Lambda^4 > V$

Corrections of the form $\delta\mathcal{L} \sim \sum_n d_n \frac{F^{2n}}{\Lambda^{4n}} (\partial\phi)^2$
give identical story after

1. Integrating out F
2. Canonically normalizing ϕ

Small Λ not always bad

String scenario with ϕ a D-brane modulus:

$$V(\phi) = \sqrt{M_1^8 + M_2^6\phi^2}$$

- $V \sim \mu^2\phi^2$ for small $\phi \leq m_{pl}$
- Large ϕ: $V \sim M^3\phi$

Silverstein and Westphal, 0803.3085

McAllister, Silverstein, and Westphal, 0808.0706

Source of corrections

ψ a field (modulus, Kaluza-Klein state, etc.) with mass M_{class}

$$\delta\mathcal{L} \sim \sum_n d_n \frac{F^{2n}}{\Lambda^{4n}} (\partial\psi)^2 + \sum_n d'_n \frac{F^{2n}}{\Lambda^{4n-2}} \psi^2 + \ldots$$

Canonically normalize ψ: effective mass

$$M^2 = M^2_{classical} + \Lambda^2 \sum_n \tilde{d}_n \frac{V^n}{\Lambda^{4n}}$$

Integrate out ψ: induce "Coleman-Weinberg" potential

$$V_{CW} \sim M(\phi)^4 \ln \frac{M^2}{\Lambda^2}$$

Must sum over all such fields with $M_{class} < \Lambda_{UV}$

Safe if $n_{species} M^2_{class} \ll \Lambda^2$; $V \ll \Lambda^4$.

Eg: $n_{species} \sim m^2_{pl}/M^2_{gut}$; $\Lambda \sim M_{gut} \Rightarrow M^2_{class} \ll H^2$.

III. Conclusions

1. Large scale inflation feasible without infinite numbers of fine tunes.
2. Compactness of field space, small μ controls corrections

Additional questions:

1. Lifetime of p due to membrane nucleation should be longer than inflation.
2. Compare to explicit string models with and without 4-forms.
3. Is $\mu^2\phi^2$ inflation possible or do we always get $V \sim \phi^{p<2}$?
4. Can we find a way to make μ naturally small?