Unitarity and Holography in Gravitational Physics

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Goals:

- Reconcile known gravitational physics with the suggested Unitary of Black Hole Evaporation
- Explain AdS/CFT from a bulk (gravitational) perspective

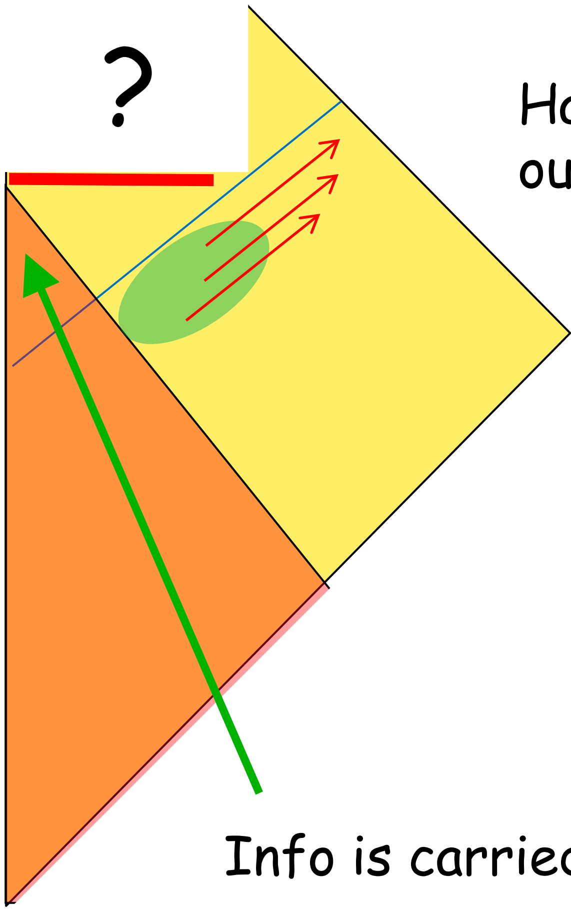
Main Tool:

H_{ADM} is a pure boundary term on shell.

Approach:

Derive properties of classical GR which, if true in the quantum theory, would (largely?) achieve these goals.

Info "Paradox" in a Nutshell



Hawking radiation forms outside the black hole.

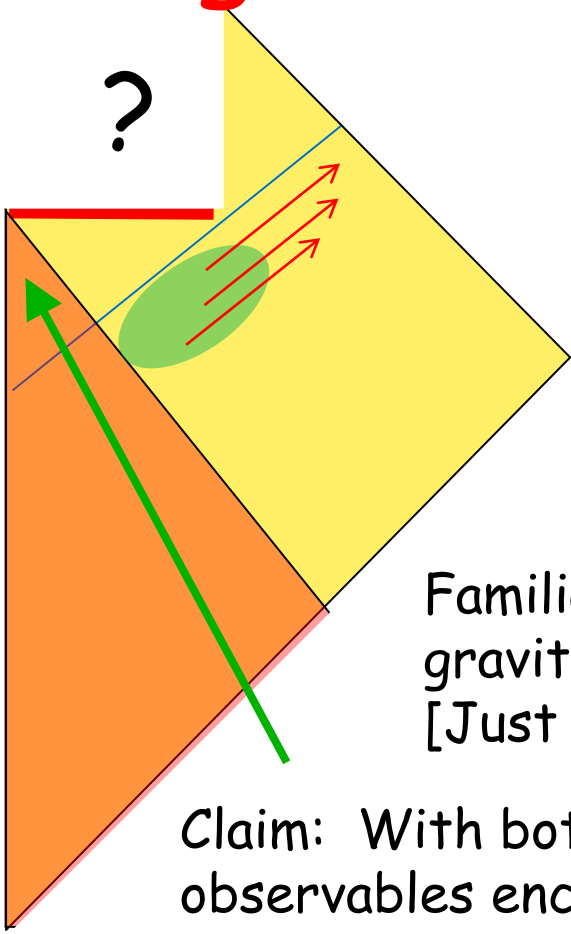
But string theory, AdS/CFT, BH pair production, & other arguments suggest that this radiation should carry the info away when the black hole evaporates.

Can this tension be resolved?
How could the info be transferred to the Hawking radiation?

Info is carried deep inside the black hole.

1) New non-locality or causality violation?
2) Infalling info is "stored" outside?

Rough Summary of Claim:



The gravitational constraints relate operators that might a priori seem different. In particular, they allow info to be "stored" in an algebra of observables associated with both the Coulomb tail of the gravitational field and propagating d.o.f. at infinity.

Familiar example: M is encoded in the gravitational flux through i^0.
[Just Coulomb Part]

Claim: With both parts, the algebra of distant observables encodes "all" info. From there, it can be transferred to the Hawking radiation.

Precise statements will follow for various boundary conditions, using the fact that H_{ADM} is a pure boundary term (on shell).

Precise Claims:

Poisson Algebras

I. Classical GR w/ AdS Boundary Conditions

Claim a) "Ads Boundary Unitarity:"

In the full theory, a cosmic censorship-like hypothesis implies that

$A_{\text{Bndy Obs}}(t_2) = A_{\text{Bndy Obs}}(t_1)$, for all t_1, t_2

Claim b) "Ads Perturbative Holography:"

At any order n > 1 of pert. theory
(i.e., where Gauss' law is non-trivial)
about a collapsing black hole background,

$A_{\text{Pert Obs}}(\text{all } t) = A_{\text{Pert Bndy Obs}}(t_1)$, for any t_1

Corresponding properties in the quantum theory would provide a mechanism for information to be "stored" outside the black hole and transferred to the Hawking radiation.

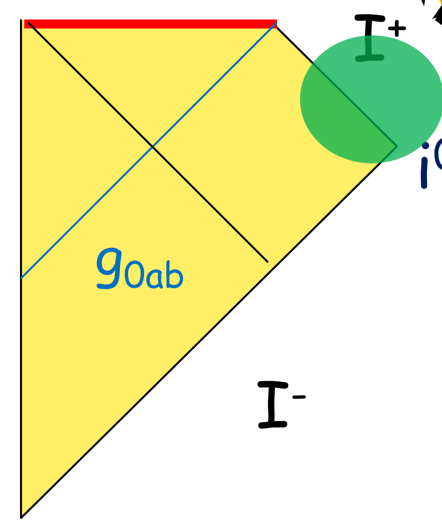
Precise Claims:

II. Classical GR w/ As. Flat Boundary Conditions

"As. Flat Perturbative Holography:"

At any order $n > 1$ of pert. theory
(i.e., where Gauss' law is non-trivial)
about a collapsing black hole background,

$$A_{\text{Pert Obs}} = A_{\text{Pert Obs}}\,(i^0 \text{ and early } \mathcal{I}^+\!: u < u_0)$$

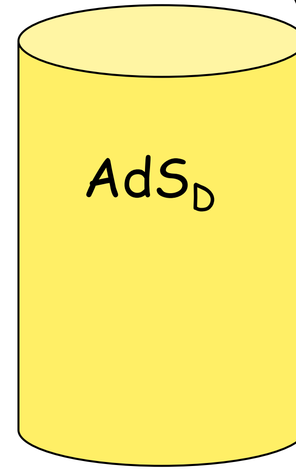


Again, a corresponding property in the (full) quantum theory would provide a mechanism for information to be "stored" outside the black hole and transferred to the Hawking radiation.

Outline

1. Intro – done!
2. Review of AdS Asymptotics & Bndy Observables.
3. AdS Boundary Unitarity
4. A physical example
5. AdS Pert Holography
6. Comments on As Flat and BH evap.
7. Appendix: As Flat argument

Background: AdS_D asymptotics and boundary observables



- Use EOMs to expand fields in asymptotic series.
- 2^{nd} order equations $\Longrightarrow$ 2^{nd} order recursion

$\Longrightarrow$ 2 independent pieces of data (up to gauge): ``Dirichlet & Neumann"

In Fefferman-Graham Gauge w/ bdny @ z=0:

$$ds^2 = z^{-2}\,(dz^2 + g_{ij}(x,z)\, dx^i dx^j. \,),$$

For even D (no conformal anomalies),

$$g_{ij} = \underbrace{g_{ij}^{(0)} + z\, g_{ij}^{(1)} + z^2 g_{ij}^{(2)} + \ldots}_{\text{Determined by } g_{ij}^{(0)}\ \text{“Dirichlet”}} + 16\pi G/(D-1)\, z^{D-1}\, T_{ij} + \overbrace{\ldots}^{\text{Determined by } g_{ij}^{(0)},\ T_{ij}}$$

Independent of $g_{ij}^{(0)}$
"Neumann"

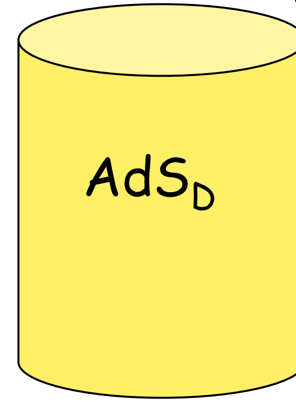
where $T_{ij} = -\, E_{ij}\,(D-1)/8\pi G$, with $E_{ij} = \lim_{z\to 0} z^{D-3}\, C_{ijkl}\, n^k n^l$.

Similar expansion for scalars in terms of ϕ_D, ϕ_N . Details depend on m^2.

Which diffeos are gauge?

- Expect fall-off faster than isometries.
- I.e., vanish at Bndy of conformal compactification

$\Rightarrow$ $g_{ij}^{(0)}$ is gauge invariant



Indeed, standard boundary conditions fix $g_{ij}^{(0)}$, even up to diffeos.

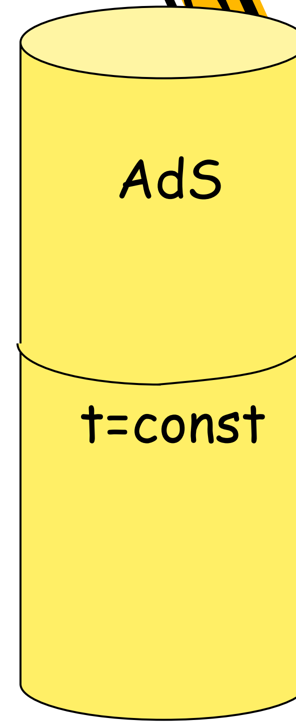
Similar for scalars. Say, fix ϕ_D.

To preserve BCs, gauge trans must fall of faster.

Pick a point x on the bndy. Any component in the FG expansion at x defines an observable. (FG fixes gauge for z, no reason to fix gauge for x.)

$T_{ij}(x)$, $\phi_N(x)$ are "Boundary Observables"

Ia. "Boundary Unitarity"



Simple case: Assume $g_{ij}^{(0)}$ has time-trans sym

1. "Boundary Fields" form a natural set of *observables.*

   Let $A_{\text{bndy obs}}(t)$ = On-shell Poisson/Peierls algebra of bndy observables [generated by ϕ_N, T_{ij}] at time t

2. Construct the Hamiltonian:

   H=H(t) is a pure boundary term on-shell.

   $H = H(t) \in A_{\text{bndy obs}}(t)$
   [weak equivalence, or action on physical phase space.]

   E.g., for above BCs fixing $g^{(0)}_{ij}$ and $\phi_D=0$, find

$$H(t) := \int_{\text{Bndy Cut w/ t =const}} T_{ij}\, \xi^i\, \underline{n}^i \, dA$$

with $\xi = \partial_t$ and $\underline{n}^i$ = normal to t= constant cut of boundary.

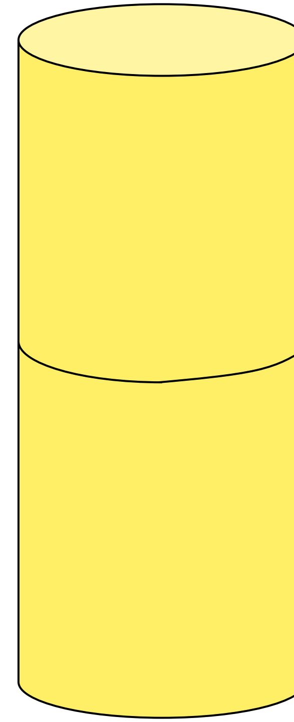
Ia. "Boundary Unitarity,"

$H = H(t)$ ♏ $A_{\text{bndy obs}}(t)$

E.g., $H := \int_{t=\text{const}} T_{ij}\, \xi^i\, \underline{n}^i \; dA$

Note: For any *observable* $\mathcal{O}$,

$$\partial_t \mathcal{O}\,(t) = -\,i\,[\mathcal{O}(t)\,, H]$$



t=0

3. *Suppose* * that we can exponentiate H to define $e^{iH\Delta t}$
as an operator on boundary observables

Then, $\mathcal{O}(t_2) = e^{-iH\Delta t}\, \mathcal{O}(t_1)\, e^{iH\Delta t}$

I.e., expresses any Bndy Obs at t_2
in terms of Bndy Fields ϕ_N, T_{ij}, at any other t_1.

$\Rightarrow$ $A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$

In QM, information present on the Bndy at any one time t_1 remains present at any other time t_2.

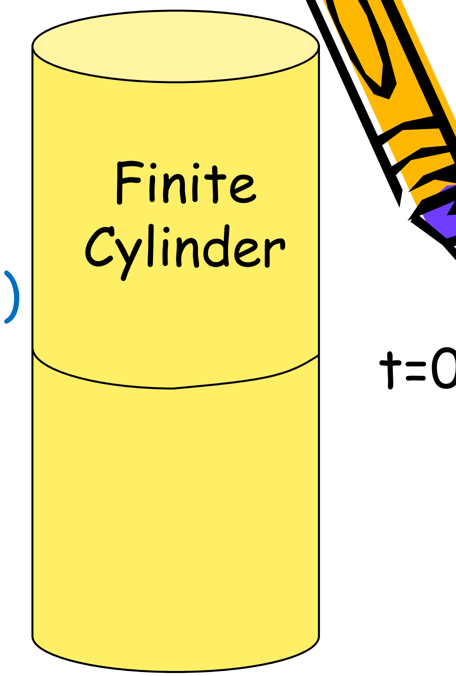
If no sym, use $\mathscr{P}$exp. Just like Q.M. w/ t-dep Hamiltonian.

Comment on Assumption:

For any *observable* $\mathcal{O}$, $\partial_t \mathcal{O}(t) = -i\,[\mathcal{O}(t), H(t)]$

3. *Suppose* * that we can exponentiate H(t) to define

$$U(t_1,t_2) = \mathcal{P}\exp\left(-i\int_{t_1}^{t_2} H(t)dt\right) \Longrightarrow A_{\text{bndy obs}}(t_1) = A_{\text{bndy obs}}(t_2)$$



Classical Interpretation on space of smooth metrics:

Assumes long-time existence of solutions to EOMs, at least in some neighborhood of the Bndy.

I.e., form of "Cosmic Censorship."

QM interpretation:

Assumes quantum Hamiltonian can still be built from ϕ_N, T_{ij}, but that Quantum Gravity "resolves any Classical failures of cosmic censorship".

Appears consistent w/ S.T. and LQG.

Note: U is Unitary if H is self-adjoint on some Hilbert space. But info is conserved so long as $U(t_1,t_2)$ exists.

Interpretation:

Classical Level:

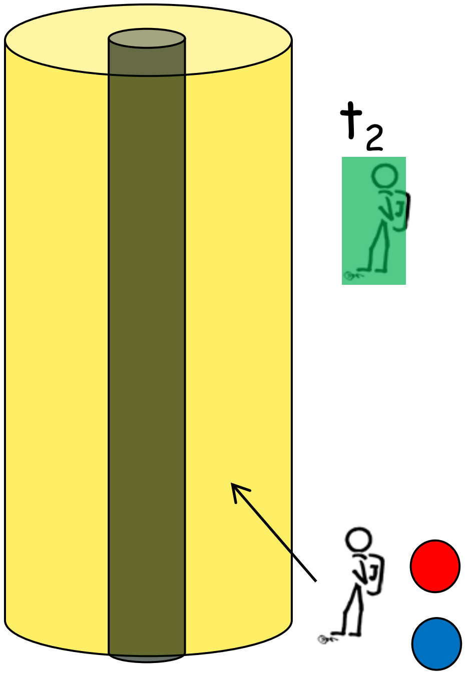
Note that Poisson/Peierls algebra of Bndy Observables is much larger than commutative algebra generated by addition and multiplication.

No real implications for information. E.g., measuring J_x and J_y need not classically tell you anything about J_z.

Quantum Level: Only one algebra!

Correlators of J_x, J_y determine correlators of J_z.

A "physical" Example



Suppose that the Green "super- observer" has access to an ensemble of AdS spaces, all prepared the same way.

[I.e., first observer made the same choice for each.]

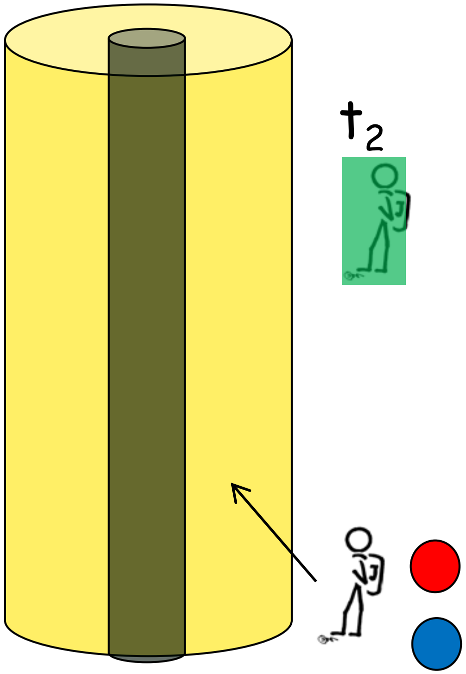
To find out what was thrown in earlier, at time t_2 the Green observer:

1. Carefully measures the energy of each system,
2. Looks for red or blue particles near infinity at time t_2 ($O(t_2)$),
3. Carefully measures each energy again, and
4. Performs a certain interference experiment.

Use this to compute (for all E,λ,E′):

$$f(E,\lambda,E') = \langle\psi|\mathcal{P}_{H=E}\mathcal{P}_{O(t_2)=\lambda}\mathcal{P}_{H=E'}|\psi\rangle$$

A "physical" Example



Now compute the probability distribution for $O(t_1)$ via

$$\langle\psi|\mathcal{P}_{O(t_1)=\lambda}|\psi\rangle = \langle\psi|e^{-iH(t_1-t_2)}\mathcal{P}_{O(t_2)=\lambda}e^{iH(t_1-t_2)}|\psi\rangle$$
$$= \int dE dE' f(E,\lambda,E') e^{-i(E-E')(t_1-t_2)}$$

1. Carefully measures the energy of each system,
2. Looks for red or blue particles near infinity at time t_2,
3. Carefully measures each energy again, and
4. Performs a certain interference experiment.

Use this to compute (for all E,λ,E'):

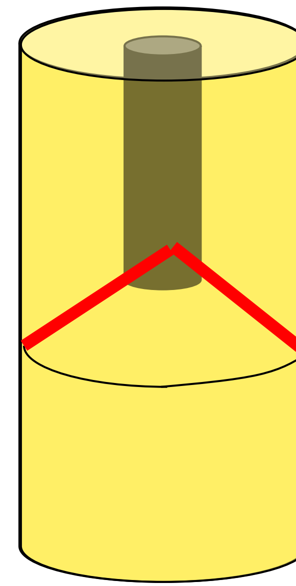
$$f(E,\lambda,E') = \langle\psi|\mathcal{P}_{H=E}\mathcal{P}_{O(t_2)=\lambda}\mathcal{P}_{H=E'}|\psi\rangle$$

Ib. AdS Perturbative Holography

Summary of Above: Any info ever present in the Bndy Fields remains encoded in Bndy Fields.

Q: Is this everything? Or is there more info "in the bulk."

A: Maybe, but not in perturbation theory.



Consider perturbation theory abt some classical solution which is largely empty before t=0.

(Though need not remain empty for time-dep BCs. E.g., can make a black hole.)

At linearized level, any h_{ab}, ϕ can be written (up to gauge) in terms of Bndy observables at early times by solving EOMs.

(Related to Holmgren's Uniqueness Thm.)

Remains true at any order in perturbation theory.

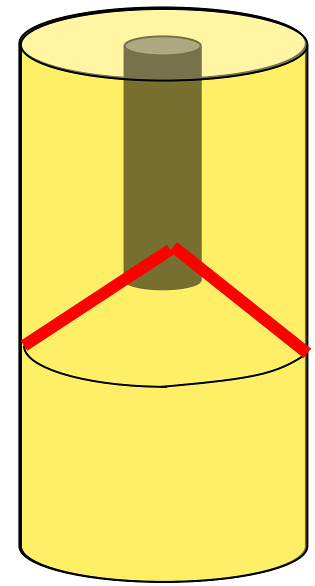
Ib. AdS Perturbative Holography

So, *any* perturbative observable can be written in terms of Bndy Observables at early times by solving EOMs.

$$A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\text{all } t < 0)$$

Not holography, just solving EOMs.

But in *gravity*, at any order beyond the linearized theory, the Hamiltonian can again be written as a boundary term!



(I.e., Gauss' Law gives a useful measure of the energy.)

Apply Above Bndy Unitarity Argument $\Rightarrow$

$$A_{\text{Bndy Obs}}(\text{all } t < 0) = A_{\text{Bndy Obs}}(\text{any single } t)$$

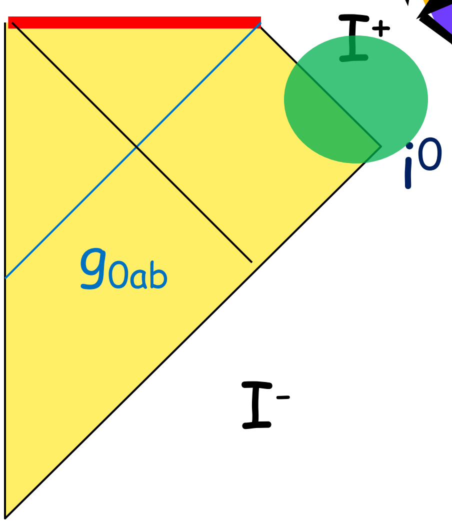
$$\Rightarrow \quad A_{\text{All Pert Obs}} = A_{\text{Bndy Obs}}(\textit{any} \text{ single } t)$$

"Perturbative Holography"

Comments on As Flat case

1. Perturbative Holography:

Consider a collapsing black hole background g_{0ab} in pure Einstein-Hilbert gravity.

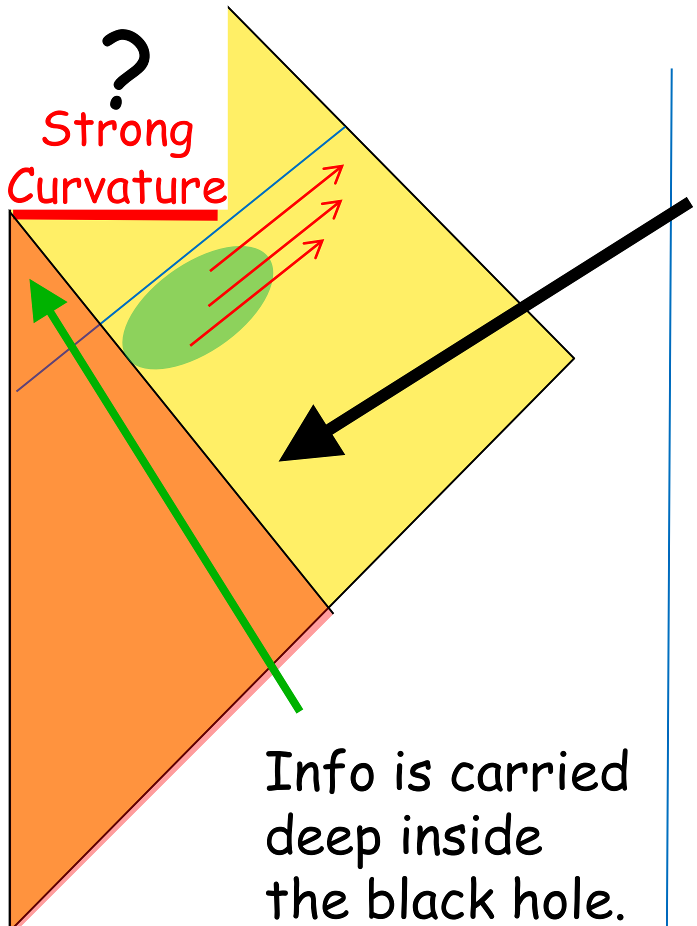


Claim: A complete set of perturbative observables is available on I^+ in any neighborhood of i^0.

2. *Suggests* Unitary S-matrix, with info imprinted in Hawking radiation (next slide).

Basic Mechanism: Constraints and local energy conservation!

Cartoon of BH evaporation



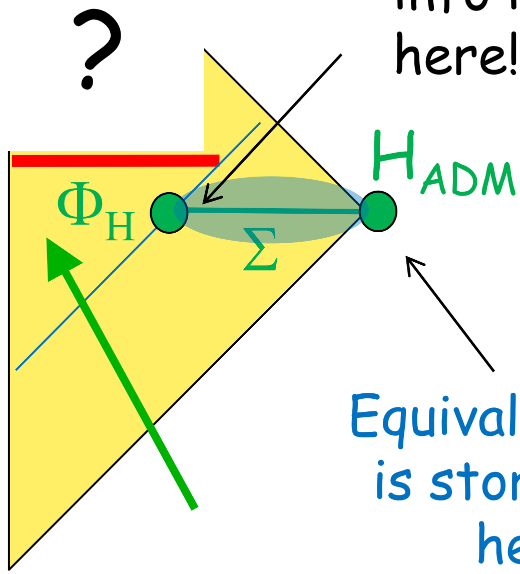
Suppose physics far from strong coupling region is essentially perturbative.

Then perturbative holography implies that all info is encoded in asymptotic fields g_{ab}, especially H_{ADM}.

Info is carried deep inside the black hole.

Cartoon of Black Hole Evaporation 2

Remaining info is stored here!



Equivalent info is stored out here

Info carried inside by infalling matter.

But constraints relate H_{ADM} to $T^{Hawking}{}_{ab}$ and a surface term "Gauss Law Grav. Flux" Φ_H at the horizon.

$$H_{ADM} - \Phi_H(h) \sim \int_\Sigma T_{ab}(h)$$

Info shared between Φ_H and T_{ab}.

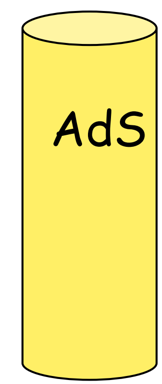
Note that $\Phi_H(h) \to 0$ as BH evaporates.
Assume finite density of states for Φ_H.
$\Rightarrow$ info transferred *locally* to T_{ab}.

Indeed, once evaporation is complete, constraint implies $H_{ADM} \sim \int_\Sigma T_{ab}(h)$.

I.e., info fully transferred to Hawking radiation.

Summary:

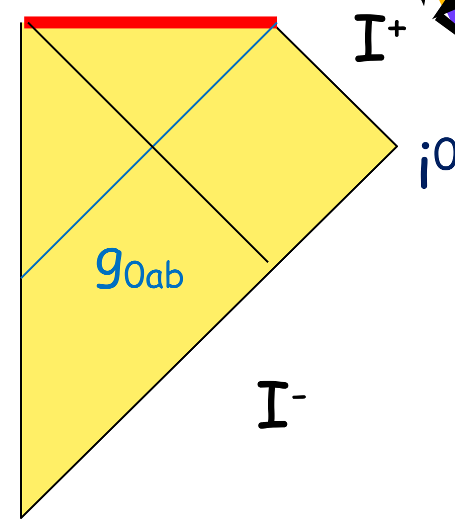
1. At the classical level, Perturbative Holography & (for AdS) Bndy Unitarity follow from gravitational constraints, gauge invariance, and a form of Cosmic Censorship.
2. Info is stored in asymptotic local fields, and throughout BH exterior.
3. If these properties also hold in the quantum theory, info can be transferred to Hawking rad via constraints and (local) Energy conservation.



No new causality violation or non-locality required.

Perturbative As. Flat gravity is holographic

Claim: A complete set of observables is available on I^+ in any neighborhood of i^0.



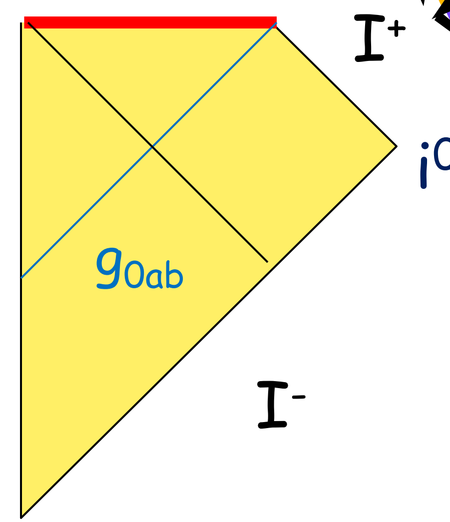
Strategy: Consider a collapsing black hole background g_{0ab} in pure Einstein-Hilbert gravity.

Fix a Coulomb-like gauge and show that *every* operator can be rewritten as an operator on I^+ near i^0 using equations of motion expanded to *any* order $n \geq 2$.

Enough to show this for every operator on I^-.

Recall: In full theory, H is a boundary term at i^0.

Perturbative As. Flat gravity is holographic



Claim: A complete set of observables is available on $\mathcal{I}^-$ in any neighborhood of i^0.

$$H = \frac{1}{2\kappa^2}\int_C dA(r^a P^{bc} D_b - r^b P^{ac} D_b) g_{ac} =: \Phi$$

where r^a, P^{bc}, D_b are defined by η_{ab}.
I.e., *linear* in g_{ac}.
No explicit corrections past n = 1.

Instead, *implicit* changes via constraints.

$$0 = \mathcal{H} = \pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2 - {}^3R\,.$$

Interactions are small near $\mathcal{I}^-$. So write:

$$g_{ab} = g_{0ab} + h_{ab} = \eta_{ab} + h_{0ab} + h_{ab}$$

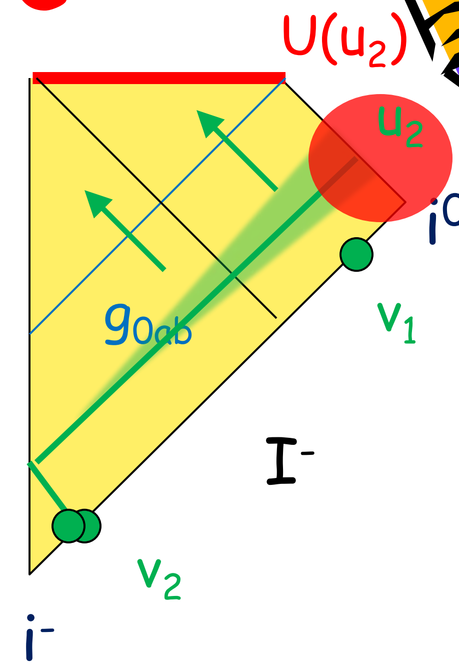
For $n \geq 2$, find:

$$\Phi = \int_{I^-} \sqrt{q}\; t^a k^b\, T^{lin}_{ab}(h + h_0)$$

- Translates $h+h_0$ along $\mathcal{I}^-$
- Operator @ i^0

Perturbative As. Flat gravity is holographic

✓ Claim: A complete set of observables is available on I^- in any neighborhood of i^0.



Consider $h_{ab}(v_1)$ on I^-.

Translate to v_2 in distant *past* using Φ and $h_{0ab}(v_2)$- $h_{0ab}(v_1)$.

Now, evolve to I^+ using perturbative EOMS.
Find operator approximately supported near u_2.

Invert to write $h_{ab}(v_2)$ $\cong$ operator in $U(u_2) \subset I^+$. Also for $h_{ab}(v_1)$.

Take limit $v_2 \Longrightarrow i^-$.

- Errors $\Longrightarrow$ 0.
- $U(u_2)$ collapses to i^0.

$$\Phi = \int_{I^-} \sqrt{q}\; t^a k^b\, T^{lin}_{ab}(h + h_0)$$