CFT\ADS & ADS/CFT

Mukund Rangamani
Durham University

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Hubeny, Marolf, MR

Aharony, Marolf, MR

OUTLINE

- Motivation
- Free fields on AdS_4: boundary conditions
- $\mathcal{N} = 4$ SYM on AdS_4
- Discussion

Motivation

Field theory motivation:

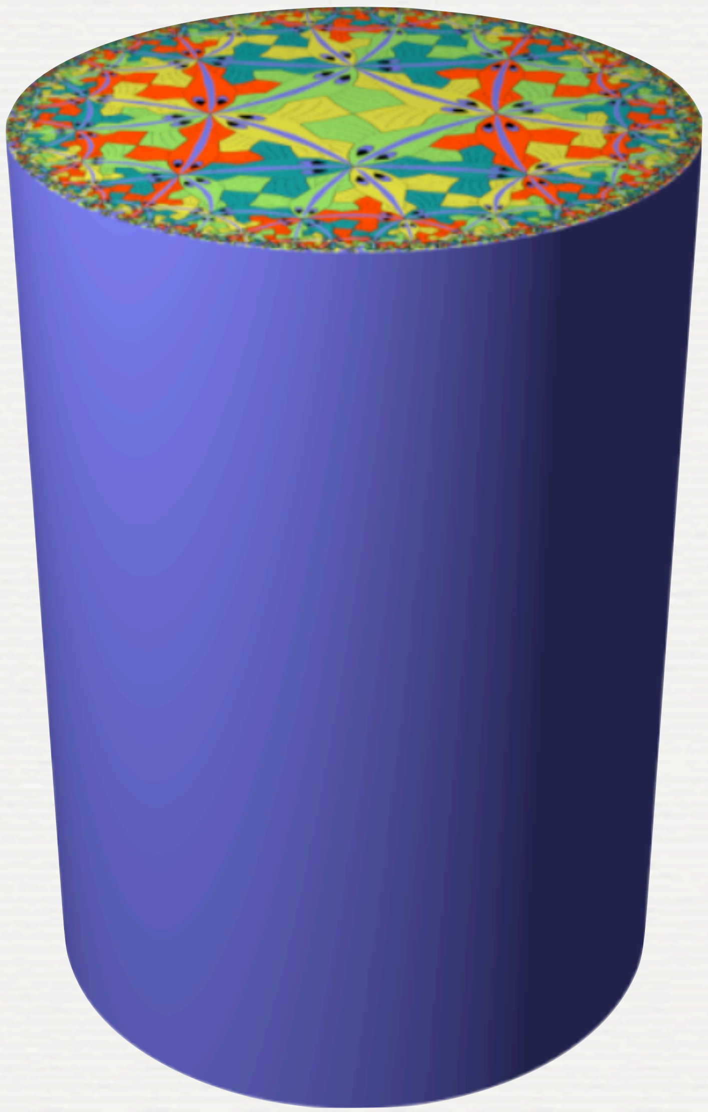
- Strongly coupled fields in curved spacetime
- AdS provides a geometric IR cut-off
- Better understanding of $\mathcal{N} = 4$ dynamics

AdS/CFT motivation:

- `Russian doll' holography:

- boundary dynamics of strongly interacting QFT + weak AdS gravity
- AdS duals of field theories on AdS

Global AdS spacetime



$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{\ell^2}\right)} + r^2\, d\Omega^2$$

Global AdS is like a cylinder with a time-like boundary which is a copy of the Einstein Static Universe (Lorentzian cylinder).

Fields on AdS: Boundary conditions

- AdS is not globally hyperbolic, and has a timelike $\mathscr{I}^{+}$ $\Rightarrow$ boundary conditions are necessary to prescribe dynamics.

- Scalars in $\mathrm{AdS_d}$: $m^2\, l^2 = \Delta(\Delta - (d\text{-}1))$

$$\phi(r) \to A(x)\, r^{\Delta-(d-1)} + B(x)\, r^{-\Delta}$$

- Usual boundary conditions: fix A (source).
- For $m_{BF}{}^2 \leq m^2 \leq m_{BF}{}^2 + 1$ can alternately fix B to be the source.

$$m_{BF}^2 = -\frac{(d-1)^2}{4\,\ell^2} \qquad m_c^2 = -\frac{d\,(d-2)}{4\,\ell^2}$$

- Conformally coupled scalars always lie in this window.

Breitenlohner, Freedman

Gauge fields on AdS: Boundary conditions

- Vector fields can also admit different boundary conditions
- Vectors in AdS_4:

$$A_r = 0 \ , \qquad\qquad A_\mu(r,x) \to a_\mu(x) + \frac{b_\mu(x)}{r}$$

- Dirichlet/standard/electric boundary conditions:

Marolf, Ross

- fix $a_\mu \Rightarrow$ source for boundary current J_μ

Ishibashi, Wald

- Neumann/modified/magnetic boundary conditions:

Witten

- fix b_μ or integrate over all sources $A_\mu \Rightarrow \langle J_\mu \rangle = 0$

Free gauge theories on AdS_4

- *SU(N)* gauge theory with Dirichlet boundary conditions:
  - No Gauss law constraint
  - $\mathcal{O}(N^2)$ excitations about the vacuum.

- *SU(N)* gauge theory with Neumann boundary conditions:
  - No charged states allowed
  - Gauss law constraint
  - $\mathcal{O}(1)$ excitations about the vacuum.
  - Theory undergoes a Hagedorn transition at $T_c\, l \sim 1$.

$\mathcal{N} = 4$ SYM on AdS_4

- Field content (all adjoint valued):
- 6 conformally coupled scalars
- 4 Weyl fermions
- Gauge fields

$$\omega\,\ell = \Delta + k + 2\,n\ , \qquad k, n \in \mathbb{Z}_+$$

- Lots of choices of boundary conditions:
- scalars can have $\Delta = 1, 2$
- gauge fields Neumann/Dirichlet

N = 4 SYM on AdS$_4$: Two puzzles

- $\mathcal{N} = 4$ has $SL(2,Z)$ S-duality.
- Expect exchange Dirichlet and Neumann bcs (true for abelian theory), *but*
  - Dirichlet: $\mathcal{O}(N^2)$ excitations about the vacuum
  - Neumann: $\mathcal{O}(1)$ excitations about the vacuum
- Phase transition as a function of coupling?

- Expect to have holographic dual in AdS$_5$
  - how does bulk dual have N^2 dofs?
  - New black holes in AdS?

$\mathcal{N} = 4$ SYM on AdS$_4$: Susy bc

- Useful to look at susy preserving bcs for $\mathcal{N} = 4$
- Have to treat scalars asymmetrically: $SO(6) \rightarrow SO(3) \times SO(3)$

Breitenlohner, Freedman

- Lots of 1/2-BPS boundary conditions.
- Classified for theory on half-space $\boldsymbol{R}^{2,1} \times \boldsymbol{R}_{+}$ (conformal to AdS$_4$)

- Neumann bc: N D3-branes ending on a single NS5-brane
- S-dual is not the Dirichlet bc.
- Rather, N D3-branes end on a single D5-brane with SU(N) breaking bcs.
- Dirichlet bc involves N D5-branes.

Gaiotto, Witten

N = 4 SYM on AdS$_4$: Susy bc

- Gaiotto-Witten bcs are characterized by a triple (ϱ, H, $\mathscr{B}$).

- ϱ (Nahm data): SU(2) → G
- H: commutant of Nahm data (preserved gauge group)
- $\mathscr{B}$: Boundary CFT living on R X S^2.

Gauge group	**Quotient**	**Dual gauge group**	**Nahm data**	**Boundary dof**
$SO(N)$	$O5^+$	$SU(N)$	0	$2+1$ SCFT
$USp(N)$	$O5^-$	$SU\left(\frac{N}{2}\right)_d \subset SU(N)$	N-dim irrep of $SU(2)$	0
$SU(N/2) \times SU(N/2)$	$\mathbb{Z}_2(-1)^{F_L}$	$USp(N)$	0	D5-hypers
$SU(p) \times SU(q)$	$\mathbb{Z}_2(-1)^{F_L}$	$USp(2q)$	$N = (p-q) + q \times 1$	0

Holographic duals of N = 4 SYM on AdS_4

- Expect holographic duals in terms of strings in $AdS_5 \times S^5$
- AdS_4 foliations AdS_5:
  - boundary is a double cover of AdS_4
  - identify two copies of AdS_4 via some orbifold/orientifold
  - alternately work with transparent bcs.

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- GW bcs with orbifolds/orientifolds preserving large enough gauge symmetry have duals in terms of strings on $AdS_5 \times S^5$ with orbifold/orientifold 5-plane wrapping $AdS_4 \times S^2$.

Summary & Open issues

- Rich dynamics for field theories on AdS_4.
- Lots of the structure of the dynamics is due to the non-trivial nature of the boundary conditions.
- Possibility of various different bcs for scalars, vectors, fermions.

- The story for $\mathcal{N} = 4$ is very intricate.
- Discussed only susy (1/2-BPS) bcs, for which already there many interesting issues.
- Could look for duals for other bcs with less/no susy.