Exotic Branes and Non-Geometric Backgrounds

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Introduction

U-duality

- Relates various objects in string/M-theory

  T: Dp ↔ D(p±1), F1 ↔ P, NS5 ↔ KKM, …

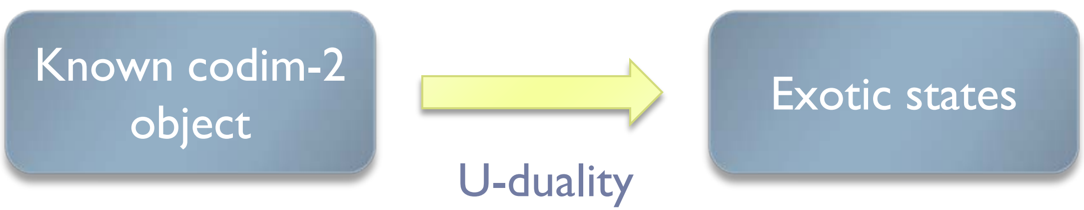
  S: F1 ↔ D1, NS5 ↔ D5, …

- Enhances in lower dims.
  - M-theory on T^k: $E_{k(k)}(Z)$ [Hull+Townsend]

k	D	G(Z)
1	10	1
2	9	$SL(2,Z) \times Z_2$
3	8	$SL(3,Z) \times SL(2,Z)$
4	7	$O(5,5,Z)$
5	6	$SL(5,Z)$
6	5	$E_{6(6)}(Z)$
7	4	$E_{7(7)}(Z)$
8	3	$E_{8(8)}(Z)$

Codimension-2 objects (1)

- U-duality on codim-2 objects produces exotic states



Known 10D/11D object wrapped on internal torus

10D/11D origin unknown!

They can have mass $\sim g_s^{-3}, g_s^{-4}$

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9809039 Obers+Pioline]

Codimension-2 objects (2)

- Example: Type II on T^2

noncompact R^7 (directions 1–7); internal T^2 (directions 8–9)

	1	2	3	4	5	6	7	8	9
NS5	·	·	O	O	O	O	O	~	~

T-duality along x^8, x^9

$$5^2_2$$

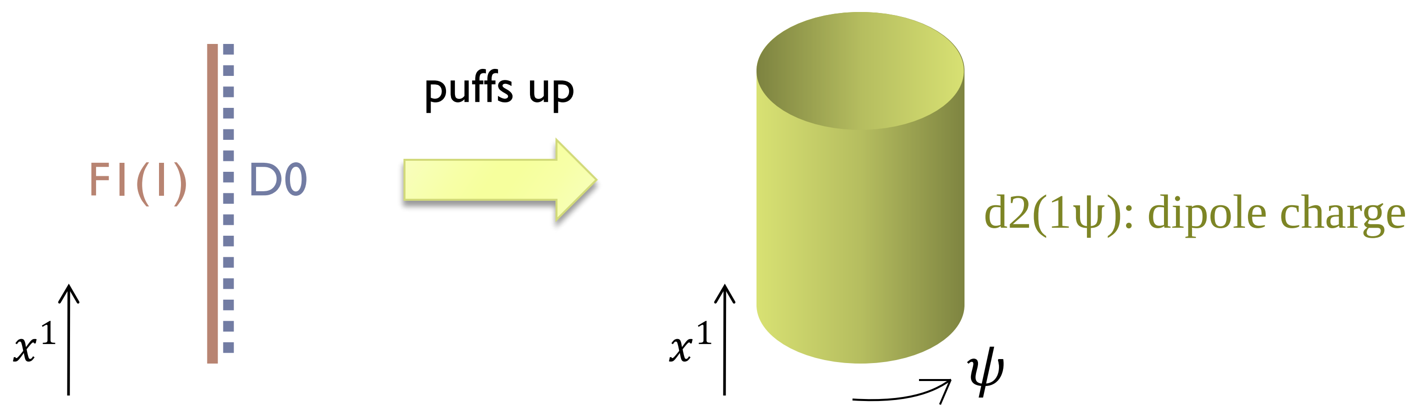
We will see that this is a "T-fold".

Supertube effect

- Codim-2 objects: problematic
  - Log divergences

$$V \sim \frac{1}{r^{d-2}} \xrightarrow{d=2} V \sim \log\left(\frac{\mu}{r}\right)$$

- Why care about exotic stuff anyway?
- Supertube effect = spontaneous polarization [Mateos+Townsend]



Relevance of exotic branes

- Non-exotic branes can puff up to produce exotic dipole charges
  - → No log divergence
  - → Exotic branes are relevant for non-exotic physics!

- Black holes: bound states of branes
  - → Generic microstates include exotic charges
  - → Microstate non-geometries?

Outline

- Introduction
- Exotic states & their higher-D origin
- Sugra description
- Supertube effect
- Conclusion

Exotic states and their higher-D origin

Exotic states in 3D (1)

- M-theory on T^8 or Type II on T^7
  - → 3D N=16 sugra
  - → 128 scalars (in 3D, scalar=vector)
  - → U-duality group $E_{8(8)}(Z)$: generated by T- and S-dualities
- Particle multiplet:
  - → Start from a point-like object

    e.g. D7 completely wrapped on T^7
  - → Take T- and S-dualities to get other states

Exotic states in 3D (2)

- Particle multiplet:

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9809039 Obers+Pioline]

Type IIA	P ($\mathbf{7}$), F1 ($\mathbf{7}$), D0 ($\mathbf{1}$), D2 ($\mathbf{21}$), D4 ($\mathbf{35}$), D6 ($\mathbf{7}$), NS5 ($\mathbf{21}$), KKM ($\mathbf{42}$), 5_2^2 ($\mathbf{21}$), 0_3^7 ($\mathbf{1}$), 2_3^5 ($\mathbf{21}$), 4_3^2 ($\mathbf{35}$), 6_3^1 ($\mathbf{7}$), $0_4^{(1,6)}$ ($\mathbf{7}$), 1_4^6 ($\mathbf{7}$)
Type IIB	P ($\mathbf{7}$), F1 ($\mathbf{7}$), D1 ($\mathbf{7}$), D3 ($\mathbf{35}$), D5 ($\mathbf{21}$), D7 ($\mathbf{1}$), NS5 ($\mathbf{21}$), KKM ($\mathbf{42}$) , 5_2^2 ($\mathbf{21}$), 1_3^6 ($\mathbf{7}$), 3_3^4 ($\mathbf{35}$), 5_3^2 ($\mathbf{21}$), 7_3 ($\mathbf{1}$), $0_4^{(1,6)}$ ($\mathbf{7}$), 1_4^6 ($\mathbf{7}$)
M-theory	P ($\mathbf{8}$), M2 ($\mathbf{28}$), M5 ($\mathbf{56}$), KKM ($\mathbf{56}$), 5^3 ($\mathbf{56}$), 2^6 ($\mathbf{28}$), $0^{(1,7)}$ ($\mathbf{8}$)

248 states

- Notation for exotic states

$$b_n^c : M = \frac{R^b (R^c)^2}{g_s^n} \qquad b_n^{(d,c)} : M = \frac{R^b (R^c)^2 (R^d)^3}{g_s^n}$$

Example: $5_2^2(34567,89)$: $M = \frac{R_3 \cdots R_7 \, (R_8 R_9)^2}{g_s^2 l_s^8}$

Duality rules

- Duality rules can be read off from:

$$T_y: \quad R_y \to \frac{l_s^2}{R_y}, \quad g_s \to \frac{l_s}{R_y} g_s \qquad\qquad S: \quad g_s \to \frac{1}{g_s}, \quad l_s \to g_s^{1/2} l_s$$

- Example:

$$\mathrm{NS5}(34567) \overset{T_8}{\to} \mathrm{KKM}(34567{,}8) \overset{T_9}{\to} 5_2^2(34567{,}89)$$

$$M = \frac{R_3 \cdots R_7}{g_s^2 l_s^6} \overset{T_8}{\to} \frac{R_3 \cdots R_7}{(g_s l_s / R_8)^2 l_s^6} = \frac{R_3 \cdots R_7 R_8^2}{g_s^2 l_s^8} : 5_2^1 = \mathrm{KKM}$$

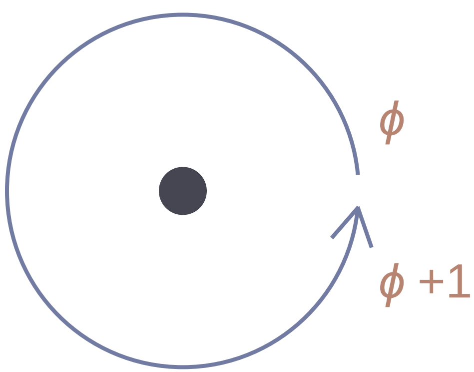
$$\overset{T_9}{\to} \frac{R_3 \cdots R_7 R_8^2}{(g_s l_s / R_8)^2 l_s^8} = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^{10}} : 5_2^2$$

Higher D origin = U-folds (1)

- Claim: higher D origin is
  U-fold = non-geometric background

E.g. D7 on T^7

- (magnetically) coupled to RR 0-form C_0
- 3D scalar $\phi = C_0$
- Monodromy: $\phi \rightarrow \phi + 1$ (part of SL(2,Z) symmetry of IIB)



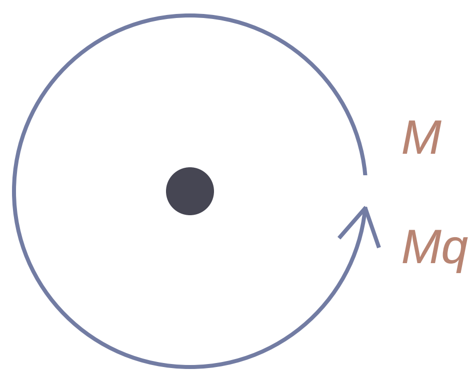
Higher D origin = U-folds (2)

- In 3D, charge = monodromy of scalar ϕ
- Shifting symmetry + S, T-dualities ⟹ $E_{8(8)}(\mathbb{Z})$
- ϕ gets combined with other scalars to form moduli matrix

$$M \in \mathcal{M} = SO(16)\backslash E_{8(8)}(\mathbb{R})/E_{8(8)}(\mathbb{Z})$$

- Can consider a particle with general U-duality monodromy

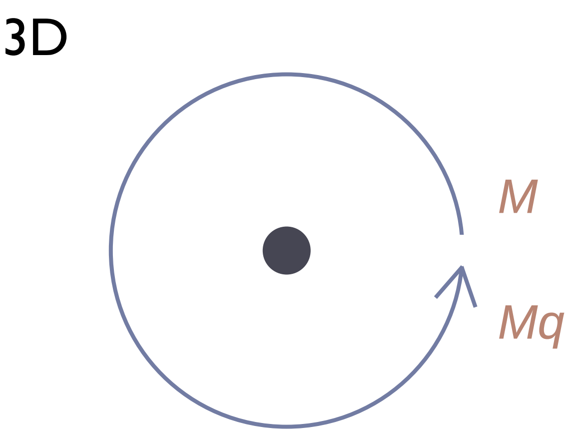
$$q \in E_{8(8)}(\mathbb{Z})$$

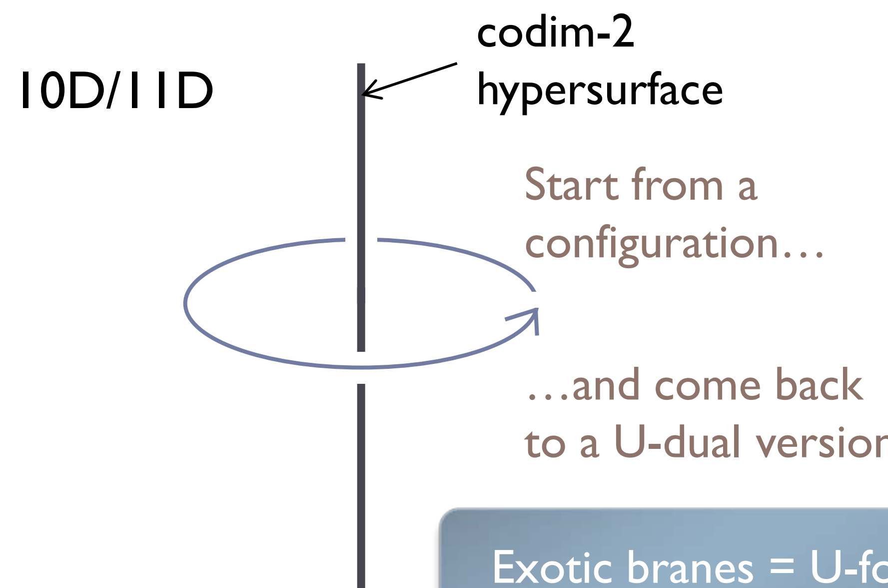


"Charge" of a 3D particle is U-duality monodromy around it!

Higher D origin = U-folds (3)

- In 10D/11D, we have a non-geometric U-fold





Exotic branes = U-folds

Cf. F-theory, U-branes & non-geom bg

[Greene+Shapere+Vafa+Yau] [Vafa] F-theory
[Kumar+Vafa] [Liu+Minasian] [Hellerman+McGreevy+Williams] contractible U-branes
[Dabholkar+Hull] [Flournoy+Wecht+Williams] … non-contractible U-branes & moduli stabl'n

Sugra description of exotic states

Sugra solution for 5^2_2 (1)

KKM/TN(56789,4):

$$ds^2 = dx^2_{056789} + H dx^2_{123} + H^{-1}(dx^4 + \omega)^2$$

$$e^{2\Phi} = 1, \;\; d\omega = *_3 \, dH,$$

$$H = 1 + \sum_p H_p \,, \quad H_p = \frac{R_4}{2|\vec{x} - \vec{x}_p|}$$

$\vec{x}_p$: positions of centers in $\mathbb{R}^3_{123}$

↓ compactify x^3

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right)$$

↓ T-dualize along x^3 (Buscher rule)

Sugra solution for 5^2_2 (2)

$5^2_2(56789,34)$ metric:

$$ds^2 = H(dr^2 + r^2 d\theta^2) + HK^{-1} dx_{34}^2 + dx_{056789}^2$$

$$B_{34}^{(2)} = -K^{-1}\theta\sigma, \quad e^{2\Phi} = HK^{-1}, \quad K \equiv H^2 + \sigma^2\theta^2$$

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right) \qquad \sigma = \frac{R_3 R_4}{2\pi\alpha'}$$

Cf. [Blau+O'Loughlin]: 6^1_3

- T-fold structure:

$$\theta = 0: \quad G_{33} = G_{44} = H^{-1},$$

$$\theta = 2\pi: \quad G_{33} = G_{44} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

→ x^3-x^4 torus size doesn't come back to itself!

Comments

- Not well-defined as stand-alone objects
  - Log divergence
    - → Superpositions (cf. F-theory 7-branes)
    - → Configs with higher codims. (next topic)
- Easy to get sugra metric for other exotic branes
  - Questionable for type II branes with $M \sim g_s^{-3}, g_s^{-4}$

Supertube effect and exotic branes

Dualizing supertube effect

Original supertube effect:

$$\mathrm{D0} + \mathrm{F1}(1) \to \mathrm{D2}(1\psi)$$

dualize

Various other known puff-ups:

$$\mathrm{F1}(1) + \mathrm{P}(1) \to \mathrm{F1}(\psi)$$ FP/DH sys

$$\mathrm{D1}(1) + \mathrm{D5}(12345) \to \mathrm{KKM}(2345\psi, 1)$$ LM geom

$$\mathrm{M2}(12) + \mathrm{M2}(34) \to \mathrm{M5}(1234\psi)$$ black ring

Puff-ups involving exotic charges

$$\mathrm{D0} + \mathrm{F1}(1) \to \mathrm{D2}(1\psi)$$

dualize

Exotic puff-ups:

$$\mathrm{D4}(6789) + \mathrm{D4}(4589) \to 5^2_2(4567\psi, 89)$$

: part of 4D BH system

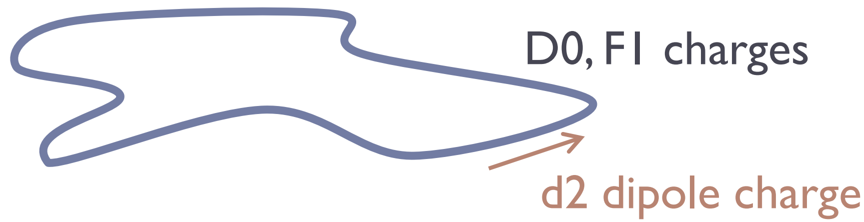
$$\mathrm{D3}(589) + \mathrm{NS5}(46789) \to 5^2_3(4567\psi, 89)$$

$$\mathrm{NS5}(46789) + \mathrm{KKM}(46789{,}5) \to 1^6_4(\psi, 456789)$$

$$g_s{}^{-a} + g_s{}^{-b} \to g_s{}^{-a-b}$$

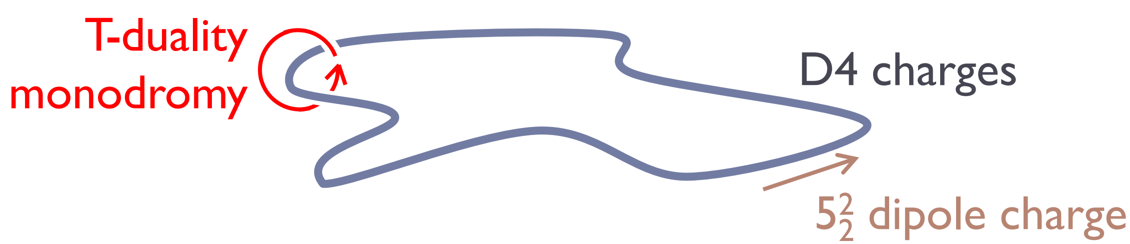
Sugra solution for D4+D4→5^2_2

- Basic sugra supertube



Dualize

- Exotic 2-charge solution



D4(6789)+D4(4589)→5_2^2 (4567ψ,89)

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}(dt - A)^2 + \sqrt{f_1 f_5}\, dx_{123}^2 + \sqrt{\frac{f_1}{f_2}}\, dx_{45}^2 + \sqrt{\frac{f_2}{f_1}}\, dx_{67}^2 + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}\, dx_{89}^2,$$

f_i , A : sourced along curve

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\vec{F}}(v)|^2}{|\vec{x} - \vec{F}(v)|}dv, \quad A_i = -\frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|}$$

$$d\beta_I = *_3\, df_I, \qquad d\gamma = *_3\, dA$$

- β_i, γ have monodromy around curve

$$\beta_I \to \beta_I - 2Q_I, \quad \gamma \to \gamma - 2q,$$

→ T-fold structure just as before

- No long distance log div — asymptotically flat 4D

D4(6789)+D4(4589)→5^2_2 (4567ψ,89)

Other fields:

$$e^{2\Phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}, \qquad B^{(2)}_{89} = \frac{\gamma}{f_1 f_2 + \gamma^2}, \qquad C^{(3)} = -\gamma\rho + \sigma$$

$$\rho = (f_2^{-1} + dt - A) \wedge dx^4 \wedge dx^5 + (f_1^{-1} + dt - A) \wedge dx^6 \wedge dx^7$$

$$\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7$$

Circular D4+D4$\rightarrow 5_2^2$

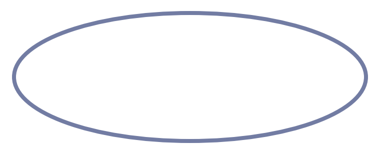
For circular profile, all functions can be explicitly written down

$$dx_{123}^2 = \frac{R^2}{(\cos\phi - y)^2}\left[\frac{dy^2}{y^2-1} + (y^2-1)d\psi^2 + d\phi^2\right]$$

$$f_I = 1 + \frac{Q_I}{R}\sqrt{\frac{\cos\phi - y}{-2y}}\,F\left(\frac{1}{4},\frac{3}{4};1;z^2\right),\quad A_\psi = -\frac{qR}{2}\frac{y^2-1}{(\cos\phi - y)^{1/2}(-2y)^{3/2}}\,F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)$$

$$\gamma = -\frac{q\sqrt{1-y}}{4\sqrt{2}(-y)^{3/2}}\left\{(1+y)\mathbf{F}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)\right.$$
$$\left. + u\mathbf{E}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)\left[3F\left(\frac{3}{4},\frac{1}{4};2;z^2\right) + F\left(\frac{3}{4};\frac{5}{4};2;z^2\right)\right]\right\}$$

$$\beta_I = \cdots$$



$$z = 1 - y^{-2}$$

Puff-ups and BH microstates (1)

- Standard 4D BH system

$$\mathrm{D0},\ \ \mathrm{D4}(6789),\ \ \mathrm{D4}(4589),\ \ \mathrm{D4}(4567)$$

: Well studied for microstate counting [MSW]

- Possible puff-ups:

	$\mathrm{D4}(6789)$		$\mathrm{NS5}(6789\psi)$	$5^2_2(6789,45\psi)$
$\mathrm{D0}$	$\mathrm{D4}(4589)$	puff up →	$\mathrm{NS5}(4589\psi)$	$5^2_2(4589,67\psi)$
	$\mathrm{D4}(4567)$		$\mathrm{NS5}(4567\psi)$	$5^2_2(4567,89\psi)$

The three 5^2_2 charges are labelled: non-geometric

puff up → More exotic charges?

Puff-ups and BH microstates (2)

- 5D BH system

$$M2(56),\ \ M2(78),\ \ M2(9A)$$

: Well studied for microstate geometry

[Mathur] [Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van de Bleeken]

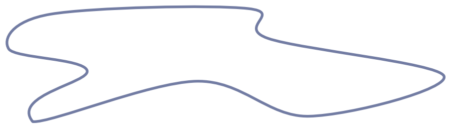
- Possible puff-ups:

$M2(56)$	puff up →	$M5(789A\psi)$	puff up →	$5^3(789A\phi, 56\psi)$
$M2(78)$		$M5(569A\psi)$		$5^3(569A\phi, 78\psi)$
$M2(9A)$		$M5(5678\psi)$		$5^3(5678\phi, 9A\psi)$
		cf. black ring		non-geometric

Puff-ups and BH microstates (3)

2-charge system

- Worldvolume theory:
  - Higgs branch coming from intersection of two stacks
- Gravity:
  - Fluctuation of 1-dimensional object

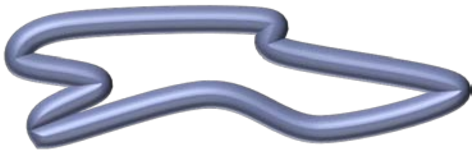


$$S_{brane} = S_{gravity}$$

Puff-ups and BH microstates (4)

3-charge system

- Worldvolume theory:
  - More complicated Higgs branch from triple intersection
- Gravity:



  - Fluctuation of 2-dimensional object?
    - — Exotic branes has just the right dimensionality
  - Explains missing entropy in sugra microstate geometries?

$S_{brane} = S_{gravity}$??

[Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van den Bleeken]

Conclusions

Conclusions

- Exotic branes = non-geometries (U-folds)
- Exotic charges = U-duality monodromies
- Relevant even for non-exotic physics by supertube effect
- Unexplored exotic land out there awaiting us!
  - Classification of exotic branes (bound states, etc.)
  - Non-Abelian anyon statistics
  - AdS/CFT
  - Microstate non-geometries
  - 4D black ring??

Enjoy!