Black holes of $\mathcal{N} = 8$ supergravity

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The simplicity of $\mathcal{N} = 8$ supergravity

- Obtained as the low energy regime of M-theory upon compactification on T^7
- Inherits $E_{7(7)}(\mathbb{R})$ symmetries broken to $E_{7(7)}(\mathbb{Z})$ at the quantum level Cremmer and Julia (1978), etc
- In perturbation theory, UV finite up to 4, maybe 8, loops Bern et al. (2009), etc
- In perturbation theory, amplitudes enjoy remarkable properties : square of $N = 4$ SYM, etc Kawai, Lewellen, Tye (1986), etc
- The theory cannot be decoupled from string theory Green, Ooguri, Schwarz (2007)
- The theory contains BPS extremal black holes with known microscopic counting in M-theory Maldacena, Strominger, Witten (1997).

Ideal setup to address black hole microscopics

- All known non-extremal black holes in string theory obey
  Cvetič and Larsen (1997)

$$S_+ S_- \in \pi^2 \mathbb{Z}$$

- All known extremal black holes have $SL(2,\mathbb{R}) \times U(1)$ $\to SL(2,\mathbb{R}) \times \mathit{Virasoro}$ symmetries and obey

$$S_+ = \frac{\pi^2}{3} c_J T_J = \frac{\pi^2}{3} c_Q T_Q$$

Are there other “IR” universal formulae ? Is there a proof from the “UV" ?

In this talk, I will present the most general asymptotically flat non-extremal black hole of $\mathcal{N} = 8$ supergravity and derive some of its properties.

(Next to simple) Gauged $\mathcal{N} = 8$ $SO(8)$ supergravity

- Obtained as the low energy regime of M-theory upon compactification on S^7
- Admits AdS_4 as a $\mathcal{N} = 8$ vacuum
- When embedded in M-theory, the dual CFT is the ABJM theory at level $k = 1$. Aharony, Bergman, Jafferis, Maldacena (2008)
- New sugra theories labelled by a continuous parameter have been recently found Dall'Agata, Inverso, Trigiante (2012)
- Much less is known on solutions and "IR" relations
- Known black holes in AdS_4 obey Cvetič, Gibbons, Pope (2010)

$$S_+ S_- S_i S_{-i} \in \pi^4 \ell^4 \mathbb{Z}$$

In this talk, I will present some black holes of this theory and discuss subtleties with their thermodynamics

Based on work done with David Chow (ULB)

- "Seed for general rotating non-extremal black holes of $\mathcal{N} = 8$ supergravity", arXiv :1310.1925
- "Dyonic AdS black holes in maximal gauged supergravity", arXiv :1311.1204
- "Black holes in $\mathcal{N} = 8$ supergravity from $SO(4,4)$ hidden symmetries", arXiv :1404.2602

Outline

1. *STU* supergravity and hidden symmetries
2. The general black hole and its properties
3. Microscopic counting for extremal branches
4. Dyonic black holes of maximal gauged supergravity

I. *STU* supergravity and hidden symmetries

From $\mathcal{N} = 8$ to *STU* supergravity

Under U-dualities, the $4d$ metric will be unchanged. The matter field will be shuffled around.

There are 28 abelian gauge fields in $\mathcal{N} = 8$ supergravity. If one starts with a dyonic rotating black hole with 3 additional electric charges (5 charges in total), one can perform U-dualities and generate the general black hole. Cvetič-Hull, 1996

A suitable sector of $\mathcal{N} = 8$ supergravity is a $\mathcal{N} = 2$ supergravity with three vector multiplets known as the *STU* supergravity Cremmer et al '85 ; Duff et al. '96. This is the theory that we will study here.

STU supergravity

In a specific U-duality frame the Lagrangian has the general form Duff et al. ’96

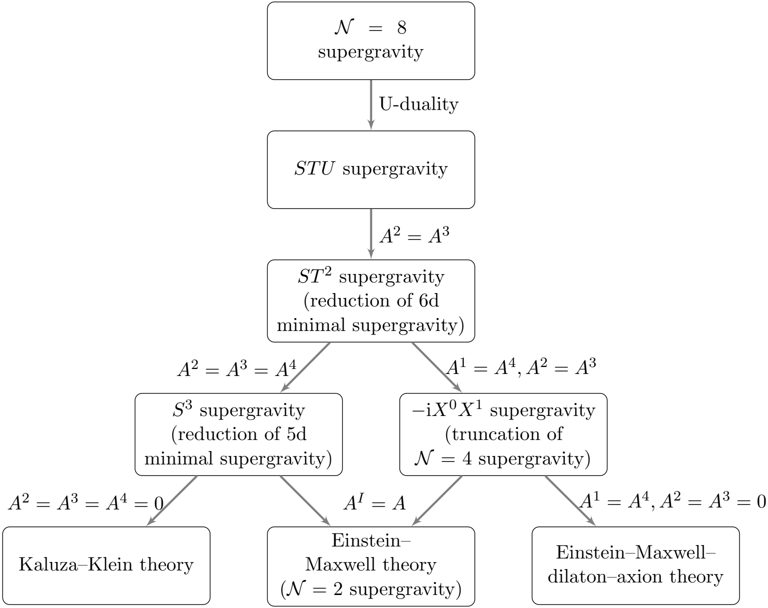
$$\begin{aligned}\mathcal{L}_4 &= d^4x\sqrt{-g}\Big(R-\frac{1}{2}f_{AB}(z)\partial_\mu z^A\partial^\mu z^B \\ &\quad -\frac{1}{4}k_{IJ}(z)F^I_{\mu\nu}F^{J\mu\nu}+\frac{1}{4}h_{IJ}(z)\epsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma}\Big)\end{aligned}$$

where

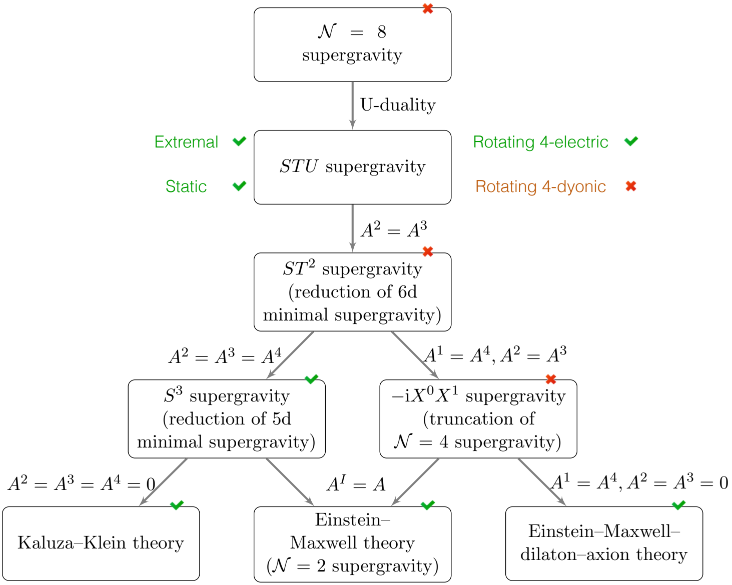
- $z_j = x_j + \mathrm{i}\, y_j$, $j = 1,2,3$ are three complex scalar fields
- $A^I = (A^1, A^2, A^3, A^4)$ are the four $U(1)$ gauge fields.

Triality symmetry : $SL(2,\mathbb{R})\times SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ and their permutations.

From $\mathcal{N} = 8$ to STU supergravity



Known solutions



Algorithm for solution generation

- Reduce on time. Get Euclidean $3d$ Einstein gravity coupled to scalars.
- Scalars form the $\frac{SO(4,4)}{SL(2,\mathbb{R})^4}$ coset model Breitenlohner, Maison, Gibbons, '88
- Take Kerr-Taub-NUT (a, m, n) as a seed
- Act with $SL(2,\mathbb{R})^4 \subset SO(4,4)$ hidden symmetries
- Generate all 4 Q_I + 4 P^I charges
- Cancel the total NUT charge at the end by tuning the initial NUT charge n
- Iterate the loop "Recognize patterns $\to$ Simplify $\to$ Rewrite $\to$ Recognize patterns $\to \ldots$"

II. The general black hole and its properties

The Kerr metric (1963)

$$ds^2 = -\frac{R-U}{W}(dt+\omega_3)^2 + W\left(\frac{dr^2}{R} + \frac{du^2}{U} + \frac{RU}{a^2(R-U)}d\phi^2\right),$$

where

$$W(r,u) = r^2 + u^2,$$
$$\omega_3(r,u) = \frac{2mrU}{a(R-U)}d\phi,$$

and

$$R(r) = r^2 - 2mr + a^2,$$
$$U(u) = a^2 - u^2,$$

The standard spherical polar angle θ is $u = a\cos\theta$.

The $\mathcal{N} = 8$ black hole metric

Input : M, N, m, n, a + 2 harmonic functions $L(r)$, $V(u)$:

$$ds^2 = -\frac{R-U}{W}\left(dt + \omega_3\right)^2 + W\left(\frac{dr^2}{R} + \frac{du^2}{U} + \frac{RU}{a^2(R-U)}\,d\phi^2\right),$$

where

$$W^2(r,u) = (R-U)^2 + (2Nu + L)^2 + 2(R-U)\,(2Mr + V)\,,$$

$$\omega_3(r,u) = \frac{2N(u-n)R + U(L + 2Nn)}{a(R-U)}\,d\phi,$$

$$R(r) = r^2 - 2mr + a^2 - n^2,$$

$$U(u) = a^2 - (u-n)^2,$$

$$L(r) = L_1 r + L_2,$$

$$V(u) = V_1 u + V_2.$$

The standard spherical polar angle θ is $u = n + a\cos\theta$.

Separability and hidden conformal symmetry

Define the string frame metric

$$\tilde{ds}^2 = \frac{r^2 + u^2}{W} ds^2.$$

It admits an irreducible Killing-Stäckel tensor K_{ab} obeying

$$\nabla_{(a} K_{bc)} = 0.$$

The massive Klein-Gordon equation on $\tilde{ds}^2$ is separable.
Therefore, the metric ds^2 admits an irreducible conformal Killing-Stäckel tensor with components $Q^{ab} = K^{ab}$ obeying

$$\nabla_{(a} Q_{bc)} = q_{(a} g_{bc)}.$$

The massless Klein-Gordon equation on ds^2 is separable.
[Subcases include : Chow, '08 ; Keeler, Larsen, '12]

Gauge fields : All is known !

We have

$$A^I = W \frac{\partial}{\partial \delta_I} \left(-\frac{1}{W}(dt + \omega_3) \right).$$

The electric and magnetic charges are

$$Q_I = 2\frac{\partial M}{\partial \delta_I}, \qquad P^I = -2\frac{\partial N}{\partial \delta_I}.$$

Scalar fields

The three axions and dilatons are :

$$\chi_i = \frac{f_i}{r^2 + u^2 + g_i}, \qquad e^{-\varphi_i} = \frac{W}{r^2 + u^2 + g_i},$$

where

$$f_i(r,u) = 2(mr + nu)\xi_{i1} + 2(mu - nr)\xi_{i2} + 4(m^2 + n^2)\xi_{i3},$$
$$g_i(r,u) = 2(mr + nu)\eta_{i1} + 2(mu - nr)\eta_{i2} + 4(m^2 + n^2)\eta_{i3},$$

are linear functions of r and u.

Thermodynamics

First law and Smarr relation hold

$$\delta M = T_+ \delta S_+ + \Omega_+ \delta J + \Phi^I_+ \delta \overline{Q}_I + \Psi^+_I \delta \overline{P}^I ,$$
$$M = 2T_+ S_+ + 2\Omega_+ J + \Phi^I_+ \overline{Q}_I + \Psi^+_I \overline{P}^I ,$$

Also at the inner horizon,

$$\delta M = T_- \delta S_- + \Omega_- \delta J + \Phi^I_- \delta \overline{Q}_I + \Psi^-_I \delta \overline{P}^I ,$$
$$M = 2T_- S_- + 2\Omega_- J + \Phi^I_- \overline{Q}_I + \Psi^-_I \overline{P}^I .$$

Quartic invariant

$$\Delta = \frac{1}{16}[4(Q_1Q_2Q_3Q_4 + P^1P^2P^3P^4) + 2\sum_{J<K} Q_JQ_KP^JP^K - \sum_J (Q_J)^2(P^J)^2].$$

The invariant is a Cayley hyperdeterminant, and is manifestly invariant under $\mathrm{SL}(2,\mathbb{R})^3$ upon rewriting as [Duff, '06]

$$\Delta = \frac{1}{32}\epsilon^{ii'}\epsilon^{jj'}\epsilon^{kk'}\epsilon^{ll'}\epsilon^{mm'}\epsilon^{nn'}a_{ijk}a_{i'j'm}a_{npk'}a_{n'p'm'}$$

with $\epsilon^{ij} = \epsilon^{[ij]}$, $\epsilon^{01} = 1$ and components a_{ijk} given by

$$(a_{000}, a_{111}) = -(Q_1, P^1), \qquad (a_{001}, a_{110}) = (P^2, Q_2),$$
$$(a_{010}, a_{101}) = (P^3, Q_3), \qquad (a_{011}, a_{100}) = (Q_4, P^4).$$

This invariant is a special case of a more general $E_{7(7)}$ quartic invariant [Kallosh, Kol, '96].

Universal properties of horizons

Product of area law : Cvetič, Gibbons, Pope, '10

$$\frac{A_+}{4}\frac{A_-}{4} = 4\pi^2 \left(J^2 + \Delta(Q_I, P^I)\right) \in \pi^2 \mathbb{Z}$$

Angular momentum law :

$$8\pi^2 J = \frac{\Omega_+}{T_+}(S_+ - S_-) \in 4\pi^2 \mathbb{Z}$$

Kinematic relationship :

$$\frac{\Omega_+}{T_+} = -\frac{\Omega_-}{T_-}$$

A new $E_{7(7)}(\mathbb{R})$ invariant

Using these properties, one can prove Cardy's form :

$$S_+ = 2\pi \left(\sqrt{\Delta + F} + \sqrt{-J^2 + F} \right)$$

Since S_+,J, Δ are $E_{7(7)}(\mathbb{R})$ invariants, then $F(M, Q_I, P^I, z^i_\infty)$ is invariant as well.

Known special cases :

- For BPS black holes, $F = J = 0$.
- In the extremal "fast" rotating limit, $F = J^2$.
- In the extremal "slow" rotating limit, $F = -\Delta$
- For Kerr-Newman : $F = M^4 - M^2 Q^2$.
- In regime $Q_{1,2,3} \to \infty$: $F = \frac{1}{2} Q_1 Q_2 Q_3 (M - M_{BPS})$ Cvetic-Larsen (2014)

F is not a rational function of the charges

For the Kaluza-Klein black hole (Rasheed-Larsen),

$$F = (M^2 - \frac{1}{4}P^2)(M^2 - \frac{1}{4}Q^2) + \frac{1}{3}(M^2 + \frac{1}{8}(P^2 + Q^2))^2 H[x]$$

where $0 \leq x \equiv \frac{54M^2(P^2-Q^2)^2}{(8M^2+P^2+Q^2)^3} \leq 1$ and

$$H[x] = 2\sqrt{1-x}\cos\frac{\arcsin\sqrt{x}}{3} + 6\sqrt{x}\sin\frac{\arcsin\sqrt{x}}{3} - 2$$

increases monotonically from 0 to 1.

It remains a challenge to write it in terms of $E_{7(7)}$ invariants and elucidate its structure.

III. Microscopic counting for extremal branches

Brane intersection table

In Type IIA frame, the black hole corresponds to

	t	r	θ	ϕ	z_1	z_2	z_3	z_4	z_5	z_6
D0	$\times$	$\cdot$	$\cdot$	$\cdot$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$
D2	$\times$	$\cdot$	$\cdot$	$\cdot$	$\times$	$\times$	$\sim$	$\sim$	$\sim$	$\sim$
D2	$\times$	$\cdot$	$\cdot$	$\cdot$	$\sim$	$\sim$	$\times$	$\times$	$\sim$	$\sim$
D2	$\times$	$\cdot$	$\cdot$	$\cdot$	$\sim$	$\sim$	$\sim$	$\sim$	$\times$	$\times$
D4	$\times$	$\cdot$	$\cdot$	$\cdot$	$\sim$	$\sim$	$\times$	$\times$	$\times$	$\times$
D4	$\times$	$\cdot$	$\cdot$	$\cdot$	$\times$	$\times$	$\sim$	$\sim$	$\times$	$\times$
D4	$\times$	$\cdot$	$\cdot$	$\cdot$	$\times$	$\times$	$\times$	$\times$	$\sim$	$\sim$
D6	$\times$	$\cdot$	$\cdot$	$\cdot$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

with total energy M and angular momentum J.

Two extremal limits

Attractor mechanism $\Rightarrow$ Moduli(Q_I, P^I)

Fast branch

$$S_+ = 2\pi\sqrt{\Delta + J^2}$$

- $1/8 - BPS\ D4 - D4 - D4$ system modeled by the MSW CFT $\Rightarrow_{IR}$ (0,4) CFT ($c = 6Q_1Q_2Q_3$).
- Attempts to deform this theory have been made to describe extremal Kerr-Newman black hole

Slow branch

$$S_+ = 2\pi\sqrt{-\Delta - J^2}$$

- non-BPS $D0 - D6$ system, not understood
- Contains rotating Kaluza-Klein 5d black hole

Universal semi-classical counting at extremality

The near-horizon solution is Kunduri, Lucietti, Reall, '07

$$ds^2 = W_+\left(-r^2\,dt^2 + \frac{dr^2}{r^2} + \frac{du^2}{U} + \Gamma^2(d\phi + kr\,dt)^2 \right),$$

$$A^I = f^I(d\phi + kr\,dt) + e^I\,d\phi/k,$$

$$\widetilde{A}_I = \widetilde{f}_I(d\phi + kr\,dt) + \widetilde{e}_I\,d\phi/k.$$

The entropy is Guica, Hartman, Song, Strominger, 2008 Hartman, Murata, Nishioka, Strominger 2008

$$S_+ = \frac{\pi^2}{3}c_J T_J = \frac{\pi^2}{3}c_{Q_1}T_{Q_1} = \dots$$

where

$$c_J = 12J, \qquad T_J = \frac{1}{2\pi k},$$

$$c_{Q_1} = 6\frac{\partial|\Delta|}{\partial Q_1}, \qquad T_{Q_1} = \frac{1}{2\pi e^1}.$$

Entropy of non-extremal black holes (?)

For static non-extremal electrically charged black holes Cvetič-Youm, 95, Horowitz-Lowe-Maldacena wrote in ’96 the mysterious formula

$$S = 2\pi(\sqrt{n_L} + \sqrt{n_R})(\sqrt{N_2} + \sqrt{N_{\bar{2}}})(\sqrt{N_5} + \sqrt{N_{\bar{5}}})(\sqrt{N_6} + \sqrt{N_{\bar{6}}})$$

in terms of free $D2, \bar{D}2, D6, \bar{D}6$, $NS5, \bar{NS}5$ and string left/right momentum n_L, n_R.

However, this formula does not generalize when a fifth charge is added.

IV. Dyonic black holes of maximal gauged supergravity

Dyonic black holes of gauged supergravity

Another theory of interest is $\mathcal{N} = 8$ SO(8) gauged supergravity that can be obtained from S^7 reduction of 11-dimensional supergravity.

A consistent truncation exists to $\mathcal{N} = 2$ $\mathrm{U}(1)^4$ gauged supergravity. The action is

$$\mathcal{L}_{\text{gauged}} = \mathcal{L}_{STU} + g^2 V[z^A] \star 1.$$

Cvetič, Duff, Hoxha, Liu, Lü, Martinez-Acosta, Pope, Sati, Tran, '99

There are very few dualities remaining. Solutions are harder to generate.

Two new classes of dyonic AdS_4 black holes

We guessed and checked :

- Most general spherical or planar static black holes of $U(1)^4$ $\mathcal{N} = 2$ gauged supergravity.
  [4 electric and 4 magnetic charges] They admit two conformal Killing tensors.
- Most general spherical rotating black holes of $\mathcal{N} = 2$ $U(1)^2$ gauged supergravity.
  [2 electric and 2 magnetic charges] They admit two Killing-Yano tensors with torsion.

This generalizes previously known subcases

Duff, Liu, '99, Chong, Cvetič, Lu, Pope, '05, Chow, '10, Lu, Pang, Pope, '13, Lu, '13.

We obtained a consistent black hole thermodynamics, except in some dyonic cases. Lü, Pang, Pope, 2013

Boundary conditions for dyons : free case

A $U(1)$ gauge field in AdS_4 obeys

$$A = A^{(0)} + \frac{1}{r}A^{(1)} + \dots = P\cos\theta d\phi + \frac{Q}{r}dt + \dots$$

A free Maxwell field has

$$\delta S = \int_{\partial\mathcal{M}} \eta^{ab} A^{(1)}_a \delta A^{(0)}_b \,.$$

Allowed boundary conditions :

- Dirichlet (P fixed) or Neumann (Q fixed).
- $SL(2,\mathbb{Z})$ family of boundary conditions compatible with a boundary CFT dual. Witten, '03
- Lorentz-violating boundary conditions exist when both Q, P vary $\Rightarrow$ Non-relativistic holography.

Boundary conditions for dyons : our case

For interacting theories, non-trivial couplings might prevent boundary conditions to be consistent. Then, the mass does not exist and the first law does not make sense.

We found two consistent classes of boundary conditions with dyons :

- $P^I = \pm Q_I \ \forall I = 1, .., 4$
- $P^1 = P^4$, $P^2 = P^3$ and $Q_1 = Q_4$, $Q_2 = Q_3$.
  This class contains the AdS-Kerr-Newman solution

In those cases, the mass exists and the first law of thermodynamics holds.

Take-home results

Asymptotically flat black holes :

- The general non-extremal stationary solution, including the matter sector, is now written in a manageable form.
- Two distinct extremal limits : BPS ($\supset$ Reissner-Nördstrom) and non-BPS ($\supset$ Rasheed-Larsen)
- Solution admits a conformal Killing tensor, implying separability and hidden conformal symmetries.
- Non-extremal entropy depends upon a new $E_{7(7)}$ invariant, $F(M, Q_I, P^I, \phi^i_\infty) \geq J^2$, as

$$S_+ = 2\pi\sqrt{\diamond + F} + 2\pi\sqrt{-J^2 + F}.$$

- The relation $\frac{\Omega_+}{T_+}(S_+ - S_-) \in 4\pi^2\mathbb{Z}$ is also universal.

Take-home results

Asymptotically AdS_4 black holes :

- General non-extremal stationary solution is beyond reach
- Static dyonic solutions with 4 electric and magnetic charges can be written in a nice form. They admit two conformal Killing tensors.
- Rotating dyonic solution with 2 electric and magnetic charges can be written in a nice form. They admit two Killing-Yano tensors with torsion.
- Mass is defined only when boundary conditions exist, which is prevented in general when both electric and magnetic charges are varied independently.