M-theoretic Matrix Models

Alba Grassi
Université de Genève

Mostly based on: A.G. , M. Mariño, 1403.4276

Outline

The ABJM theory: it has been possible to compute exactly the full partition function which includes the full series of instanton corrections.

M-theoretic matrix models: generalize the ideas behind ABJM. They are excellent laboratories to understand the structure of non-perturbative effects in the large N expansion, and also, via gauge/string dualities, non-perturbative effects in M-theory and string theory.

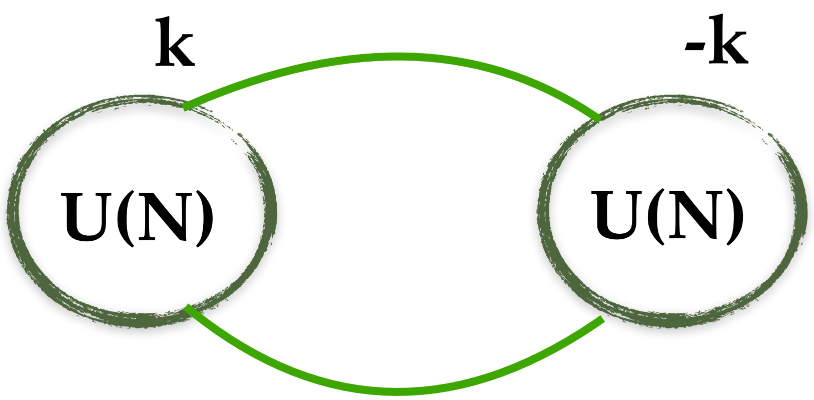
The N_f Matrix Model: is an M-theoretic matrix model with large N dual which displays a similar structure to ABJM.

Review of ABJM

ABJM:

- 2 Super Chern-Simons nodes with opposite coupling
- matter in bifundamental of $U(N)_k \times U(N)_{-k}$

Aharony, Bergman,
Jafferis, Maldacena.



1) **It has large N string / M theory dual:**

- M-theory dual on $AdS_4 \times S^7/Z_k$
- Type IIA string dual on $AdS_4 \times CP^3$

Maldacena

Review of ABJM

2) Localization: the partition function on $\mathbb{S}^3$ reduces to a matrix model

$$Z_{ABJM}(N,k) = \frac{1}{N!}\int \prod_{i=1}^{N} \frac{dx_i}{4\pi k} \frac{1}{2\cosh\frac{x_i}{2}} \prod_{i<j}\left(\tanh\left(\frac{x_i - x_j}{2k}\right)\right)^2 .$$

Kapustin, Willett, Yaakov

A related quantity: The *Grand Potential* $\mathbf{J}(\mu,\mathbf{k})$

$$Z(N,k) = \frac{1}{2\pi i}\int d\mu e^{J(\mu,k)-N\mu}$$

3) ABJM is deeply related to topological strings on local $\mathbb{P}_1 \times \mathbb{P}_1$

Aganagic, Klemm, Mariño, Vafa - Mariño, Putrov

Review of ABJM

ABJM can be studied in two different regimes:

1) The 't Hooft regime:

- **N large, k large:** $\lambda = \frac{\mathbf{N}}{\mathbf{k}}$ **fixed**

- **Make contact with IIA theory**

$$\left(\frac{L}{l_s}\right)^4 \sim \lambda, \quad g_{st} \sim \frac{1}{k}$$

- **The grand potential has a genus expansion**

$$J^{'tHooft} = \sum_g k^{2-2g} J_g\left(\frac{\mu}{k}\right)$$

2) M theory regime:

- **N large, k fixed**

- **Make contact with M-theory**

$$\left(\frac{L}{l_p}\right)^6 \sim Nk, \quad k: \ \text{geometrical}$$

- Z_{ABJM} **is the partition function of a Fermi gas and this regime is the thermodynamic limit .**

Mariño, Putrov

Review of ABJM

1) The 't Hooft regime:

The Grand potential reads

$$J^{'t\ Hooft} = J^p\left(\frac{\mu}{k}\right) + J^{WS}\left(\frac{\mu}{k}\right)$$

Polynomial part

GOPAKUMAR-VAFA

$$J^{WS}\left(\frac{\mu}{k}\right) = \sum_{m\geq 1} c_m(k) e^{-4m\frac{\mu}{k}}$$

- **Type IIA: worldsheet wrapping $\mathbb{CP}^1 \subset \mathbb{CP}^3$**
- **Invisible in Fermi gas approach around k=0.**
- **poles for finite k**

2) M theory regime:

The Grand potential reads

$$J(\mu, k) = J(\mu, k)^p + J^{np}(\mu, k)$$

Polynomial part

HATSUDA, MARIÑO, MORIYAMA, OKUYAMA

$$J^{np}(\mu, k) = \sum_{\ell\geq 1} d_m(k, \mu) e^{-2\ell\mu}$$

- **M-Theory: M2 Membrane wrapping 3-cycle**
- $\mu \sim k\sqrt{\lambda}$ **: Invisible in 't Hooft expansion.**
- **poles for finite k**

Review of ABJM

The point:

1) There are two types of instanton corrections:

- One visible in ’t Hooft expansion
- One visible in M-theory expansion / Fermi gas

2) Thanks to Topological strings we have an exact expression for n.p. corrections. Each of them has poles at physical value of the coupling constant. But when we sum them the poles cancel (HMO cancellation).

Hatsuda, Moriyama, Okuyama

- The ’t Hooft expansion is fundamentally incomplete
- The Fermi gas approach makes possible to go beyond ’t Hooft expansion

Generalization to others models.

We define "M-theoretic matrix models" as these models that can be studied in the two following expansions:

- The 't Hooft expansion: N large and the others parameters scale with N.
- The M-theory expansion: N large and the others parameters are kept fixed. From a statistical mechanics point of view this corresponds to the thermodynamic limit.

Examples

- Models with a large N dual: the N_f Model

  BENINI, CLOSSET, CREMONESI

- Models appearing in 2D statistical physics: the polymer matrix model

  ZAMOLODCHIKOV

The N_f Matrix Model

We consider the $\mathcal{N}=4$ SYM in 3d with the following matter content:

- **one vectormutliplet with gauge group U(N)**
- **one adjoint hypermultiplet**
- **N_f fundamental hypermultiplets**

➔ **Localization : the partition function on $\mathbb{S}^3$ reduces to a matrix model:**

$$Z(N,N_f)=\frac{1}{N!}\int\prod_{i=1}^{N}\frac{dx_i}{4\pi}\frac{1}{\left(2\cosh\frac{x_i}{2}\right)^{N_f}}\prod_{i<j}\left(\tanh\left(\frac{x_i-x_j}{2}\right)\right)^2$$

Kapustin, Willett, Yaakov

➔ **$N_f=1$: this is the partition function of ABJM for k=1**

The N_f Matrix Model

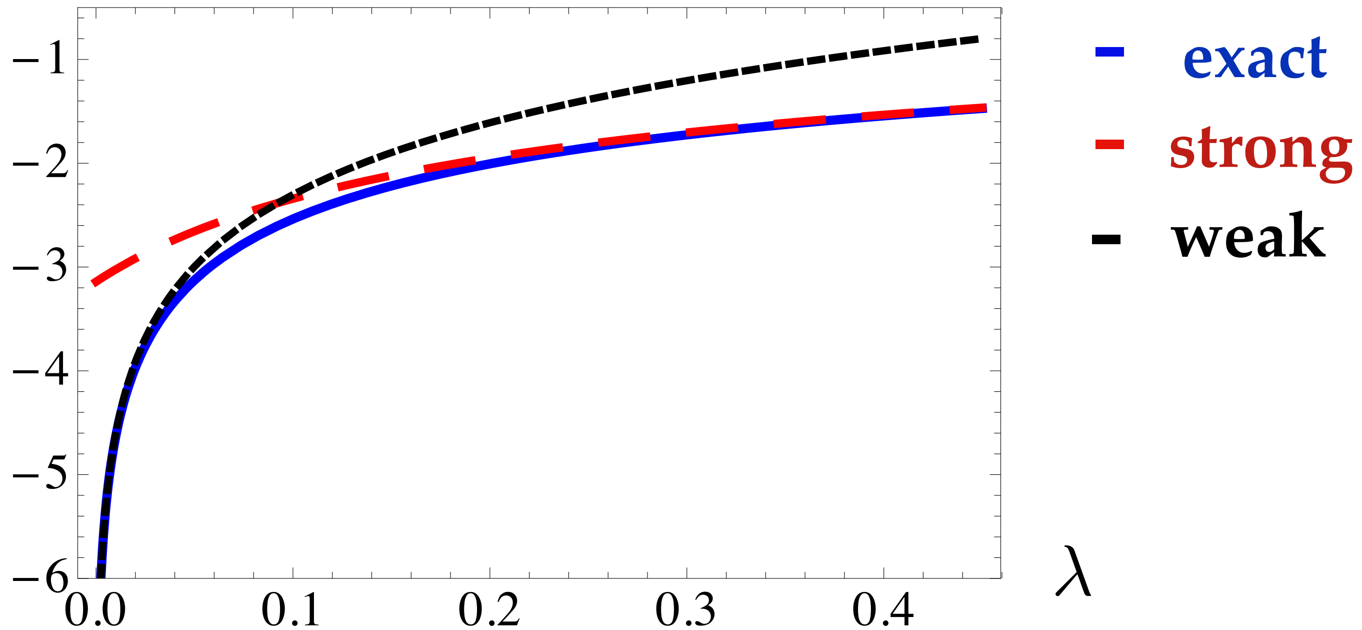
The partition function on $\mathbb{S}^3$ can be expressed as a O(2) matrix model:

$$Z(N, N_f) = \int \prod_{i=1}^{N} dz_i \, e^{-N_f \log\left(z^{1/2}+z^{-1/2}\right)} \prod_{i<j} (z_i - z_j)^2 \prod_{i,j} (z_i + z_j)^{-1}$$

By exploiting the O(2) formalism we solved the planar and genus one limit:

Example: $F_0 = J_0 - \mu N / N_f^2$

$$\frac{d^2 F_0}{d^2 \lambda}$$



The N_f Matrix Model

From the point of view of M theory:

- **Describes N M2 branes probing the space** $\mathbb{C}^2 \times \left(\mathbb{C}^2 / \, \mathbb{Z}_{N_f}\right)$, **where the** $\mathbf{N_f}$ **action is**

$$e^{2\pi i/N_f} \cdot (a, b) = \left(e^{2\pi i/N_f} a, e^{-2\pi i/N_f} b\right)$$

This corresponds to an $\mathbf{A_{N_f-1}}$ **singularity / multi-Taub-NUT space**

BENINI, CLOSSET, CREMONESI

- **In the large N limit we have the M-theory on** $AdS_4 \times \mathbb{S}_7/\mathbb{Z}_{N_f}$

The N_f Matrix Model

Summary

't Hooft expansion	M-theory/Fermi gas expansion
N large, N_f large, λ fixed	N large, N_f fixed
The N_f model is a O(2) model	$Z_{N_f} = Z_{\text{Fermi Gas}}$
The polynomial part of the Grand Potential is the same .	
$e^{-2\pi\sqrt{2\lambda}}$ Dual WS instantons?	$e^{-2\pi N_f \sqrt{\frac{\lambda}{2}}}$ Dual membrane instantons?
non perturbative: membrane instantons	non perturbative: ws instantons

! Very similar to ABJM structure, but without underlying topological strings

The N_f Matrix Model

Hard task without topological strings

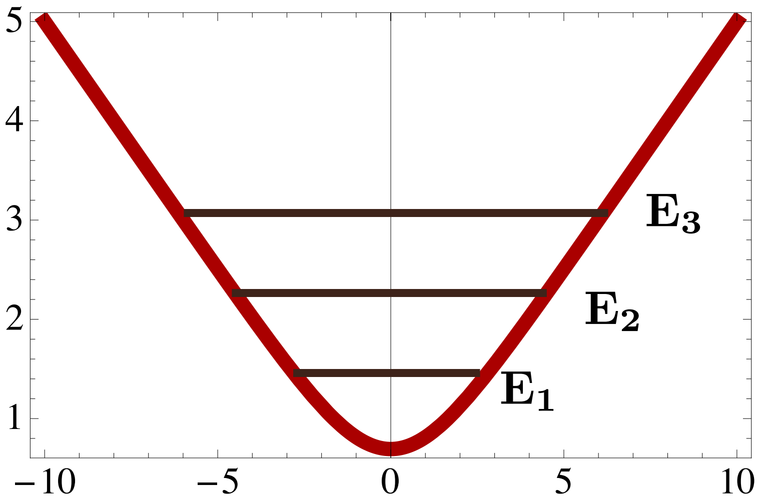
- ’t Hooft regime: we solved the planar limit and the genus one, but we lack an efficient method to go beyond
(Holomorphic anomaly equation?)

- How to resum the whole series of WS/Membrane instantons?

Recent numerical work by Hatsuda & Okuyama shows that there seems to be a pole cancellation mechanism also for this model.

The N_f Matrix Model

A possible solution



We have an Ideal Fermi gas, hence the full information is contained in knowledge of the the energy levels.

The energy levels are determined by the this integral equation:

$$\int_{-\infty}^{\infty} \rho(x_1, x_2)\phi_n(x_2)dx_2 = e^{-E_n}\phi_n(x_1), \qquad n \geq 0$$

Problem: We lack an analytic method to solve it.

Nevertheless we can treat this integral equation numerically and check our previous analytic results.

Conclusions

- **There are many models that share the characteristics of ABJM.**

  Kazakov, Kostov ,Nekrasov Jafferis, Tomasiello Nekrasov, Shatashvili Honda,Okuyama - Kallen

- **ABJ(M) is privileged because of the connection with topological strings.**

- **For these other theories, in order to fully solve the problem, we need a method to solve analytically the integral equation appearing in the spectral problem.**