Towards a Geometry of α' Corrections

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- O. H., B. Zwiebach arXiv: 1407.0708, 1407.3803
- O. H., W. Siegel, B. Zwiebach arXiv: 1306.2970
- W. Siegel, hep-th/9305073, O. H., C. Hull, B. Zwiebach arXiv: 1003.5027, 1006.4823
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Motivation:

- α' corrections encode truly stringy effects beyond supergravity
- usually written with higher powers of R_{mnkl} and $H = \mathrm{d}b$,
  e.g. determined by string S-matrix calculations
- *very* messy. [Metsaev &Tseytlin (1987), Gross & Sloan (1987), Hull & Townsend (1987)]
  Is there some principle? T-dualty/U-duality invariance?
  [A. Sen (1991), K. Meissner (1996)]
- Use double field theory to make T-duality manifest
  $\Rightarrow$ novel (duality-covariant) gauge principle

Plan of the talk:

Part I: Two-derivative DFT

- Efficient reformulation of supergravity (‘generalized geometry’)
- Gauge structure of DFT:

  generalized diffeomorphisms, duality-covariantized Courant bracket

Part II: Higher-derivative deformations

- exact deformation of gauge structure
- physical interpretation on physical subspace

  $\Rightarrow$ Green-Schwarz mechanism and α'-deformed Courant bracket
- further deformations from bosonic closed SFT
- Further constraints from $E_{8(8)}$ Exceptional Field Theory ?
- Conclusions and Outlook

Part I: Two-derivative Double Field Theory

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\mathsf{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4}\alpha' R^{ijkl} R_{ijkl} + \cdots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D,D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi}\sqrt{-g}$):

$$S_{\mathsf{DFT}} = \int d^{2D} X \, e^{-2d} \, \mathcal{R}(\mathcal{H}, d) \quad \xrightarrow{\tilde{\partial}^i = 0} \quad S_{\mathsf{NS}}\big|_{\alpha'=0}$$

generalized curvature scalar

$$\begin{aligned} \mathcal{R} \;\equiv\; & 4\,\mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\,\mathcal{H}^{MN} \partial_M d \, \partial_N d + 4 \partial_M \mathcal{H}^{MN} \, \partial_N d \\ & + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \, \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \, \partial_K \mathcal{H}_{NL} \end{aligned}$$

Gauge transformations and generalized Lie derivatives

In DFT gauge invariance governed by generalized Lie derivatives

$$\widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + \left(\partial_M \xi^P - \partial^P \xi_M\right) \mathcal{H}_{PN} + \left(\partial_N \xi^P - \partial^P \xi_N\right) \mathcal{H}_{MP}$$

$$\widehat{\mathcal{L}}_\xi \left(e^{-2d}\right) = \partial_M \left(\xi^M e^{-2d}\right)$$

Invariance and closure, $[\widehat{\mathcal{L}}_{\xi_1}, \widehat{\mathcal{L}}_{\xi_2}] = \widehat{\mathcal{L}}_{[\xi_1,\xi_2]_C}$,

$$\left[\, \xi_1 \,,\, \xi_2 \,\right]_C^M \;=\; \xi_1^N \partial_N \xi_2^M - \xi_2^N \partial_N \xi_1^M - \frac{1}{2} \xi_{1N} \partial^M \xi_2^N + \frac{1}{2} \xi_{2N} \partial^M \xi_1^N$$

modulo strong constraint

$$\eta^{MN} \partial_M \partial_N = 2 \tilde{\partial}^i \partial_i = 0 \qquad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

solved by

$$\partial_M = \begin{cases} \partial_i & \text{if } M = i \\ 0 & \text{else} \end{cases}$$

$O(D,D)$ covariant, captures IIA/M-theory & IIB simultaneously

Conventional gauge transformations and Courant bracket

Setting $\tilde{\partial}^i = 0$ gauge transformations imply for $\xi^M = (\tilde{\xi}_i, \xi^i)$

$$\delta g \;=\; \mathcal{L}_\xi g\;, \qquad \delta b \;=\; d\tilde{\xi} + \mathcal{L}_\xi b$$

Viewing $\xi + \tilde{\xi}$ as section in $T \oplus T^*$ ('generalized geometry')

C-bracket reduces to Courant bracket

$$\Big[\,\xi_1 + \tilde{\xi}_1, \xi_2 + \tilde{\xi}_2\Big] \;=\; \big[\,\xi_1\,, \xi_2\big] \,+\, \mathcal{L}_{\xi_1}\tilde{\xi}_2 - \mathcal{L}_{\xi_2}\tilde{\xi}_1 - \tfrac{1}{2}\mathrm{d}\big(i_{\xi_1}\tilde{\xi}_2 - i_{\xi_2}\tilde{\xi}_1\big)$$

exact term not fixed by closure but by gauge covariance of C-bracket

or 'B automorphism' of Courant bracket

Part II: Higher-derivative deformations

Geometrical structures for generalized vector ξ^M in $\alpha' = 0$ DFT:

$$\langle \xi_1 | \xi_2 \rangle \;=\; \xi_1^M \xi_2^N \eta_{MN}\,, \qquad \big[\xi_1, \xi_2\big]_C^M \;=\; 2\,\xi_{[1}^N \partial_N \xi_{2]}^M \;-\; \frac{1}{2}\,\xi_1^K \overleftrightarrow{\partial}^M \xi_{2K}$$

$$\widehat{\mathcal{L}}_\xi V^M \;=\; \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P$$

All receive non-trivial higher-derivative corrections:

$$\langle \xi_1 | \xi_2 \rangle \;=\; \xi_1^M \xi_2^N \eta_{MN} - (\partial_N \xi_1^M)(\partial_M \xi_2^N)$$

$$\big[\xi_1, \xi_2\big]_C^M \;=\; 2\,\xi_{[1}^N \partial_N \xi_{2]}^M \;-\; \frac{1}{2}\,\xi_1^K \overleftrightarrow{\partial}^M \xi_{2K} \;+\; \frac{1}{2}\,(\partial_K \xi_1^L) \overleftrightarrow{\partial}^M (\partial_L \xi_2^K)$$

$$\mathbf{L}_\xi V^M \;=\; \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P - (\partial^M \partial_K \xi^L) \partial_L V^K$$

Closure and gauge invariance exact! ($\mathbf{L}_\xi \langle V, W \rangle = \xi^N \partial_N \langle V, W \rangle$, etc.)

Not removable by $O(D, D)$ covariant redefinitions

DFT relations for $\mathcal{H} \in O(D,D)$

$$(\mathcal{H}^2)_{MN} \equiv \mathcal{H}_{MK}\mathcal{H}^{K}{}_{N} = \eta_{MN} \qquad \text{Tr}\,\mathcal{H} \equiv \eta^{MN}\mathcal{H}_{MN} = 0$$

get deformed $\Rightarrow$ dynamical equations!

$$(\mathcal{M}\star\mathcal{M})_{MN} \equiv 2(\mathcal{M}^2)_{MN} - \frac{1}{2}\partial_M\mathcal{M}^{PQ}\partial_N\mathcal{M}_{PQ} + \cdots = 2\eta_{MN}$$

$$\text{tr}\,\mathcal{M} \equiv \eta^{MN}\mathcal{M}_{MN} - 3\partial_M\partial_N\mathcal{M}^{MN} + \cdots = 0$$

In derivative expansion:

$$\mathcal{M}_{MN} = \mathcal{H}_{MN} + \frac{1}{2}\{\mathcal{H},\mathcal{V}^{(2)}\}_{MN} + \cdots\,, \qquad \mathcal{H}^2 = \eta$$

$\Rightarrow$ dilaton eq. & gravity eq. plus α' corrections!

Exact gauge invariant action (with deformed products)

$$S = \int e^{\phi}\big(\,\text{Tr}\mathcal{M} - \frac{1}{3}\mathcal{M}^3 + \cdots\big)$$

Interpretation on physical subspace?

(Perturbative) analysis shows that b-field transforms as

$$\delta_{\xi+\tilde{\xi}}\, b \;=\; \mathrm{d}\tilde{\xi} \;+\; \mathcal{L}_{\xi} b \;+\; \tfrac{1}{2}\,\mathrm{tr}\big(\mathrm{d}(\partial\xi) \wedge \Gamma\big)$$

with (Christoffel) connection 1-form $(\Gamma)^{k}{}_{l} \equiv \Gamma^{\,k}_{\,il}\, dx^{i}$

deformed gauge invariant 3-form curvature

$$\widehat{H}(b,\Gamma) \;=\; \mathrm{d}b + \tfrac{1}{2}\,\Omega(\Gamma)\,, \quad \Omega(\Gamma) \;=\; \mathrm{tr}\big(\Gamma \wedge \mathrm{d}\Gamma + \tfrac{2}{3}\,\Gamma \wedge \Gamma \wedge \Gamma\big)$$

$\Rightarrow$ Green-Schwarz anomaly cancellation mechanism of heterotic string but with deformed diffeomorphisms rather than deformed Lorentz

Deformation of Courant bracket

Deformed gauge transformations close according to bracket

$$\Big[\,\xi_1+\tilde\xi_1,\xi_2+\tilde\xi_2\,\Big]' \;=\; \big[\,\xi_1\,,\xi_2\,\big] \;+\; \mathcal{L}_{\xi_1}\tilde\xi_2-\mathcal{L}_{\xi_2}\tilde\xi_1-\tfrac{1}{2}\mathsf{d}\big(i_{\xi_1}\tilde\xi_2-i_{\xi_2}\tilde\xi_1\big)$$
$$-\,\tfrac{1}{2}\big(\tilde\varphi(\xi_1,\xi_2)-\tilde\varphi(\xi_2,\xi_1)\big)$$

with the map $\tilde\varphi$ that produces a ‘one-form’ from 2 vectors

$$\tilde\varphi(V,W)\;\equiv\;\mathsf{tr}\big(\mathsf{d}(\partial V)\partial W\big)\;\equiv\;\partial_i\partial_k V^l\partial_l W^k dx^i$$

not genuine 1-form $\;\Rightarrow\;$ anomalous transformation under diffeomorphisms

Bracket covariant under *deformed* diffeomorphisms

$$\delta_{\xi+\tilde\xi}\tilde V\;\equiv\;\mathcal{L}_\xi\tilde V-i_V\mathsf{d}\tilde\xi\;-\;\tilde\varphi(\,\xi,V)$$

α' Corrections for Bosonic Strings and Closed SFT

α' corrections for bosonic string (Riemann-sq.) ? ($\mathbb{Z}_2$ invariant $b \to -b$)

Closed bosonic SFT $\Rightarrow$ deformed gauge algebra for *cubic* theory

$$\big[\,\xi_1,\,\xi_2\,\big]_+^M \;=\; \big[\,\xi_1,\,\xi_2\,\big]_C^M \;+\; \tfrac{1}{2}\,\bar{\mathcal{H}}^{KL}\,K_{[1\,K}{}^{P}\,\partial^M K_{2]\,LP}$$

with $K_{MN} = 2\partial_{[M}\xi_{N]}$ and background generalized metric $\bar{\mathcal{H}}_{MN}$

$\Rightarrow$ α'-deformed diffeomorpisms as implied by (perturbative) redefinition

$$h'_{ij} \;=\; h_{ij} - \tfrac{1}{4}\,\alpha'\,\partial_k h_i{}^p\,\partial^k h_{jp} + \cdots\;,$$

agrees with earlier results on duality-invariant Riemann-sq.

[Meissner (1996), Hohm & Zwiebach (2011)]

More general $\mathbb{Z}_2$ even/odd deformations (with parameters $\gamma^{\pm}$)

$$\big[\,\xi_1,\,\xi_2\,\big]_{\alpha'}^M \;=\; \big[\,\xi_1,\,\xi_2\,\big]_C^M \;+\; \tfrac{1}{2}\big(\gamma^+\bar{\mathcal{H}}^{KL} - \gamma^-\eta^{KL}\big)\,K_{[1\,K}{}^{P}\,\partial^M K_{2]\,LP}$$

Cubic Action

$$
\begin{aligned}
S \;=\; & S^{(2,2)} + S^{(3,2)} \\
& + \tfrac{1}{4}\,\mathcal{R}^{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}} + \tfrac{1}{4}\,\phi\,\mathcal{R}^{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}} \\
& - \tfrac{1}{8}\Big(\Gamma^{\underline{P}\bar{M}\bar{N}}\,\Gamma_{\bar{M}}{}^{\underline{K}\,\underline{L}}\,\partial_{\underline{P}}\Gamma_{\bar{N}\,\underline{K}\,\underline{L}} \;-\; \Gamma^{\bar{P}\,\underline{M}\,\underline{N}}\,\Gamma_{\underline{M}}{}^{\bar{K}\bar{L}}\,\partial_{\bar{P}}\Gamma_{\underline{N}\bar{K}\bar{L}} \\
& \qquad - \Gamma^{\bar{M}}{}_{\underline{K}\,\underline{L}}\,\Gamma^{\bar{N}\,\underline{K}\,\underline{L}}\,\partial_{\bar{M}}\Gamma_{\bar{N}} \;+\; \Gamma^{\underline{M}}{}_{\bar{K}\bar{L}}\,\Gamma^{\underline{N}\bar{K}\bar{L}}\,\partial_{\underline{M}}\Gamma_{\underline{N}}\Big) \\
& - \tfrac{1}{2}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\Gamma^{\bar{K}\,\underline{M}\,\underline{P}}\,\Gamma^{\bar{L}\,\underline{N}}{}_{\underline{P}} + \tfrac{1}{2}\,\mathcal{R}_{\underline{K}\,\underline{L}\bar{M}\bar{N}}\,\Gamma^{\underline{K}\bar{M}\bar{P}}\,\Gamma^{\underline{L}\bar{N}}{}_{\bar{P}} \\
& - \tfrac{1}{2} m_{\underline{M}\bar{N}}\,\mathcal{R}^{\underline{M}\,\underline{K}\bar{P}\bar{Q}}\,\partial^{\bar{N}}\Gamma_{\underline{K}\bar{P}\bar{Q}} + \tfrac{1}{2} m_{\underline{M}\bar{N}}\,\mathcal{R}^{\underline{P}\,\underline{Q}\bar{N}\bar{K}}\,\partial^{\underline{M}}\Gamma_{\bar{K}\,\underline{P}\,\underline{Q}} \\
& + \tfrac{1}{2}\,\mathcal{R}_{\underline{M}\,\underline{N}\bar{K}\bar{L}}\,\partial^{\underline{P}} m^{\underline{M}\bar{K}}\,\partial_{\underline{P}} m^{\underline{N}\bar{L}}\;.
\end{aligned}
$$

$E_{8(8)}$ Exceptional Field Theory

[Hohm & Samtleben (2013–2014), completing earlier work by de Wit & Nicolai (1986),

Hillmann (2009), Berman & Perry (2011), Strickland-Constable, Coimbra & Waldram (2011)]

Field content: $\left\{ e_\mu{}^a\,,\ \mathcal{V}_{\underline{M}}{}^{M}\,,\ A_\mu{}^M\,,\ B_{\mu\,M} \right\}$ $\qquad M = 1,\ldots,248$

depending on coordinates (x^μ, Y^M), $\mu = 0,1,2$, subject to

$$\left(\mathbb{P}_{1+248+3875}\right)_{MN}{}^{KL} \partial_K \otimes \partial_L \;=\; 0$$

Generalized Lie derivative

$$\mathbb{L}_{(\Lambda,\Sigma)} V^M \;\equiv\; \Lambda^K \partial_K V^M - f^M{}_{NP}\, f^{PK}{}_L\, \partial_K \Lambda^L\, V^N - \Sigma_L f^{LM}{}_N V^N$$

Vectors gauge fields for local Λ, Σ symmetries

$$D_\mu \;\equiv\; \partial_\mu - \mathbb{L}_{(A_\mu, B_\mu)}$$

Complete $E_{8(8)}$ covariant bosonic action

$$S = \int d^3x\, d^{248}Y\, e\left(\widehat{R} + e^{-1}\mathcal{L}_{\mathsf{CS}} + \frac{1}{240}\, g^{\mu\nu} D_\mu \mathcal{M}^{MN} D_\nu \mathcal{M}_{MN} - V(\mathcal{M}, g)\right)$$

(generalized) Chern-Simons term ($\mathcal{F}^M = dA^M + \cdots$, $\mathcal{G}_M = dB_M + \cdots$)

$$S_{\mathsf{CS}} \propto \int_{\Sigma_4} d^4x \int d^{248}Y \left(\mathcal{F}^M \wedge \mathcal{G}_M - \frac{1}{2} f_{MN}{}^K \mathcal{F}^M \wedge \partial_K \mathcal{F}^N\right)$$

'Potential' term:

$$\begin{aligned} V(\mathcal{M}, g) &= -\frac{1}{240}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\,\partial_N\mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_L\mathcal{M}_{NK} \\ &\quad + \frac{1}{7200}\, f^{NQ}{}_P f^{MS}{}_R\, \mathcal{M}^{PK}\partial_M\mathcal{M}_{QK}\mathcal{M}^{RL}\partial_N\mathcal{M}_{SL} + \cdots \end{aligned}$$

reduces to 1) $D = 11$ supergravity or 2) type IIB supergravity

dual graviton components drop out   No dual graviton problem!

Summary & Outlook

- DFT provides strikingly economic reformulation of supergravity
- Beyond supergravity (non-zero α'): duality covariance requires novel field variables with *non-standard* diffeomorphisms
- However, usual diffeomorphism covariance replaced by duality-covariant gauge principle
- so far only partial results:
  background-independent extension for bosonic strings?
  Field-dependent gauge algebra? Higher order in α'?
  Type II Strings and M-theory extensions?
- Extension to 'Exceptional Field Theory'
  with exceptional duality groups $E_{6(6)}$, $E_{7(7)}$, $E_{8(8)}$, $\ldots$?