Logarithmic Corrections to Black Hole Entropy v. 2.0

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Logarithmic Corrections

- The leading corrections to the area law for black hole entropy are logarithmic
$$\delta S = \frac{1}{2} D_0 \log A \; .$$

- These corrections can be computed from the low energy theory: only massless fields contribute.

- In some situations the corrections give non-trivial support for a known microscopic description.

- In other situations they offer clues to the nature of the unknown microscopic theory.

Updates in v. 2.0

In principle: computations are straightforward applications of techniques from the 70's.

In recent years, Sen (and collaborators) did what we do, and more.

In practice: computations are cumbersome and intransparent.

Updates in v 2.0 focus on short-cuts that add clarity:

- Interactions with background gravity and graviphoton: employ AdS/CFT, specifically organize fluctuations as ***chiral primaries***.
- Contributions from ***on-shell states only*** (no ghosts).
- Remnant of unphysical states: ***simple boundary states*** .
- Careful with ***4D zero-modes*** (done incorrectly until recent years).

Reference: C. Keeler, FL, P. Lisbao, arXiv: 1404.1379

Setting

- Consider matter in a general theory with $\mathcal{N} \geq 2$ SUSY.

- In terms of $\mathcal{N} = 2$ fields: one SUGRA multiplet, $\mathcal{N} - 2$ (massive) gravitini, n_V vector multiplets, n_H hyper multiplets.

- Setting: focus on extremal black holes where it is sufficient to consider the $\text{AdS}_2 \times S^2$ near horizon region.

- The final result:

$$\delta S = \frac{1}{12}\left[23 - 11(\mathcal{N} - 2) - n_V + n_H\right] \log A_H \ .$$

- Example (relevant for microscopics): no correction in $\mathcal{N} = 4$ theory with an arbitrary number of $\mathcal{N} = 4$ matter multiplets.

Prelude: Chiral Primaries

- Massless fields in $\mathrm{AdS}_2 \times S^2$ organize themselves in short representations of the $SU(2|1,1)$ supergroup.
- CFT language: consider chiral multiplets where (h,j) are
$$(k,k), 2(k+\frac{1}{2}, k-\frac{1}{2}), (k+1, k-1) \ .$$
Possible values of $k = \frac{1}{2}, 1, \frac{3}{2}, \ldots$. ($k = \frac{1}{2}$ extra short).
- In the early days of AdS/CFT three groups independently solved linearized equations of motion and computed spectra.
- They all found the same spectrum for the $\mathcal{N}=2$ SUGRA multiplet.
- We used an indirect argument and found a different result.
- ***We are right***.

Spherical Harmonics

- Expansion on S^2 of single field component with helicity λ: angular momenta $j = |\lambda|, |\lambda| + 1, \ldots$.
- Example: for a gauge field ***all*** components organize themselves into two towers with $j = 1, 2, \ldots$ and two towers with $j = 0, 1, \ldots$
- The ***physical*** components of the vector field components organize themselves into two towers with $j = 1, 2, \ldots$.
- So: the set of physical angular momenta in each $\mathcal{N} = 2$ is unambiguous.
- Example: the $\mathcal{N} = 2$ vector multiplet has one vector field and two real scalars so the ***physical*** boson towers are: two with $j = 1, 2, \ldots$ and two with $j = 0, 1, \ldots$.
- Mixing is allowed (for same j) but assembly of towers into chiral multiplets uniquely determine conformal weights.

The Spectrum of Chiral Primaries

- Result: the spectrum of (h, j) for all chiral primaries:

$$\begin{aligned} \text{Supergravity}: &\quad 2[(k+2,k+2)\,,2(k+\tfrac{5}{2},k+\tfrac{3}{2})\,,(k+3,k+1)] \\ \text{Gravitino}: &\quad 2[(k+\tfrac{3}{2},k+\tfrac{3}{2})\,,2(k+2,k+1)\,,(k+\tfrac{5}{2},k+\tfrac{1}{2})] \\ \text{Vector}: &\quad 2[(k+1,k+1)\,,2(k+\tfrac{3}{2},k+\tfrac{1}{2})\,,(k+2,k)] \\ \text{Hyper}: &\quad 2[(k+\tfrac{1}{2},k+\tfrac{1}{2})\,,2(k+1,k)\,,(k+\tfrac{3}{2},k-\tfrac{1}{2})] \end{aligned}$$

Each tower has $k = 0, 1, \ldots$.

- Discrepancy: previous work had ***one*** more entry in the SUGRA multiplet

$$(1,1)\,,2(\frac{3}{2},\frac{1}{2}),(2,0)\,.$$

- Clarification: this field exists ***only*** as a boundary mode.

Example: Constraints for Gravity

- The graviton in D dimensions has $D(D+1)/2$ components, D gauge symmetries (from diffeomorphisms), D constraints (eom's left after gauge fixing).

- So: a graviton has $D(D-3)/2$ physical components.

- In 2D a graviton has -1 degrees of freedom so ***a graviton and a scalar combined has no degrees of freedom***.

- Details: after gauge fixing some "equations of motion" are in fact constraints (there are no time derivatives).

- Exception: the constraint is solved by one specific spatial profile (the zero-mode on AdS_2) so one boundary degree of freedom can be freely specified.

- These ***boundary modes are physical*** (standard in AdS/CFT).

Quantum Fluctuations: Strategy

- All contributions from quadratic fluctuations around the classical geometry take the form
$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \, .$$
- The quantum corrections are encoded in the heat kernel
$$D(s) = \mathrm{Tr} \, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} \, .$$
- The effective action becomes
$$W = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} D(s) = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} \int d^D x K(s) \, .$$
- The constant D_0 (or K_0) we need is (essentially) the 2nd Seeley-deWitt coefficient or equivalently the trace anomaly of the EM-tensor.

Simple Heat Kernels in 2D

- The heat kernel for a scalar field on S^2 is elementary:

$$K_S^s(s) = \frac{1}{4\pi a^2} \sum_{k=0}^{\infty} e^{-sk(k+1)}(2k+1) = \frac{1}{4\pi a^2 s} \left(1 + \frac{1}{3}s + \frac{1}{15}s^2 + \ldots \right)$$

- A massless scalar field on AdS_2 involves a continuous spectrum:

$$K_A^s(s) = \frac{1}{2\pi a^2} \int_0^\infty e^{-(p^2+\frac{1}{4})s} p \tanh \pi p \, dp \, .$$

- The local terms in the AdS_2 heat kernel is identical to S^2 except for the sign of the curvature:

$$K_A^s(s) = \frac{1}{4\pi a^2 s} \left(1 - \frac{1}{3}s + \frac{1}{15}s^2 + \ldots \right) \, .$$

- The heat kernel for a fermion on S^2 is also elementary:

$$K_S^f(s) = \frac{1}{4\pi a^2} \sum_{k=0}^{\infty} e^{-s(k+1)^2}(2k+2) = \frac{1}{4\pi a^2 s} \left(1 - \frac{1}{6}s - \frac{1}{60}s^2 + \ldots \right)$$

Simple Heat Kernels on AdS$_2 \times S^2$

- For a product space heat kernels multiply so for a scalar on AdS$_2 \times S^2$:

$$K_4^s(s) = K_S^s(s) K_A^s(s) = \frac{1}{16\pi^2 a^4 s^2}\left(1 + \frac{1}{45}s^2 + \ldots\right) .$$

- For a Dirac fermion on AdS$_2 \times S^2$:

$$K_4^f(s) = 4K_S^f(s) K_A^f(s) = -\frac{1}{4\pi^2 a^4 s^2}\left(1 - \frac{11}{180}s^2 + \ldots\right) .$$

- A benchmark for results in $\mathcal{N} = 2$ theory: a "free hyper"

$$K_4^{\rm min}(s) = 4K_4^s(s) + K_4^f(s) = \frac{1}{4\pi^2 a^4 s^2} \cdot \frac{1}{12}s^2 .$$

- The leading $1/s^2$ singularity cancels: no cosmological constant for equal number of fermion and bosons.

- The $1/s$ order also cancels: this is an accident.

The AdS$_2$ Perspective

- The canonical heat kernel on AdS$_2$ of for a massless field.

- A field with conformal weight h (mass $m^2 = h(h-1)$) and $SU(2)$ quantum number j (degeneracy $2j+1$):

$$K_A(h, j; s) = K_A(h = 1, j = 0; s)\, e^{-h(h-1)s}(2j + 1)\ .$$

- A free $4D$ boson is a tower of $2D$ bosons with $(h, j) = (k + 1, k)$ with $k = 0, 1, \ldots$ so

$$\begin{aligned} K_4^s(s) &= K_A^s(s) \cdot \frac{1}{4\pi a^2} \sum_{k=0}^{\infty} e^{-sk(k+1)}(2k + 1) \\ &= \frac{1}{16\pi^2 a^4 s^2} \left(1 + \frac{1}{45} s^2 + \ldots\right)\ . \end{aligned}$$

- The sum over the tower of AdS$_2$ fields computes the factor from the heat kernel on S^2.

The Vector-Multiplet: Bulk

- The conformal weights for fields in supergravity are "shifted" from the free values.
- The fermions in the vector multiplet are canonical but bosons interact: this is the ***attractor mechanism***.
- The "shifted" sum on S^2 for all four physical bosons:

$$K_4^{V,b}(s) = \frac{2K_A^s(s)}{4\pi a^2} \sum_{k=0}^{\infty} \left(e^{-sk(k+1)}(2k+3) + e^{-s(k+1)(k+2)}(2k+1) \right)$$
$$= \frac{1}{4\pi^2 a^4 s^2} \left(1 + \frac{1}{45}s^2 + \ldots + \frac{1}{2}s(1 - \frac{1}{3}s) + \ldots \right) .$$

- Heat kernel for the full vector multiplet including fermions:

$$K_4^V(s) = \frac{1}{4\pi^2 a^4} \left(\frac{1}{2s} - \frac{1}{12} + \ldots \right) .$$

- A 1/s term was generated by interactions.
- The constant term changed sign due to interactions.

The Hyper-Multiplet

- The bosons in the hyper multiplet are canonical – just four free fields.
- The fermions interact with the graviphoton so the conformal weights differ from a free field.
- The S^2 tower of fermions is shifted relative to a free fermion.
- Heat kernel for the complete hyper-multiplet:

$$K_4^H(s) = \frac{1}{4\pi^2 a^4}\left(-\frac{1}{s} - \frac{1}{12} + \ldots\right) \ .$$

- A 1/s term was generated by interactions.
- The constant term changed sign due to interactions.

The Vector-Multiplet: Boundary

- The vector multiplet has a feature not yet discussed: gauge invariance.
- Two auxiliary towers cancel: unphysical states (violate gauge condition) and physical (but pure gauge).
- The boundary state: one of the would-be gauge functions is not normalizable so ***one*** state survives.
- Alternatively: one equation of motion is a ***constraint*** so one spatial profile survives.
- The ***boundary state is a massless boson on S^2***:
$$-\nabla^I \delta \mathcal{A}_I = -\nabla^2 \Lambda = 0$$
- Final result for the heat kernel:
$$K_4^V(s) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s} - \frac{1}{12}\right) + \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s} + \frac{1}{6}\right) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{s} + \frac{1}{12}\right).$$

The (Massive) Gravitino Multiplet

- Bulk modes: bosons and fermions all have conformal weight shifted from the free value.
- Boundary modes: two vectors each have a gauge symmetry and so a boundary scalar.
- The SUSY variation is a fermionic gauge symmetry of the gravitino that gives a boundary fermion

$$\gamma^I \nabla_I \epsilon = 0 \ .$$

- The boundary heat kernel is constant because of boson-fermion degeneracy

$$K^{(3/2)}_{\text{bndy}} = \frac{1}{4\pi^2 a^4} \cdot \frac{1}{2} \ .$$

- The full heat kernel:

$$K^{(3/2)} = \frac{1}{4\pi^2 a^4} \cdot \left((-\frac{1}{s} + \frac{5}{12}) + \frac{1}{2} \right) = \frac{1}{4\pi^2 a^4} \cdot \left(-\frac{1}{s} + \frac{11}{12} \right)$$

The Graviton Multiplet

- ***Five bosonic boundary modes***: four from diffeomorphisms and one from gauge symmetry.

- Boundary modes for diffeomorphisms ***acquire a mass***

$$(g_{IJ}\nabla^2 + R_{IJ})\xi^J = 0 \ .$$

- The S^2 vectors have helicity $\lambda = \pm 1$ so angular momenta $j = 1, 2, \ldots$

- The mass of modes due to S^2 diffeomorphisms

$$m^2 = k(k+1) - 2 \ ; \quad k = 1, 2, \ldots$$

- The mass of modes due to AdS_2 diffeomorphisms

$$m^2 = k(k+1) + 2 \ , m^2 = k(k+1) \ ; \quad k = 0, 1, 2, \ldots$$

- ***Four fermionic boundary modes*** (two preserved SUSYs) with contribution to mass from background graviphoton

$$m^2 = (k+1)^2 - 1\,, \quad k = 0, 1, \ldots$$

- The heat kernel for all ***boundary modes*** in the graviton multiplet

$$K^{\rm grav}_{\rm bndy} = \frac{1}{4\pi^2 a^4} \cdot \frac{5}{2}(\frac{1}{s} + \frac{1}{3}) - \frac{1}{4\pi^2 a^4}\left(\frac{2}{s} + \frac{5}{3}\right) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{2s} - \frac{5}{6}\right)$$

- ***Bulk modes***: bosons and fermions all have conformal weight shifted from the free value.

- Full heat kernel

$$K^{\rm grav} = \frac{1}{4\pi^2 a^4}\left((\frac{1}{2s} - \frac{1}{12}) + (\frac{1}{2s} - \frac{5}{6})\right) = \frac{1}{4\pi^2 a^4}\left(\frac{1}{s} - \frac{11}{12}\right)$$

The Quadratic Divergence

- Heat kernel for all multiplets, including physical states in bulk and on boundary

$$K_{\rm phys} = \frac{1}{4\pi^2 a^4}\left[(\frac{1}{s} - \frac{11}{12}) + (\mathcal{N}-2)\cdot(-\frac{1}{s} + \frac{11}{12}) + n_V(\frac{1}{s} + \frac{1}{12}) + n_H(-\frac{1}{s} - \frac{1}{12}) \right]$$

- Contributions to the quadratic divergence (the 1/s term): interactions in bulk and counting boundary degrees of freedom.

- Net result: alternating sign.

- Special case $\mathcal{N} \geq 4$ theory (with any matter): ***quadratic divergence cancels*** (a consistency check).

- For $\mathcal{N} = 3$: ***all divergences cancel*** for any $n_V = n_H$.

- For $\mathcal{N} = 2$: ***a new result***.

4D Zero Modes: General

- 4D zero modes: AdS_2 ***boundary states and also massless*** on S^2.
- Physical origin: the ***global part*** of each unbroken gauge symmetry.
- Zero-modes play a special role in the 4D heat kernel:

$$D(s) = \sum_i e^{-s\lambda_i} = \sum_{\lambda_i \neq 0} e^{-s\lambda_i} + N_0$$

- The path integral reduces to an ***ordinary*** integral

$$e^{-W} = \int \mathcal{D}\phi_0 = \mathrm{Vol}[\phi_0] \sim \epsilon^{-N_0 \Delta} \, .$$

- The correct zero-mode contribution: ***larger than the naïve result by a factor of the scaling dimension*** Δ.

4D Zero Modes: Computation

- Vector fields: no new issue since $\Delta = 1$ for a vector field.

- Bosonic 0-modes in SUGRA multiplet: 6 diff's on S^2 (two with $j = 1$) and scaling dimension $\Delta_2 = 2$. (Heat kernel counts as if $\Delta_2 = 1$).

- Fermionic 0-modes in SUGRA multiplet: 8 preserved SUSYs $\Delta_{3/2} = \frac{3}{2}$. (Heat kernel counts as if $\Delta_{3/2} = \frac{1}{2}$).

- Correction due to 0-modes

$$K_{zm} = \frac{1}{8\pi^2 a^4} \cdot \left[6 \cdot (2-1) - 8 \cdot (\frac{3}{2} - \frac{1}{2}) \right] = \frac{1}{4\pi^2 a^4} (-1) \ .$$

- Note: ***much of the literature accounts incorrectly for 0-modes.***

Example: Reissner-Nordström

Consider a purely bosonic solution: gravity+Maxwell.

Contributions are the bosonic terms from the $\mathcal{N} = 2$ SUGRA multiplet:

- Four free bulk bosons (2 gravity + 2 gauge field): $\delta S = -\frac{1}{45} \log A_H$.
- Interactions (bulk bosons not quite free): $\delta S = -\frac{3}{2} \log A_H$.
- 5 Boundary modes (4 gravity+1 gauge field): $\delta S = -\frac{5}{6} \log A_H$.
- Zero-modes: $\delta S = -3 \log A_H$.

Total: $\delta S = -\frac{241}{45} \log A_H$.

(Fermions in SUGRA multiplet add $\delta S = \frac{1309}{180} \log A_H$)

Summary

We re-computed quadratic fluctuation determinants around an $\text{AdS}_2 \times S^2$ near horizon geometry.

Some features of our strategy:

- Setting: a general theory with $\mathcal{N} \geq 2$ SUSY.
- Focus on states that are on-shell.
- Interactions due to background: encoded in chiral primaries.
- Compute also the renormalization of the gravitational coupling constant (quadratic divergence, 1/s term in the heat kernel).
- Contributions from bulk (4D), Boundary (2D), and Zero-mode (0D).