Non-Associativity, Double Field Theory and Applications

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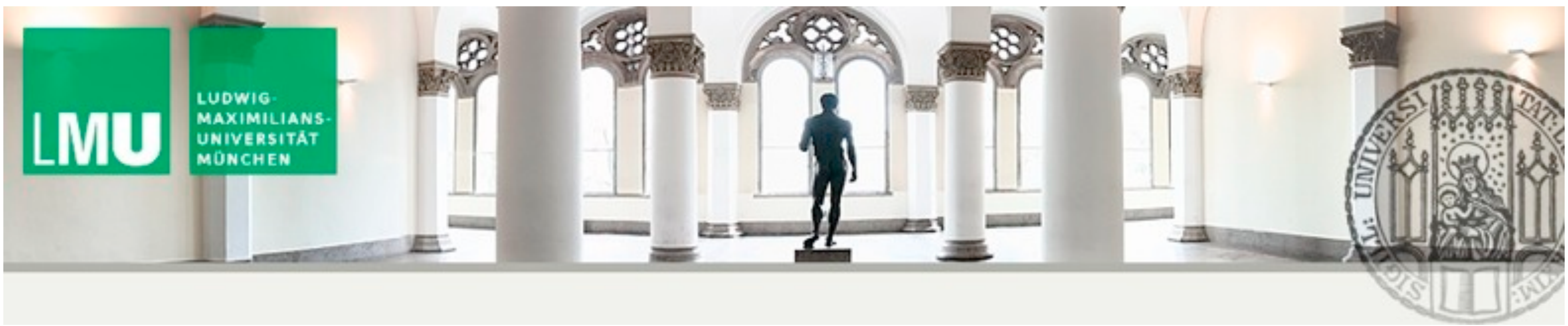


Recent Developments in String Theory, Monte Verita, Ascona , July 25, 2014

Outline:

I) Introduction

II) Non-geometric Backgrounds &
Non-Commutativity/Non-Associativity
(world sheet point of view)

III) Double field theory (target space point of view)

IV) Dimensional Reduction of DFT

V) Application: De Sitter and Inflation (if time)

I) Introduction

Non-geometric backgrounds:

- They are only consistent in string theory.

- Make use of string symmetries, T-duality $\Rightarrow$ T-folds,

- Left-right asymmetric spaces $\Rightarrow$ Asymmetric orbifolds

(Kawai, Lewellen, Tye, 1986; Lerche, D.L. Schellekens, 1986, Antoniadis, Bachas, Kounnas, 1987; Narain, Sarmadi, Vafa, 1987; Ibanez, Nilles, Quevedo, 1987; ......, Faraggi, Rizos, Sonmez, 2014)

- Are related to non-commutative/non-associative geometry

(Blumenhagen, Plauschinn; Lüst, 2010; Blumenhagen, Deser, Lüst, Rennecke, Pluaschin, 2011; Condeescu, Florakis, Lüst; 2012, Andriot, Larfors, Lüst, Patalong, 2012))

Non-associativity in physics:

- Jordan & Malcev algebras, octonions

  M. Günaydin, F. Gürsey (1973); M. Günaydin, D. Minic, arXiv:1304.0410.

- Nambu dynamics

  Y. Nambu (1973); D. Minic, H. Tze (2002); M. Axenides, E. Floratos (2008)

- Magnetic monopoles R. Jackiw (1985); M. Günaydin, B. Zumino (1985)

- Closed string field theory A. Strominger (1987), B. Zwiebach (1993)

- T-duality and principle torus bundles

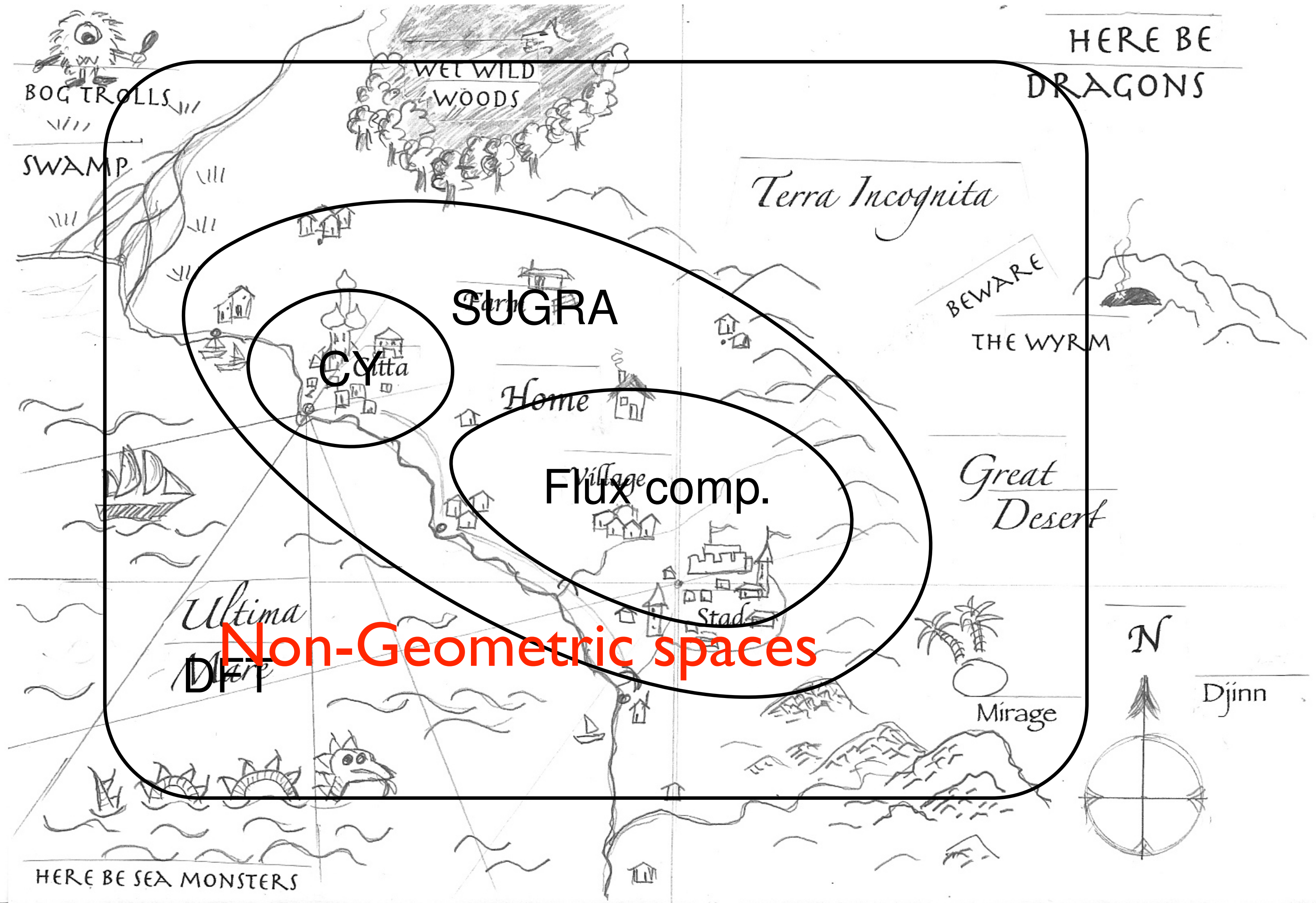
  P. Bouwknegt, K. Hannabuss, Mathai (2003)

- D-branes in curved backgrounds

  L. Cornalba, R. Schiappa (2001)

- Multiple M2-branes and 3-algebras J. Bagger, N. Lambert (2007)

Non-geometric spaces ⇔ Double Field Theory



II) Non-geometric backgrounds & non-commutativity/non-associativity (word-sheet)

Consider D-dimensional toroidal string backgrounds:

Doubling of closed string coordinates and momenta:

- Coordinates: O(D,D) vector $X^M = (\tilde{X}_i, X^i)$

$$(X^i = X_L^i(\tau+\sigma) + X_R(\tau-\sigma) \quad \tilde{X}_i = X_L^i(\tau+\sigma) - X_R(\tau-\sigma))$$

- Momenta: O(D,D) vector $p^M = (\tilde{p}^i, p_i)$

winding ←T-duality→ momentum

- O(D,D) transformations: Mix in general X^i with $\tilde{X}_i$.

□ Act asymmetrically on X_L and X_R.

$$\begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix} \rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X^i \\ \tilde{X}_i \end{pmatrix}, \quad \Lambda = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in O(D,D)$$

Generalized metric: $\mathcal{H}_{MN} = \begin{pmatrix} G^{ij} & -G^{ik}B_{kj} \\ B_{ik}G^{kj} & G_{ij} - B_{ik}G^{kl}B_{lj} \end{pmatrix}$

$$\mathcal{H}_{MN} \rightarrow \Lambda_M^P \, \mathcal{H}_{PQ} \, \Lambda_N^Q$$

Non-geometric backgrounds & non-geometric fluxes:

- **Non-geometric Q-fluxes:** spaces that are locally still Riemannian manifolds but not anymore globally.

(Hellerman, McGreevy, Williams (2002); C. Hull (2004); Shelton, Taylor, Wecht (2005); Dabholkar, Hull, 2005)

Transition functions between two coordinate patches are given in terms of O(D,D) T-duality transformations:

$$\mathrm{Diff}(M_D) \quad \rightarrow \quad O(D,D)$$

C. Hull (2004)

Q-space will become non-commutative:

- **Non-geometric R-fluxes:** spaces that are even locally not anymore manifolds.

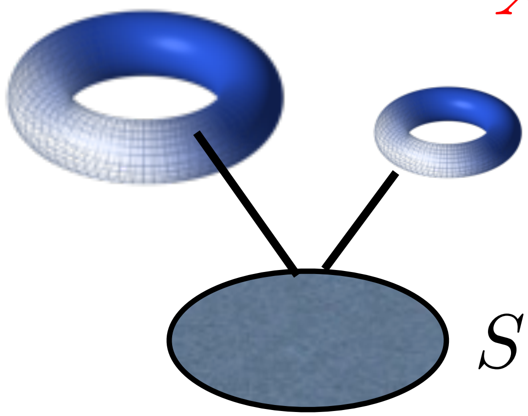
R-space will become non-associative:

$$\begin{aligned}[X^i, X^j, X^k] \quad &:= \quad [[X^i, X^j], X^k] + \text{cycl. perm.} = \\ &= \quad (X^i \cdot X^j) \cdot X^k - X^i \cdot (X^j \cdot X^k) + \cdots \neq 0\end{aligned}$$

Example: Three-dimensional flux backgrounds:

Fibrations: 2-dim. torus that varies over a circle:

$$T^2_{X^1,X^2} \hookrightarrow M^3 \hookrightarrow S^1_{X^3}$$



S^1

Metric, B-field of T^2 : depends on X^3

$$\Rightarrow \quad \mathcal{H}_{MN}(X^3) \quad (M, N = 1, \ldots, 4)$$

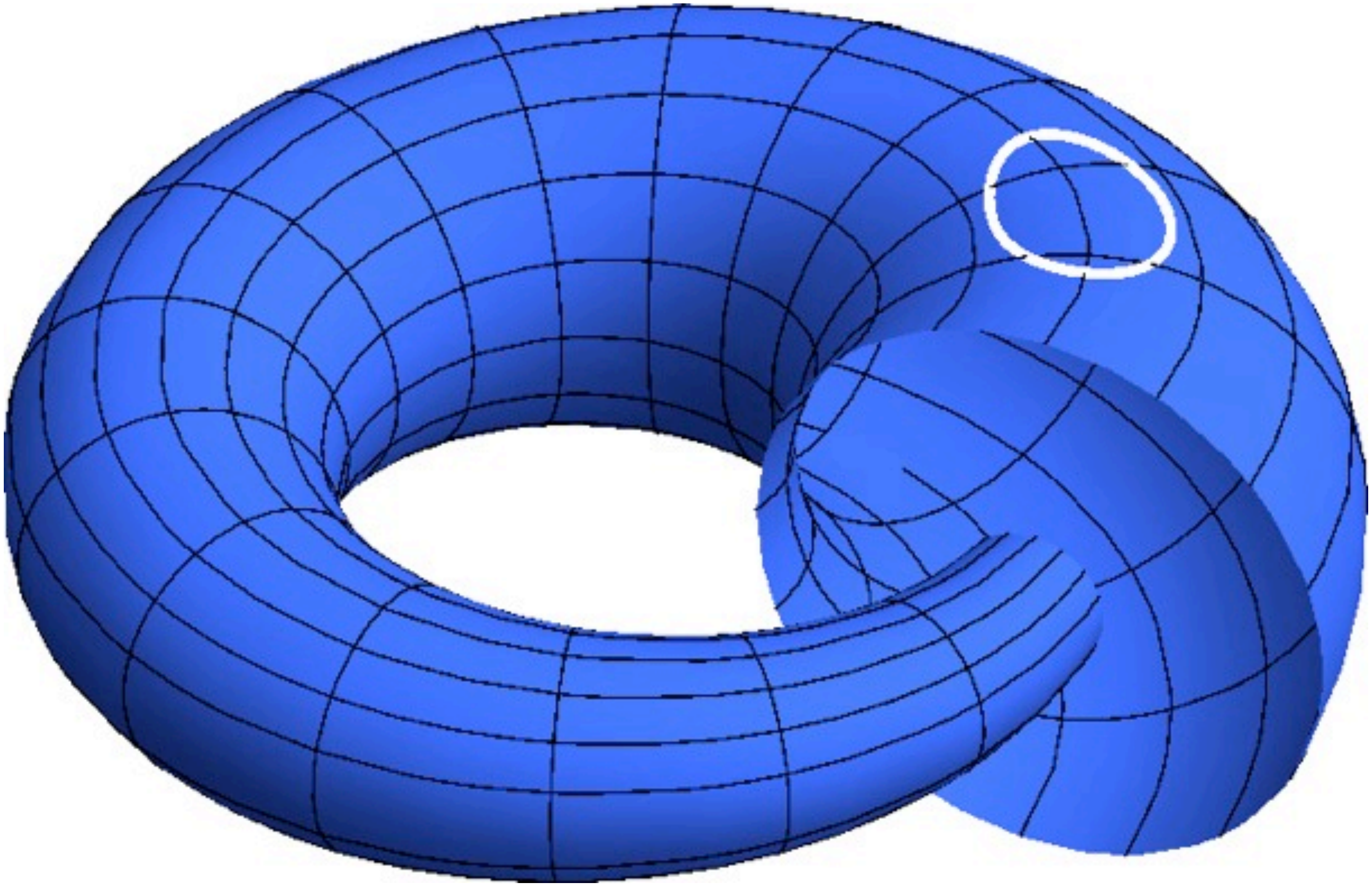
The fibration is specified by its monodromy properties.

O(2,2) monodromy:

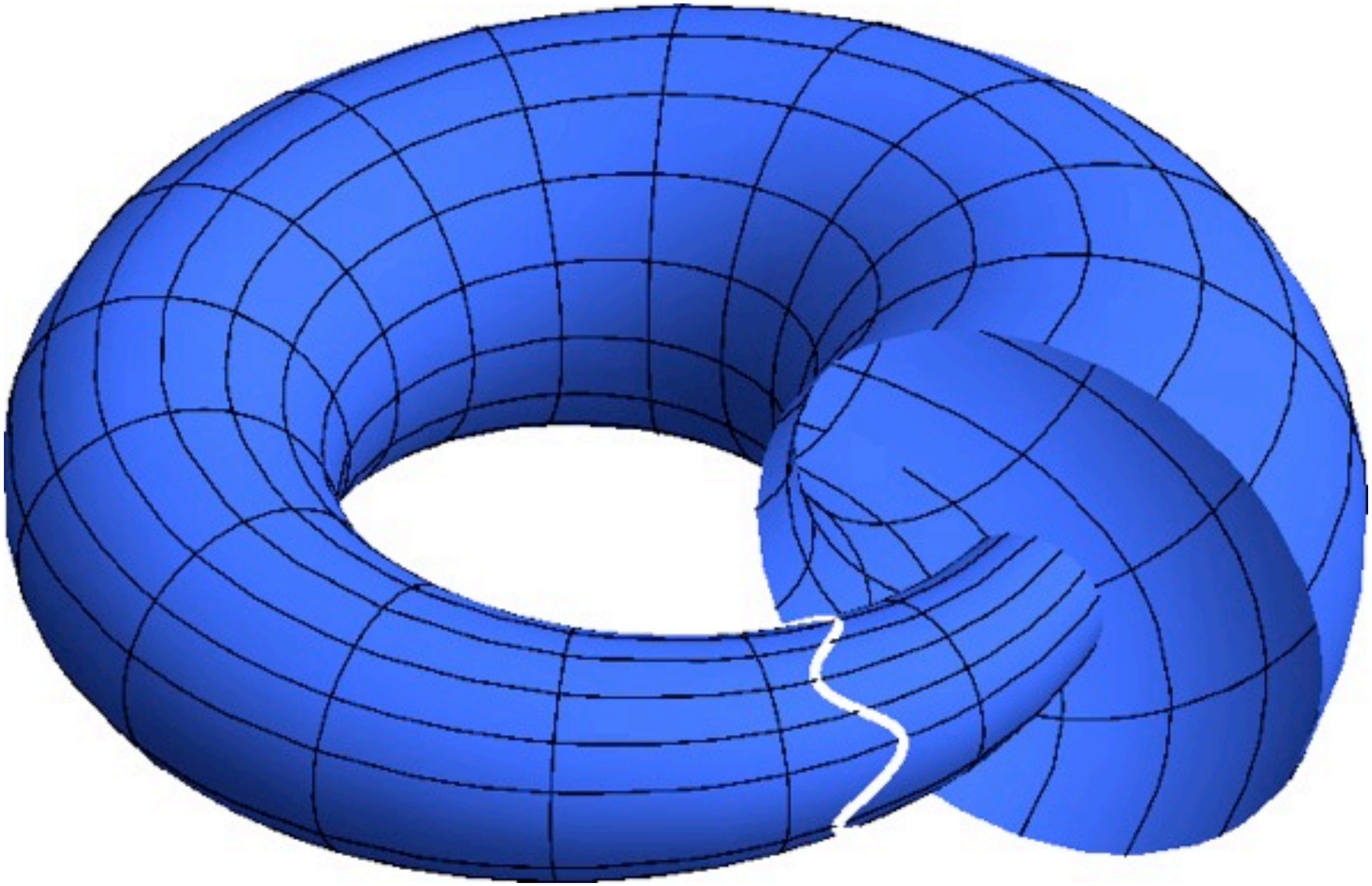
$$\mathcal{H}_{MN}(X^3 + 2\pi) \; = \; \Lambda_{O(2,2)} \, \mathcal{H}_{PQ}(X^3) \, \Lambda^{-1}_{O(2,2)}$$

Non-geometric spaces:

- Monodromy mixes $X^i \leftrightarrow \tilde{X}_i$
- Acts asymmetrically on $X^i_L, X^i_R \quad (i = 1, 2)$



Non geometric torus, metric is patched together by a T-duality transformation: $G_{ij} \;\rightarrow\; G^{ij}$



Non geometric torus, metric is patched together by a T-duality transformation: $G_{ij} \;\rightarrow\; G^{ij}$

(Non-)geometric backgrounds with parabolic monodromy and single 3-form fluxes:

Flat torus with H-flux $\overset{T_{x_1}}{\longleftrightarrow}$ Twisted torus with f-flux $\overset{T_{x_2}}{\longleftrightarrow}$ Non-geometric space with Q-flux $\overset{T_{x_3}}{\longleftrightarrow}$ Non-geometric space with R-flux

$$[X^i_{H,f}, X^j_{H,f}] = 0$$

$$[X^1_Q, X^2_Q] \simeq Q\ \tilde{p}^3$$

$$[[X^1_R, X^2_R], X^3_R] \simeq R$$

They can be computed by

- standard world-sheet quantization of the closed string

D. Andriot, M. Larfors, D. L., P. Patalong, arXiv:1211.6437

- CFT & canonical T-duality

C. Blair, arXiv:1405.2283

I. Bakas, D.L. to appear soon

Q-flux: O(2,2) monodromy ⇒

mixed closed string boundary (DN) conditions:

$$X^3_Q(\tau,\sigma+2\pi) \;=\; X^3_Q(\tau,\sigma) + 2\pi\,\tilde p^3 \quad\Longrightarrow$$

winding number along base direction

$$O(2,2)\left\{\begin{aligned} X^1_Q(\tau,\sigma+2\pi) &= X^1_Q(\tau,\sigma) - 2\pi\,\tilde p^3\, Q\,\tilde X_{Q2}(\tau,\sigma)\,,\\ X^2_Q(\tau,\sigma+2\pi) &= X^2_Q(\tau,\sigma) + 2\pi\,\tilde p^3\, Q\,\tilde X_{Q1}(\tau,\sigma)\,,\\ \tilde X_{Q1}(\tau,\sigma+2\pi) &= \tilde X_{Q1}(\tau,\sigma)\,,\\ \tilde X_{Q2}(\tau,\sigma+2\pi) &= \tilde X_{Q2}(\tau,\sigma)\,.\end{aligned}\right.$$

$$[X^1_Q(\tau,\sigma), X^2_Q(\tau,\sigma')] =$$

$$-\frac{i}{2}Q\,\tilde p^3\left(\sum_{n\neq 0}\frac{1}{n^2}e^{-in(\sigma'-\sigma)} - (\sigma'-\sigma)\sum_{n\neq 0}\frac{1}{n}e^{-in(\sigma'-\sigma)} + \frac{i}{2}(\sigma'-\sigma)^2\right)$$

$$\sigma\to\sigma' : \qquad [X^1_Q(\tau,\sigma), X^2_Q(\tau,\sigma)] = -i\frac{\pi^2}{6}Q\,\tilde p^3$$

The non-commutativity of the torus (fibre) coordinates is determined by the winding in the circle (base) direction.

Corresponding uncertainty relation:

$$(\Delta X_Q^1)^2(\Delta X_Q^2)^2 \geq L_s^6\, Q^2 \ \langle \tilde{p}^3 \rangle^2$$

The spatial uncertainty in the X_1, X_2 - directions grows with the dual momentum in the third direction: non-local strings with winding in third direction.

R-flux background: T-duality in x^3-direction $\Rightarrow$ R-flux

$$\tilde{p}^3 \longleftrightarrow p_3\,, \qquad \tilde{X}_{Q,3} \equiv X^3_R$$

$\Rightarrow$ For the case of non-geometric R-fluxes one gets:

$$[X^1_R, X^2_R] = -i\frac{\pi^2}{6}\, R\; p_3 \qquad\qquad R \equiv Q$$

Use $[X^3_R, p_3] = i \qquad \Longrightarrow$ Non-associative algebra:

$$[[X^1_R(\tau,\sigma), X^2_R(\tau,\sigma)], X^3_R(\tau,\sigma)] + \text{perm.} = \frac{\pi^2}{6}\, R$$

Corresponding classical „uncertainty relations":

$$(\Delta X^1_R)^2 (\Delta X^2_R)^2 \geq L_s^6\, R^2\, \langle p^3 \rangle^2$$

Volume: $(\Delta X^1_R)^2 (\Delta X^2_R)^2 (\Delta X^3_R)^2 \geq L_s^6\, R^2$

(see also: D. Mylonas, P. Schupp, R.Szabo, arXiv:1312.1621)

The algebra of commutation relation looks different in each of the four duality frames.

Non-vanishing commutators and 3-brackets:

T-dual frames	Commutators	Three-brackets
H-flux	$[\tilde{x}^1, \tilde{x}^2] \sim H\tilde{p}^3$	$[\tilde{x}^1, \tilde{x}^2, \tilde{x}^3] \sim H$
f-flux	$[x^1, \tilde{x}^2] \sim f\tilde{p}^3$	$[x^1, \tilde{x}^2, \tilde{x}^3] \sim f$
Q-flux	$[x^1, x^2] \sim Q\tilde{p}^3$	$[x^1, x^2, \tilde{x}^3] \sim Q$
R-flux	$[x^1, x^2] \sim Rp^3$	$[x^1, x^2, x^3] \sim R$

However: R-flux & winding coordinates:

$$[\tilde{x}^i, \tilde{x}^j, \tilde{x}^k] = 0$$

Mathematical framework to describe non-geometric string backgrounds and the non-associative algebras:

$\Rightarrow$ 3-Cocycles, 2-cochains and 3-products

Open string non-commutativity:

Constant Poisson structure: $[x_i, x_j] = \theta_{ij}$

Moyal-Weyl star-product:

$$(f_1 \star f_2)(\vec{x}) = e^{i\theta^{ij}\,\partial_i^{x1}\,\partial_j^{x2}} f_1(\vec{x}_1)\, f_2(\vec{x}_2)|_{\vec{x}}$$

2-cyclicity: $\int d^n x\,(f\star g) = \int d^n x\,(g\star f)$

Non-commutative gauge theories: $S \simeq \int d^n x\,\mathrm{Tr}\hat{F}_{ab}\star\hat{F}^{ab}$

(N. Seiberg, E. Witten (1999); J. Madore, S. Schraml, P. Schupp, J. Wess (2000); ....)

Closed strings: Non-associative algebra:

$$[x^i,\ x^j] = \epsilon^{ijk} p_k$$

$$[x^i,\ p^j] = i\hbar\delta^{ij}\ , \qquad [p^i,\ p^j] = 0$$

$$[x^i,\ x^j,\ x^k] = [[x^i,\ x^j],\ x^k] + \text{cycl. perm.} = R^{ijk}$$

Non-commutativity $\Rightarrow$ $\star_p$ 2-product:

D. Mylonas, P. Schupp, R.Szabo, arXiv:1207.0926, arXiv:1312.162, arXiv:1402.7306.

I. Bakas, D.Lüst, arXiv:1309.3172

$$(f_1 \star_p f_2)(\vec{x},\vec{p}) = e^{\frac{i}{2}\theta^{IJ}(p)\,\partial_I \otimes \partial_J}\,(f_1 \otimes f_2)\big|_{\vec{x};\ \vec{p}}$$

6-dimensional Poisson tensor:

$$\theta^{IJ}(p) = \begin{pmatrix} R^{ijk}p_k & \delta^i_j \\ -\delta^j_i & 0 \end{pmatrix}\ ;\ R^{ijk} = \frac{\pi^2 R}{6}\,\epsilon^{ijk}$$

3-product:

R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, arXiv:1106.0316

D. Mylonas, P. Schupp, R.Szabo, arXiv:1207.0926, arXiv:1312.162, arXiv:1402.7306.

I. Bakas, D.Lüst, arXiv:1309.3172

$$(f_1 \triangle_3 f_2 \triangle_3 f_3)(\vec{x}) = ((f_1 \star_p f_2) \star_p f_3)\,(\vec{x})$$

$$(f_1 \triangle_3 f_2 \triangle_3 f_3)(\vec{x}) = e^{iR^{ijk}\,\partial_i^{x1}\,\partial_j^{x2}\,\partial_k^{x3}}\,f_1(\vec{x}_1)\,f_2(\vec{x}_2)\,f_3(\vec{x}_3)|_{\vec{x}}$$

This 3-product is non-associative.

It is consistent with the 3-bracket among the coordinates:

$$f_1 = X^i\,,\ f_2 = X^j\,,\ f_3 = X^k\,:$$

$$f_1 \triangle_3 f_2 \triangle_3 f_3 = [X^i, X^j, X^k] = R^{ijk}$$

It obeys the 3-cyclicity property:

$$\int d^n x\,(f_1 \triangle_3 f_2) \triangle_3 f_3 = \int d^n x\, f_1 \triangle_3 (f_2 \triangle_3 f_3)$$

Three point function in CFT:

R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, arXiv:1106.0316

$$\langle X^i(z_1,\overline{z}_1)\, X^j(z_2,\overline{z}_2)\, X^c(z_3,\overline{z}_3)\rangle = R^{ijk}\Big[\mathcal{L}\big(\tfrac{z_{12}}{z_{13}}\big) + \mathcal{L}\big(\tfrac{\overline{z}_{12}}{\overline{z}_{13}}\big)\Big]$$

$$\Rightarrow \quad [X^i, X^j, X^k] := \lim_{z_i \to z} \Big[X^i(z_1,\overline{z}_1), \big[X^b(z_2,\overline{z}_2), X^c(z_3,\overline{z}_3)\big]\Big] + \text{cycl.} = R^{ijk}$$

$\triangle_3$: Scattering of 3 momentum states in R-background:

(corresponds to 3 winding states in H-background)

$$V_i(z,\bar{z}) =: \exp\big(ip_i X^i(z,\bar{z})\big) :$$

$$\langle\, V_{\sigma(1)}\, V_{\sigma(1)}\, V_{\sigma(1)}\,\rangle_R = \langle\, V_1\, V_2\, V_3\,\rangle_R \times \exp\Big(-i\eta_\sigma R^{ijk}\, p_{1,i} p_{2,j} p_{3,k}\Big).$$

$$(\eta_\sigma = 0, 1)$$

However this non-associative phase is vanishing, when going on-shell in CFT and using momentum conservation:

$$p_1 = -(p_2 + p_3)$$

On-shell CFT amplitudes are associative!

III) Double field theory (target space point of view)

W. Siegel (1993); C. Hull, B. Zwiebach (2009); C. Hull, O. Hohm, B. Zwiebach (2010,...)

- O(D,D) invariant effective string action containing momentum and winding coordinates at the same time:

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2\phi'} \mathcal{R} \qquad X^M = (\tilde{x}_m, x^m)$$

$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\phi'\partial_N\phi' - \partial_M\partial_N\mathcal{H}^{MN} \quad -4\mathcal{H}^{MN}\partial_M\phi'\partial_N\phi' + 4\partial_M\mathcal{H}^{MN}\partial_N\phi'$$
$$+\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} \quad -\tfrac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

- Covariant fluxes of DFT:

(Geissbuhler, Marques, Nunez, Penas; Aldazabal, Marques, Nunez)

$$\mathcal{F}_{ABC} = \mathcal{D}_{[A}E_B{}^M \, E_{C]M} \, , \quad \mathcal{D}^A = E^A{}_M \, \partial^M \, .$$

Comprise all fluxes (Q,f,Q,R) into one covariant expression:

$$\mathcal{F}_{abc} = H_{abc} \, , \quad \mathcal{F}^a{}_{bc} = F^a{}_{bc} \, , \quad \mathcal{F}_c{}^{ab} = Q_c{}^{ab} \, , \quad \mathcal{F}^{abc} = R^{abc}$$

DFT action in flux formulation:

$$S_{\rm DFT} = \int dX\, e^{-2d}\Big[\mathcal{F}_A\mathcal{F}_{A'}S^{AA'} + \mathcal{F}_{ABC}\mathcal{F}_{A'B'C'}\Big(\frac{1}{4}S^{AA'}\eta^{BB'}\eta^{CC'} - \frac{1}{12}S^{AA'}S^{BB'}S^{CC'}\Big) - \frac{1}{6}\mathcal{F}_{ABC}\,\mathcal{F}^{ABC} - \mathcal{F}_A\mathcal{F}^A\Big]$$

(Looks similar to scalar potential in gauged SUGRA.)

- Strong constraint (string level matching condition):

(CFT origin of the strong constraint: A. Betz, R. Blumenhagen, D. Lüst, F. Rennecke, arXiv:1402.1686)

$$\partial_M\partial^M\cdot = 0\,, \quad \partial_M f\,\partial^M g = \mathcal{D}_A f\,\mathcal{D}^A g = 0$$

Functions depend only on one kind of coordinates.

The strong constraint defines a D-dim. hypersurface (brane) in 2D-dim. double geometry.

Non-associative deformations in double field theory:

(R. Blumenhagen, M. Fuchs, F. Hassler, D.Lüst, R. Sun, arXiv:1312.0719)

DFT generalization of the 3-product:

$$\big(f \triangle_3 g \triangle_3 h\big)(X) = f\,g\,h + \frac{\ell_s^4}{6}\mathcal{F}_{ABC}\,\mathcal{D}^A f\,\mathcal{D}^B g\,\mathcal{D}^C h + O(\ell_s^8)$$

(For general functions $f(X,P)$ the phase space is 4D-dimensional.)

Non-vanishing R-flux:

I. Bakas, D. L., arXiv:1309.3172

$$f = x^i\,,\ g = x^j\,,\ h = x^k\ :$$

$$f \triangle_3 g \triangle_3 h = [x^i, x^j, x^k] = \ell_s^4\, R^{ijk}$$

$$f = \tilde{x}_i\,,\ g = \tilde{x}_j\,,\ h = \tilde{x}_k\ : \quad f \triangle_3 g \triangle_3 h = [\tilde{x}_i, \tilde{x}_j, \tilde{x}_k] = 0$$

Non-vanishing H-flux:

$$f = \tilde{x}_i\,,\ g = \tilde{x}_j\,,\ h = \tilde{x}_k\ :$$

$$f \triangle_3 g \triangle_3 h = [\tilde{x}_i, \tilde{x}_j, \tilde{x}_k] = \ell_s^4\, H_{ijk}$$

$$f = x^i\,,\ g = x^j\,,\ h = x^k\ : \quad f \triangle_3 g \triangle_3 h = [x^i, x^j, x^k] = 0$$

General functions f, g and h (conformal fields in CFT):

Consider the additional term in the DFT tri-product:

$$\mathcal{F}_{ABC}\,\mathcal{D}^A f\,\mathcal{D}^B g\,\mathcal{D}^C h$$

Imposing the strong constraint on f, g and h the additional term vanishes and the tri-product becomes the normal product.

IV) Dimensional Reduction of DFT

O. Hohm, D. Lüst, B. Zwiebach, arXiv:1309.2977;
F. Hassler, D. Lüst, arXiv:1401.5068.

see also: A. Dabholkar, C. Hull, 2002, 2005;
C. Hull, R. Reid-Edwards, 2005, 2006, 2007

- Consistent DFT solutions: $\mathcal{R}_{MN} = 0$

- 2(D-d) linear independent Killing vectors:

$$\mathcal{L}_{K_I{}^J}\mathcal{H}^{MN} = 0$$

- DFT and generalized Scherk-Schwarz ansatz (O(D,D) twists) gives rise to effective theory in D-d dimensions:

$$S_{\text{eff}} = \int \mathrm{d}x^{(D-d)}\sqrt{-g}e^{-2\phi}\Big(\mathcal{R} + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{4}\mathcal{H}_{MN}F^{M\mu\nu}F^{N}_{\mu\nu} + \frac{1}{8}D_\mu\mathcal{H}_{MN}D^\mu\mathcal{H}^{MN} - V\Big)$$

The corresponding backgrounds are in general non-geometric and go beyond dimensional reduction of SUGRA.

(i) DFT on spaces satisfying the strong constraint (SC)

Rewriting of SUGRA, geometric spaces

(ii) Mild violation of SC:

- Killing vectors violate the SC.
- Patching of coordinate charts correspond to generalized coordinate transformations that violate the SC.

(iii) Strong violation of SC:

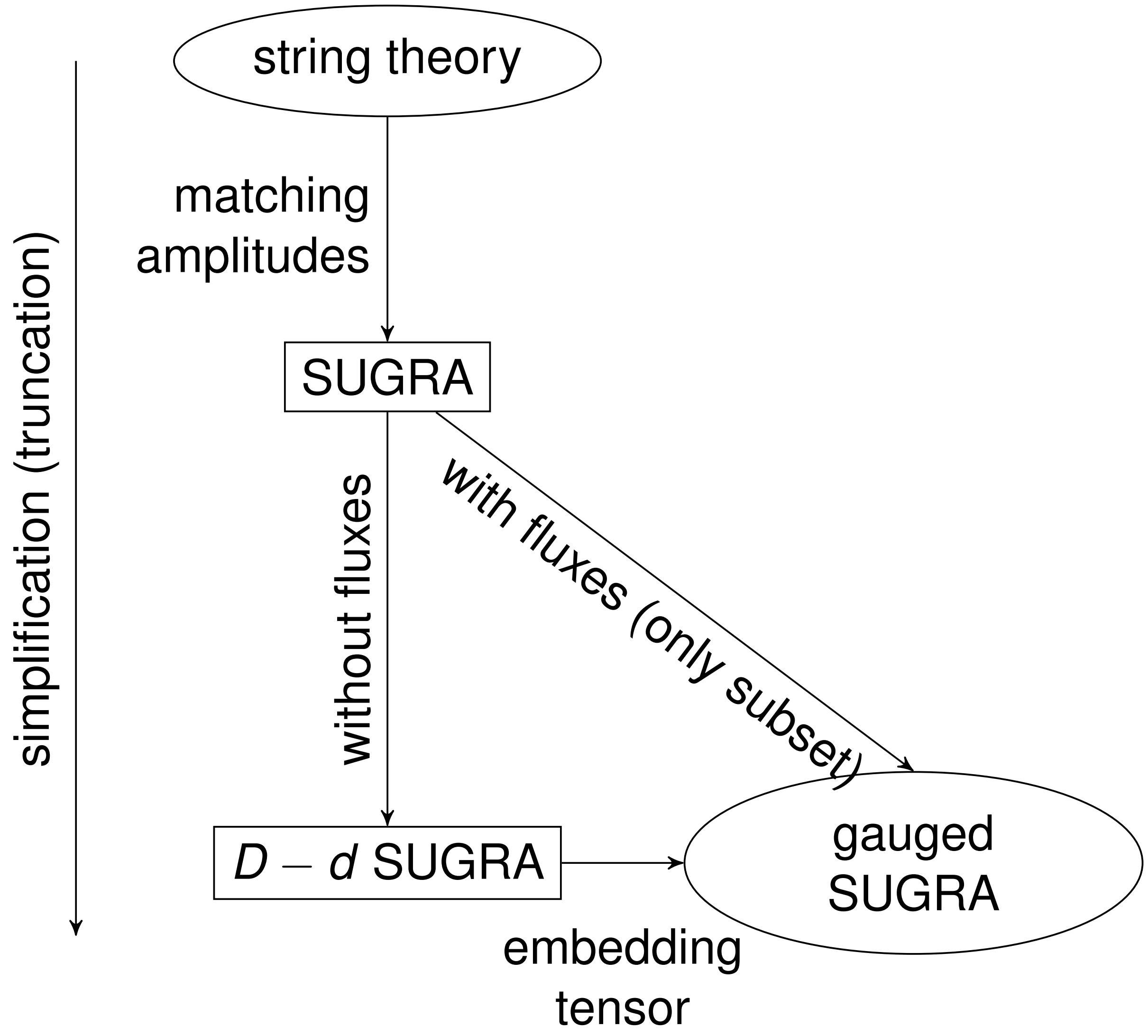
- Background fields violate the SC.

However the fluxes have to obey the closure constraint - consistent gauge algebra in the effective theory.

M. Grana, D. Marques, arXiv:1201.2924

Dimensional reduction of double field theory:

Generalized Scherk-Schwarz compactifications



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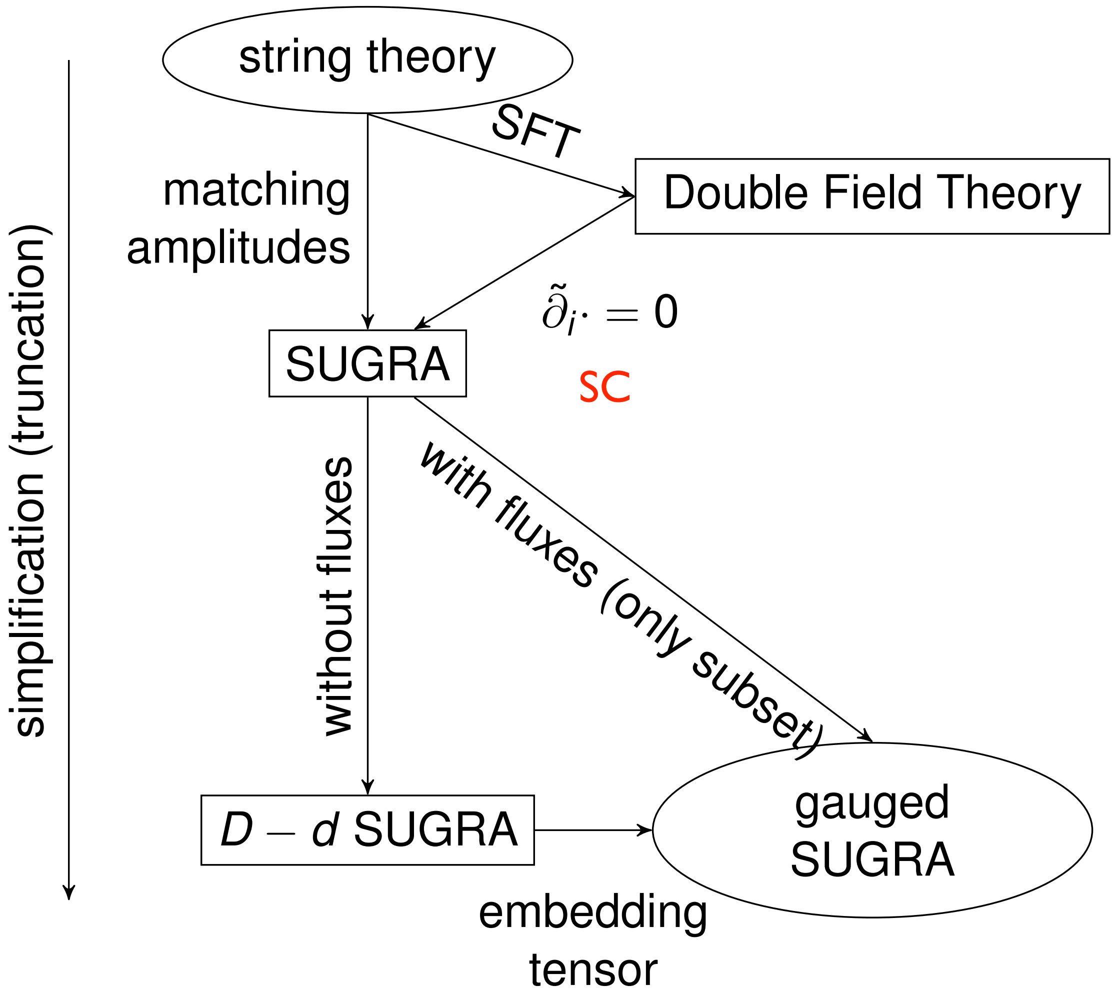

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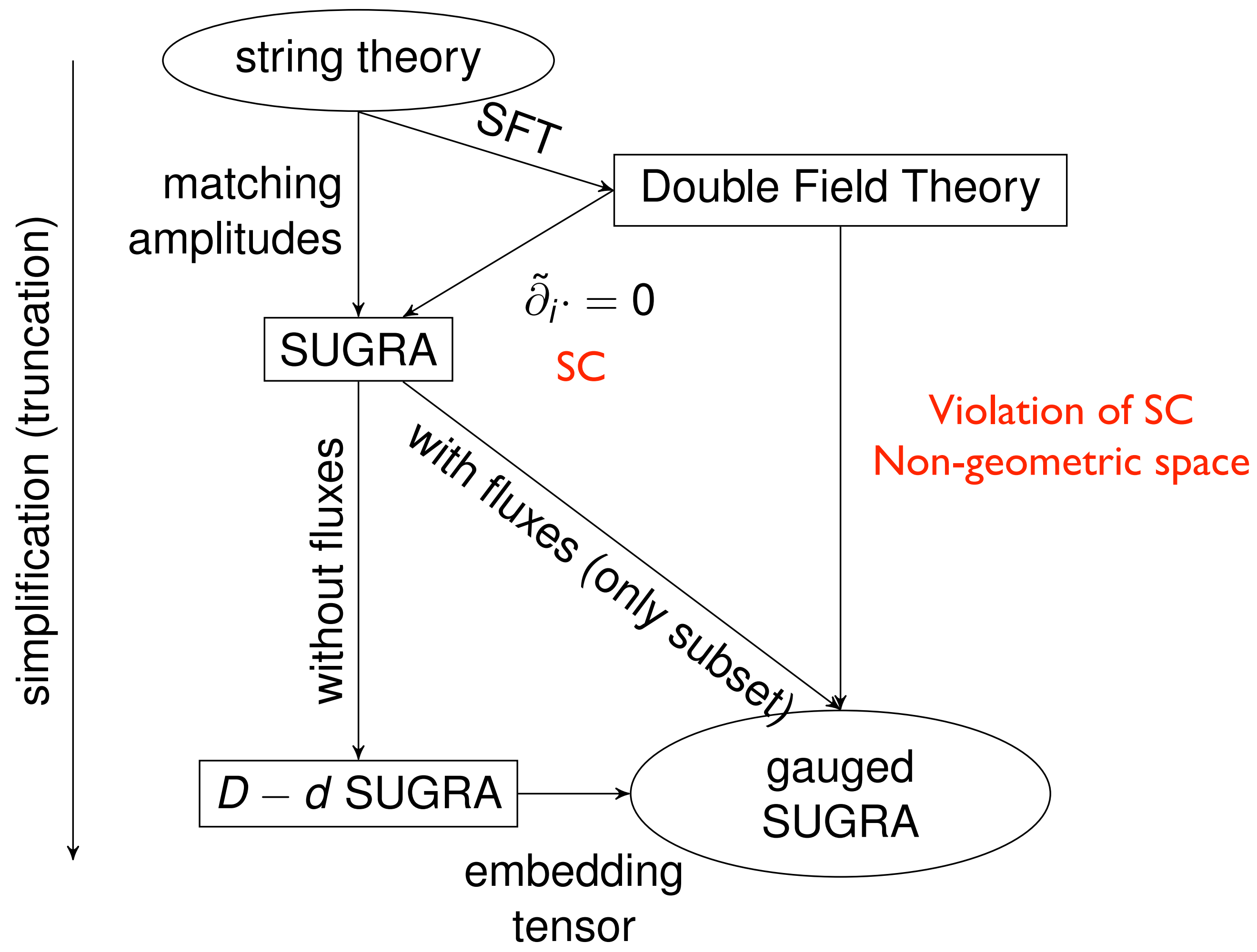

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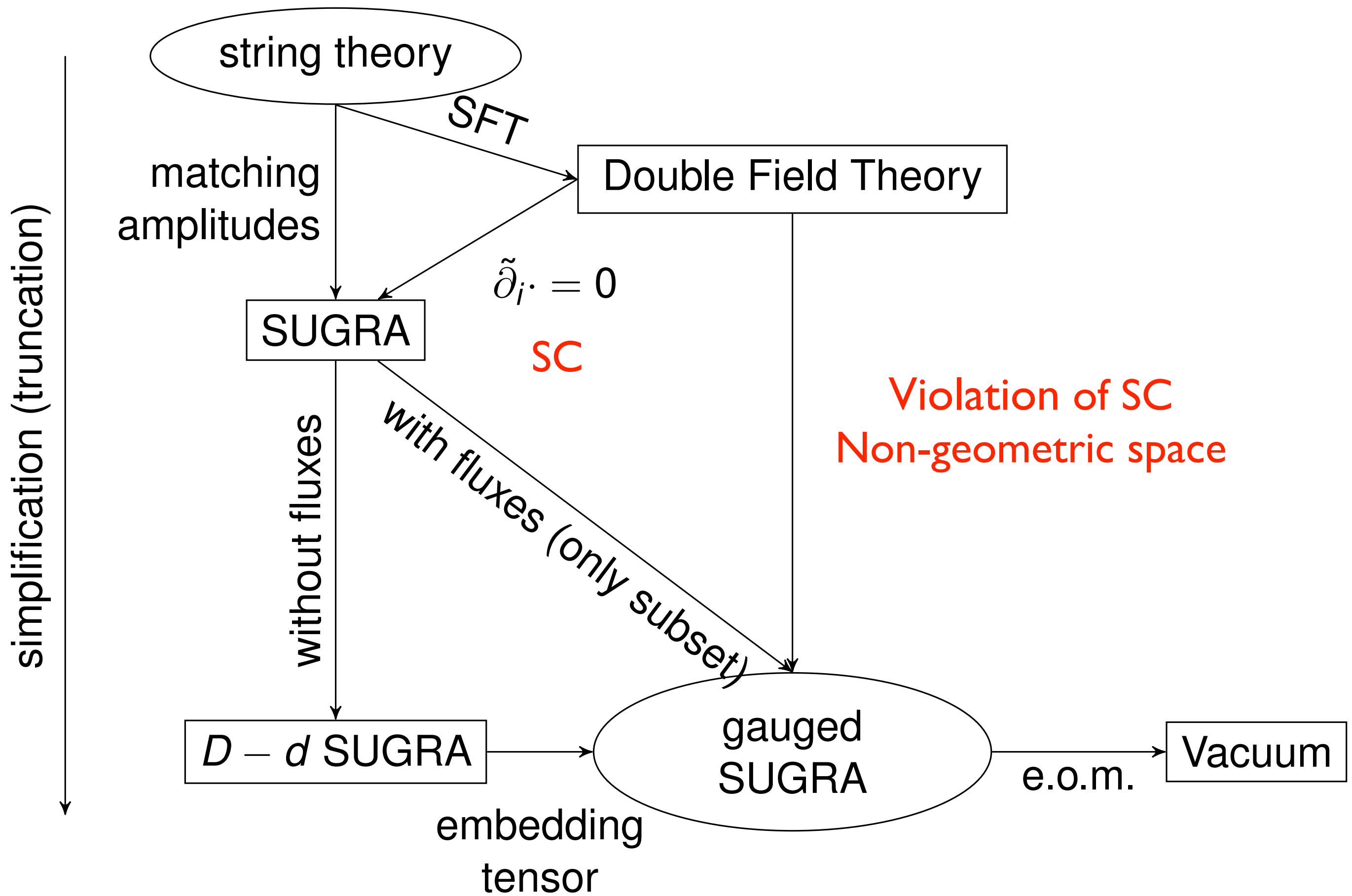

Dimensional reduction of double field theory:

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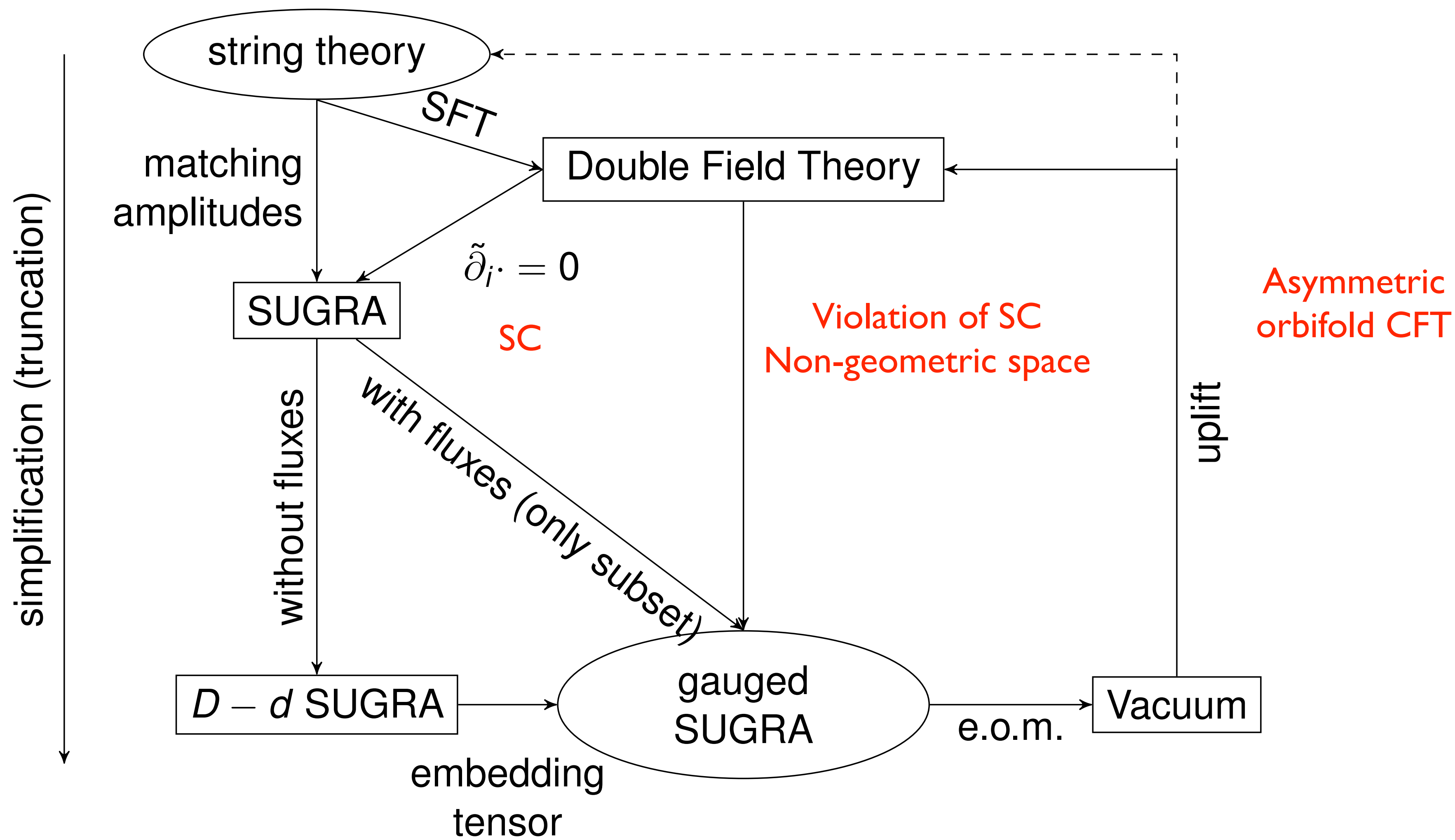


- Effective scalar potential:

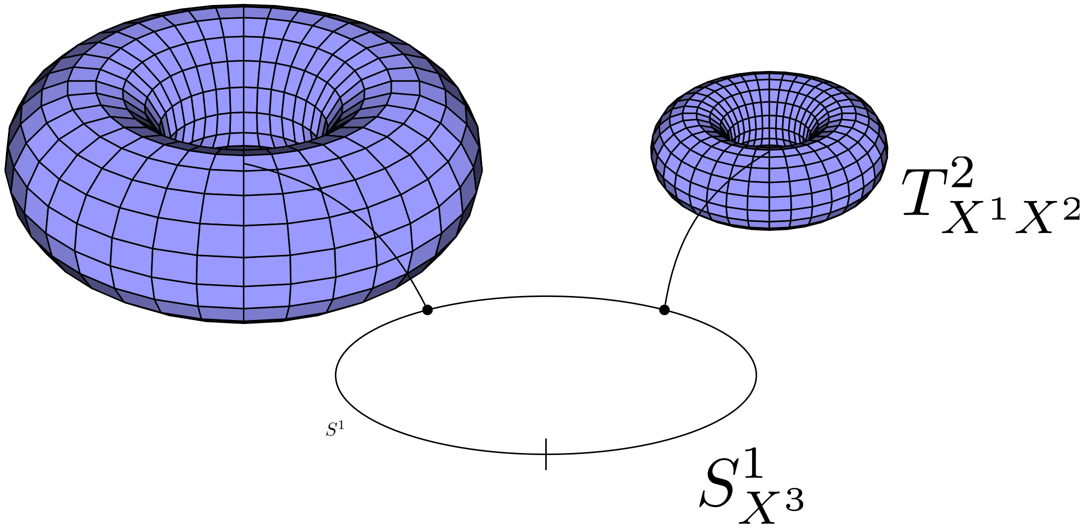
$$V = -\frac{1}{4}\mathcal{F}_I^{KL}\mathcal{F}_{JKL}\mathcal{H}^{IJ} + \frac{1}{12}\mathcal{F}_{IKM}\mathcal{F}_{JLN}\mathcal{H}^{IJ}\mathcal{H}^{KL}\mathcal{H}^{MN}$$

- $R_{MN} = 0 \quad \Rightarrow$ Minkowski vacua:

$$V = 0 \quad \text{and} \quad \mathcal{K}^{MN} = \frac{\delta V}{\delta \mathcal{H}_{MN}} = 0$$

This leads to additional conditions on the fluxes $\mathcal{F}_{IKM}$.

Simplest non-trivial solutions: d=3 dim. backgrounds:



Parabolic background spaces: Single fluxes:

$$H_{123} \quad \text{or} \quad f^1_{23} \quad \text{or} \quad Q^{12}_3 \quad \text{or} \quad R^{123}$$

These backgrounds do not satisfy $R^{MN} = 0\,.$

- CFT: beta-functions are non-vanishing at quadratic order in fluxes.

- Effective scalar potential: no Minkowski minima ($\Rightarrow$ AdS)

Elliptic background spaces: Multiple fluxes:

These backgrounds do satisfy $R^{MN} = 0\,.$

- Single elliptic geometric space (Solvmanifold):

$$f_{13}^{2} = f_{23}^{1} = f \quad \Rightarrow \quad \text{Symmetric} \quad \mathbb{Z}_4^L \times \mathbb{Z}_4^R \text{ orbifold.}$$

- Single elliptic T-dual, non-geometric space:

$$H_{123} = Q_3^{12} = H$$

$$\Rightarrow \quad \text{Asymmetric} \quad \mathbb{Z}_4^L \times \mathbb{Z}_4^R \text{ orbifold.}$$

- Double elliptic, genuinely non-geometric space:

$$H_{123} = Q_3^{12} = H\,, \qquad f_{13}^{2} = f_{23}^{1} = f$$

$$\Rightarrow \quad \text{Asymmetric} \quad \mathbb{Z}_4^L \text{ orbifold.}$$

Monodromy of double elliptic backgroun

Background satisfies strong constraint

c.s.: $$\tau(x_3) = \frac{\tau_0 \cos(f x_3) + \sin(f x_3)}{\cos(f x_3) - \tau_0 \sin(f x_3)},$$

Kahler: $$\rho(x_3) = \frac{\rho_0 \cos(H x_3) + \sin(H x_3)}{\cos(H x_3) - \rho_0 \sin(H x_3)}, \quad H \in \frac{1}{4} + \mathbb{Z}.$$

$$\Longrightarrow \quad \tau(2\pi) = -\frac{1}{\tau(0)}, \qquad \rho(2\pi) = -\frac{1}{\rho(0)}$$

Corresponding Killing vectors of background:

$$K_{\hat{I}}^{\hat{J}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2}(Hx^3 + f\tilde{x}^3) & \frac{1}{2}(Hx^2 + f\tilde{x}^2) & -\frac{1}{2}(fx^3 + H\tilde{x}^3) & \\ 0 & 0 & 1 & 0 & & \\ 0 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & & \end{pmatrix}$$

Killing vectors do not satisfy strong constraint However their algebra closes!

- There situations, where the strong constraint even for the background can be violated. - This seems to be the case for certain very asymmetric orbifolds. C. Condeescu, I. Florakis, C. Kounnas, D.Lüst, arXiv:1307.0999

$$\tau(x_3,\tilde{x}_3)=\frac{\tau_0\cos(f_4x_3+f_2\tilde{x}_3)+\sin(f_4x_3+f_2\tilde{x}_3)}{\cos(f_4x_3+f_2\tilde{x}_3)-\tau_0\sin(f_4x_3+f_2\tilde{x}_3)}\,,\qquad f_4,g_4\in\frac{1}{8}+\mathbb{Z}$$

$$\rho(x_3,\tilde{x}_3)=\frac{\rho_0\cos(g_4x_3+g_2\tilde{x}_3)+\sin(g_4x_3+g_2\tilde{x}_3)}{\cos(g_4x_3+g_2\tilde{x}_3)-\rho_0\sin(g_4x_3+g_2\tilde{x}_3)}\,,\qquad f_2,g_2\in\frac{1}{4}+\mathbb{Z}$$

Fluxes:

Parameter	Fluxes
f_4	$f,\tilde{f}$
f_2	$Q,\tilde{Q}$
g_4	H,Q
g_2	$\tilde{f},R$

Asymmetric $\mathbb{Z}_4^L\times\mathbb{Z}_2^R$ orbifold with H, f ,Q,R-fluxes.

This (partially?) solves a so far existing puzzle between effective SUGRA and uplift/string compactification.

V) De Sitter and Inflation

F. Hassler, D. Lüst, S. Massai, arXiv:1405.2325

Effective scalar potential of double elliptic backgrounds:

$$V(\tau,\rho) = \frac{1}{R^2}\left[\frac{f_1^2 + 2f_1 f_2(\tau_R^2 - \tau_I^2) + f_2^2|\tau|^4}{2\tau_I^2} + \frac{H^2 + 2HQ(\rho_R^2 - \rho_I^2) + Q^2|\rho|^4}{2\rho_I^2}\right] \geq 0$$

The potential is positive semi-definite.

No up-lift is needed!

Vacuum structure:

- Minkowski vacua: $HQ > 0$

$$\rho_R^\star = 0\,, \quad \rho_I^\star = \sqrt{\frac{H}{Q}}\,, \qquad V_{\min} = 0$$

e.g. $H = Q = 1/4 \Rightarrow$ Asymmetric $\mathbb{Z}_4^L$ orbifold.

- de Sitter vacua: $HQ < 0$

$$(\rho_R^\star)^2 + (\rho_I^\star)^2 = -\frac{H}{Q}\,, \qquad V_{\rm min} = -4HQ > 0$$

However here the radius R is not stabilized.

Another option: SO(2,2) gauging

$$V(\rho,\tau) = \frac{H^2}{2\rho_I^2}\left(1 + 2(\rho_R^2 - \rho_I^2) + |\rho|^4\right) + \frac{H^2}{\rho_I \tau_I}(1+|\rho|^2)(1+|\tau|^2) + \frac{H^2}{2\tau_I^2}\left(1 + 2(\tau_R^2 - \tau_I^2) + |\tau|^4\right)$$

$$\rho^\star = \tau^\star = i \quad \text{with} \quad V_{\rm min} = 4H^2$$

All moduli τ and ρ have positive mass square.

Inflation from non-geometric backgrounds:

There are some attractive features for inflation:

- The potentials are positive with quadratic and quartic couplings that depend on the (non)-geometric fluxes.

No up-lift is needed!

One needs to tune fluxes to obtain slow roll inflation.

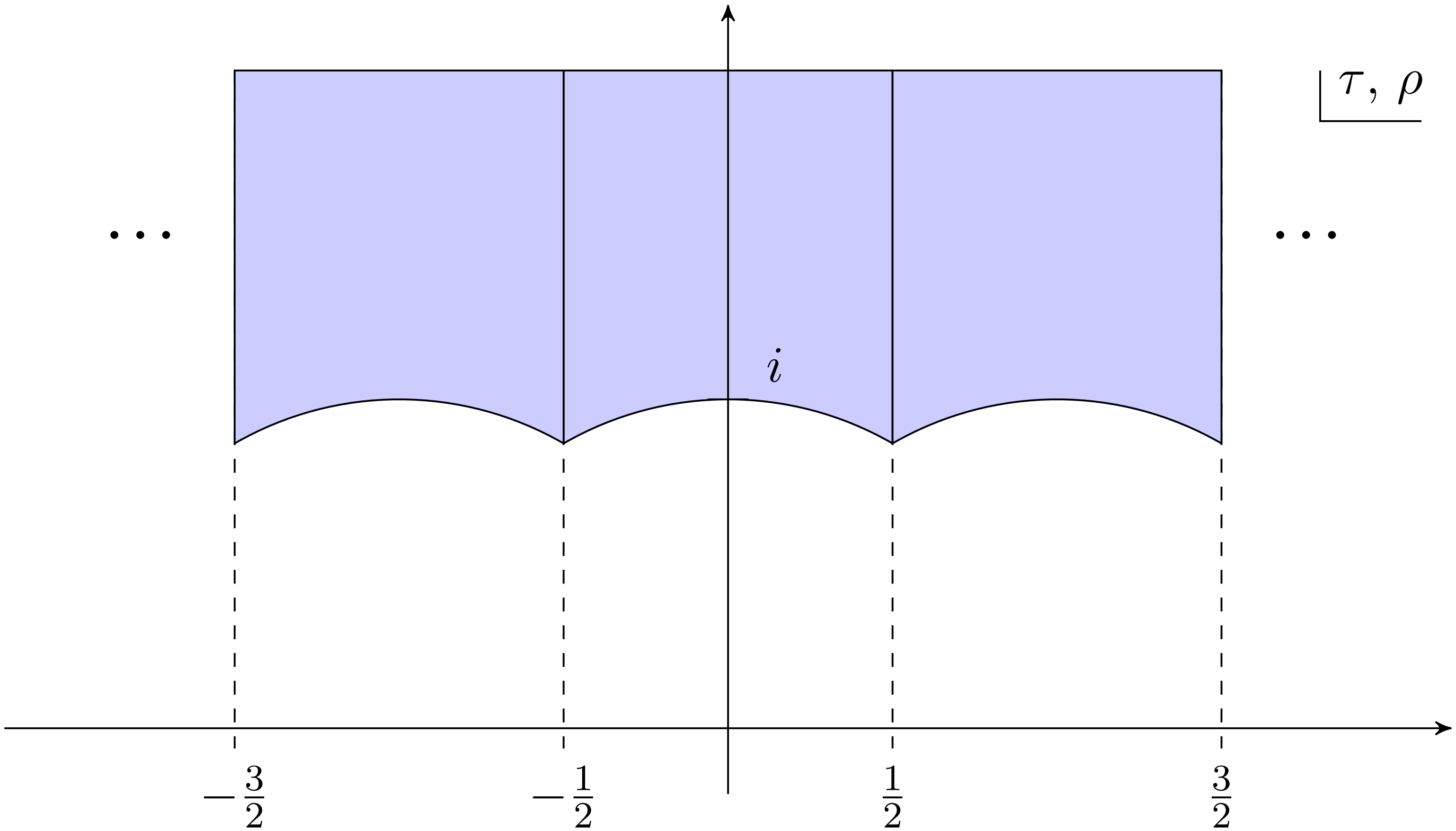
($\Rightarrow$ Orbifolds with high order of twist!)

- The non-trivial monodromies allow for enlarged field range of the inflaton field. (McAllsiter, Silverstein, Westphal, 2008)

Realization of monodromy inflation in order to obtain a visible tensor to scalar ratio (gravitational waves).

Enlarged field range for parabolic monodromy

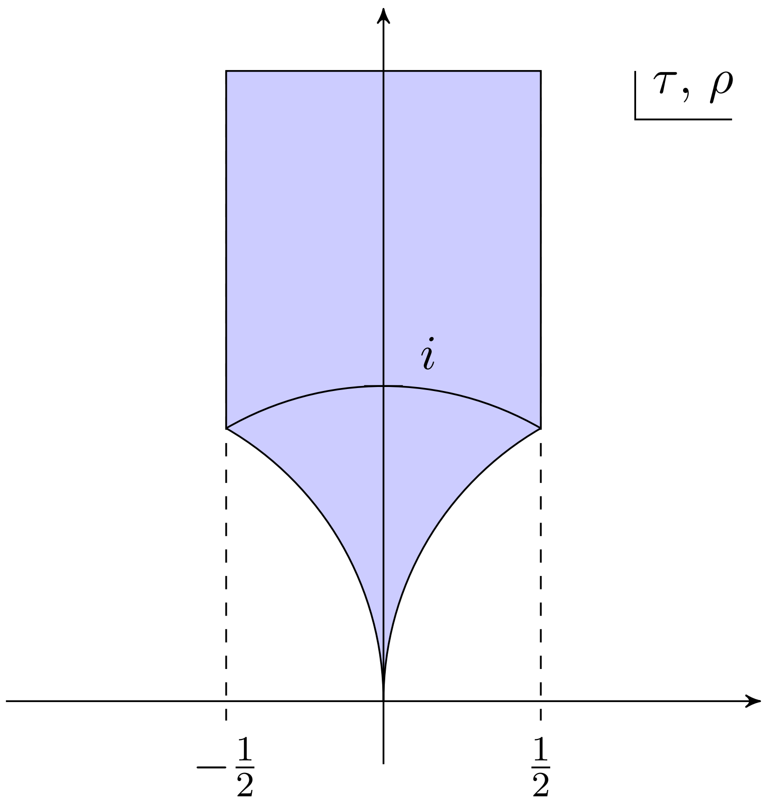
$$\rho \to \rho + 1 \quad \text{or} \quad \tau \to \tau + 1 \quad :$$



$\Rightarrow$ Infinite field range for τ_R or ρ_R .

Enlarged field range for elliptic $\mathbb{Z}_4$ monodromy

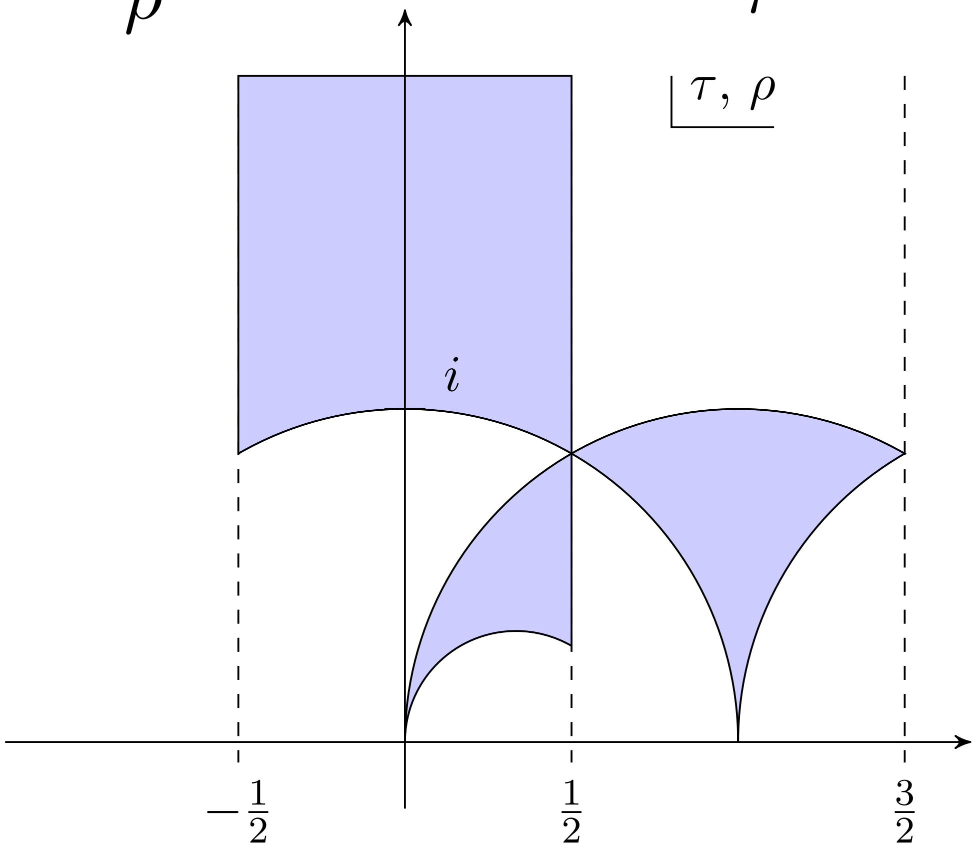
$$\rho \to -\frac{1}{\rho} \quad \text{or} \quad \tau \to -\frac{1}{\tau} \quad :$$



$\Rightarrow$ Infinite field range for combinations of τ_R and τ_I or combinations of ρ_R and ρ_I .

Enlarged field range for elliptic $\mathbb{Z}_6$ monodromy

$$\rho \to -\frac{1}{\rho} + 1 \quad \text{or} \quad \tau \to -\frac{1}{\tau} + 1 \ :$$



$\Rightarrow$ Infinite field range for combinations of

τ_R and τ_I or combinations of ρ_R and ρ_I .

Simple elliptic model for non-geometric inflation:

Expect fluxes $H, Q \sim \frac{1}{N}$

Kinetic energy: $$\mathcal{L}_{\rm kin} = \frac{1}{4\rho_I^2}\Big[(\partial\rho_R)^2 + (\partial\rho_I)^2\Big]$$

Inflaton field: $$\phi = \frac{\rho_R}{2\rho_I}$$

Inflaton potential:

$$V(\phi,\rho_I) = V_0(\rho_I) + m^2(\rho_I)\,\phi^2 + \lambda(\rho_I)\,\phi^4$$

$$V_0(\rho_I) = \frac{H^2 - 2HQ\rho_I^2 + Q^2\rho_I^4}{2\rho_I^2}\,, \quad m^2(\rho_I) = 4HQ + 4Q^2\rho_I^2\,, \quad \lambda(\rho_I) = 8Q^2\rho_I^2$$

Minimization with respect to ρ_I : $\Rightarrow \quad V_0 = 0$

Inflaton mass and self-coupling:

$$m^2 = 4HQ\left(\frac{1}{\rho_I^\star} + \rho_I^\star\right) M_s^2\,, \qquad \lambda = 8HQ\rho_I^\star$$

$$g_s^2 M_P^2 \frac{\lambda}{m^2} = \frac{2\,(\rho_I^\star)^3}{1 + (\rho_I^\star)^2} \qquad (M_P^2 = \frac{1}{g_s^2} M_s^2 \rho_I^\star)$$

Small λ $\Rightarrow$ small value for $\rho_I^\star$.

Slow roll inflation with 60 e-foldings and

$$n_s \sim 0.967\,, \quad r \sim 0.133 \quad \text{(BICEP2)}$$

$$n_s = 1 - 6\epsilon + 2\eta\,, \quad r = 16\epsilon$$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2\,, \qquad \eta = M_P^2\left(\frac{\partial_\phi^2 V}{V}\right)$$

$$m \simeq 6\times 10^{-6} M_P\,, \quad V_0^{1/4} \simeq 10^{-2} M_P \;\Rightarrow\; \phi \simeq 15 M_P$$

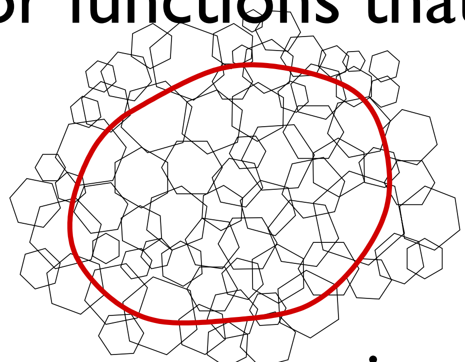
$$H' \simeq Q' \simeq 10^{-5}\,, \quad \rho_I^\star \leq 10^{-2}$$

$\Rightarrow$ Need very small fluxes (large monodromy $N \simeq 10^5$)

and sub-stringy value for the volume of the fibre.

V) Outlook & open questions

- Non-commutative & non-associative closed string geometry arises in the presence of non-geometric fluxes (like open string non-commutativity on D- branes with gauge flux). This leads to a non-associative tri-product (like the star-product).
- However the non-associativity is not visible in on-shell CFT amplitudes/ for functions that satisfy strong constraint in DFT.



Non-associativity is an off-shell phenomenon!

- Is there a non-commutative (non-associative) theory of gravity?

(A. Chamseddine, G. Felder, J. Fröhlich (1992), J. Madore (1992); L. Castellani (1993) P. Aschieri, C. Blohmann, M. Dimitrijevic, F. Meyer, P. Schupp, J. Wess (2005), L. Alvarez-Gaume, F. Meyer, M. Vazquez-Mozo (2006))

- DFT allows for consistent reduction on non-geometric backgrounds that go beyond Sugra and also beyond generalized geometry $\Rightarrow$ interesting applications for cosmology.

Dear Organizers, many thanks for this excellent and stimulating meeting at this wonderful conference site.

So let us give a long applause to Marcos, Matthias, Matthias & Niklas !