

Symmetries of $N=4$ higher spin holography

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Based on: [M. Gaberdiel and CP](#) [JHEP 1405 \(2014\) 152](#)

(Ascona, Switzerland, 07/24/2014)

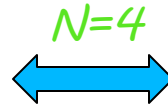
Why Higher Spin Theory

- Higher Spin theories are interesting:
 - Tensionless/High energy limit of string theory
 - AdS/CFT correspondence

- We focus on the higher spin supergravity theory on AdS₃
 - Simple in many aspects
 - Capture many important features of general higher spin theories

Higher spin/Minimal model Holography

$shs_2[\mu]$ Higher spin
theory in AdS_3

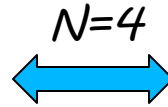


2d Wolf space
coset models

(Gaberdiel & Gopakumar 2012)

Higher spin/Minimal model Holography

$shs_2[\mu]$ Higher spin theory in AdS_3



2d Wolf space coset models

(Gaberdiel & Gopakumar 2012)

- Defined by the Chern-Simons action with infinitely dimensional super-higher-spin algebra $shs_2[\mu]$: $I_{HS} = I_{CS}(A, k_{CS}) - I_{CS}(\bar{A}, k_{CS})$

$$I_{CS}(A, k_{CS}) = \frac{k_{CS}}{4\pi} \int_{\mathcal{M}} \text{Tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A), \quad k_{CS} = \frac{\ell}{8G_3} \frac{1}{(-\text{Tr} L_0^2)}, \quad \Lambda = -\frac{2}{\ell^2}$$

- $A = A_\mu^a dx^\mu T^a$ takes value in $shs_2[\mu]$:

$$sB_2[\mu] = shs_2[\mu] \oplus \mathbb{C}, \quad sB_2[\mu] \equiv sB[\mu] \otimes \text{Mat}_2(\mathbb{C}), \quad sB[\mu] = \frac{U(\mathfrak{osp}(1|2))}{\langle C^{\mathfrak{osp}} - \frac{1}{4} \mu(\mu-1) \mathbf{1} \rangle}$$

- N=4 supersymmetry

$$D(2, 1|\alpha), \quad \alpha = \frac{\mu}{1-\mu}$$

$$shs_2[\mu] = D(2, 1|\alpha) \oplus \bigoplus_{s=1}^{\infty} R^{(s)},$$

$$L_0, L_{\pm 1},$$

$$G_{\pm \frac{1}{2}}^a, A_0^{+i}, A_0^{-i}$$

$$R^{(s)} :$$

$$s : (1, 1)$$

$$s + \frac{1}{2} : (2, 2)$$

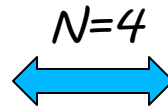
$$s + 1 : (3, 1) \oplus (1, 3)$$

$$s + \frac{3}{2} : (2, 2)$$

$$s + 2 : (1, 1).$$

Higher spin/Minimal model Holography

$shs_2[\mu]$ Higher spin theory in AdS_3



2d Wolf space coset models

(Gaberdiel & Gopakumar 2012)

- Defined by the coset (I)

$$\frac{\mathfrak{su}(N+2)_\kappa^{(1)}}{\mathfrak{su}(N)_\kappa^{(1)}} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}}, \quad \kappa = k + N + 2$$

(Van Proeyen 1989, Sevrin & Theodoridis 1990)

➤ N=4 linear A_γ ($\gamma = \mu$) algebra generated by $L_m, G_r^{\pm\pm}, A_m^{+,i}, A_m^{-i}, Q_r^{\pm\pm}, u_m$

- An alternative coset (II)

$$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa},$$

➤ N=4 nonlinear \tilde{A}_γ algebra generated by $L_m, G_r^{\pm\pm}, A_m^{+,i}, A_m^{-i}$

➤ The two algebras are related by factoring out the free fermions. (Goddard & Schwimmer 1999)

✓ The spectra:

checked

✓ The symmetries:

Asymptotic symmetry algebra = Chiral algebra ?

Asymptotic symmetry

- **Asymptotic symmetry algebra (ASA)**
 - With boundary, $G(\gamma) = \frac{k_{CS}}{4\pi} \int_M Tr(F\gamma) + Q(\gamma)$, $Q(\gamma) = -\frac{k_{CS}}{2\pi} \int_{\partial M} Tr(a\gamma)$
 - The residual gauge symmetries that preserve some boundary conditions.
 - The asymptotic symmetry of gravity on AdS₃ is 2d virasoro algebra with central charge $c_{BH} = \frac{3l}{2G_3}$. (Brown & Henneaux 1986)
- **Boundary conditions**
 - Vanishing boundary term in the variation: $A_-|_{\partial M} = 0$
 - Asymptotic AdS: $(A - A_{AdS})|_{\partial M} = \mathcal{O}(1)$
- *Directly* imposing the above boundary conditions and gauge fixing to $Q(\gamma)$ leads to the asymptotic symmetry that contains \tilde{A}_γ .

Asymptotic symmetry: non-linear

- Matches the chiral algebra of coset (II) $\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa}$ in the 't Hooft limit

$$N, k \rightarrow \infty \quad \text{while} \quad \frac{N}{N+k} = \lambda \equiv \gamma \quad \text{fixed}$$

- E. g.

$$V_{\text{CFT}}^{(2)++} \equiv \frac{iV_{\text{bulk}}^{(2)++}}{2a_{(2)++}}$$

$$\begin{aligned} \text{ASA:} \quad V_{\text{bulk}}^{(2)++}(z)V_{\text{bulk}}^{(2)++}(w) &\sim -\frac{128(\gamma-2)\gamma^2 a_{(2)++}^2 A^{++}(w)A^{++}(w)}{k_{\text{cs}}(w-z)^2} \\ &+ \frac{128(\gamma-2)\gamma^2 a_{(2)++}^2 A^{++}(w)\partial A^{++}(w)}{k_{\text{cs}}(w-z)} \end{aligned}$$

$$\begin{aligned} \text{Coset:} \quad V_{\text{CFT}}^{(2)++}(z)V_{\text{CFT}}^{(2)++}(w) &\sim \frac{-\frac{32N(2k+N+2)}{k(k+N+2)^2}(A^{++}A^{++})(w)}{(z-w)^2} \\ &- \frac{\frac{32N(2k+N+2)}{k(k+N+2)^2}(A^{++}\partial A^{++})(w)}{(z-w)} \end{aligned}$$

$$\begin{aligned}
V^{(2)++}(z)V^{(2)+-}(w) &\sim -\frac{128k_{cs}(\gamma-2)a_{(2)++}^2}{(z-w)^4} \\
&+ \frac{128(\gamma-2)(\gamma-1)A^{--}(w)\partial A^{+-}(w)a_{(2)++}^2}{3k_{cs}(w-z)} + \frac{128(\gamma-2)(\gamma-1)A^{-+}(w)\partial A^{--}(w)a_{(2)++}^2}{3k_{cs}(w-z)} \\
&+ \frac{256(\gamma-2)(\gamma-1)A^{-3}(w)\partial A^{-3}(w)a_{(2)++}^2}{3k_{cs}(w-z)} + \frac{128(\gamma-2)\gamma(2\gamma-1)A^{+-}(w)\partial A^{++}(w)a_{(2)++}^2}{3k_{cs}(w-z)} \\
&+ \frac{128(\gamma-2)(\gamma-1)\gamma A^{++}(w)\partial A^{+-}(w)a_{(2)++}^2}{3k_{cs}(w-z)} + \frac{512(\gamma-2)\gamma A^{+3}(w)T(w)a_{(2)++}^2}{3k_{cs}(w-z)} \\
&+ \frac{256(\gamma-2)\gamma(3\gamma-1)A^{+3}(w)\partial A^{+3}(w)a_{(2)++}^2}{3k_{cs}(w-z)} + \frac{64(\gamma-2)J(w)J(w)a_{(2)++}^2}{3k_{cs}(w-z)^2} \\
&+ \frac{16(\gamma-2)(5\gamma-7)G^{-+}(w)G^{+-}(w)a_{(2)++}^2}{9k_{cs}(w-z)} + \frac{64(\gamma-2)^2G'^{-+}(w)G'^{++}(w)a_{(2)++}^2}{9k_{cs}(w-z)} \\
&+ \frac{512(\gamma-2)(\gamma-1)\gamma A^{--}(w)A^{-+}(w)A^{+3}(w)a_{(2)++}^2}{3k_{cs}^2(w-z)} + \frac{512(\gamma-2)(\gamma-1)\gamma(A^{-3}(w))^2A^{+3}(w)a_{(2)++}^2}{3k_{cs}^2(w-z)} \\
&+ \frac{512(\gamma-2)(\gamma-1)\gamma^2 A^{+3}(w)a_{(2)++}^2}{3k_{cs}^2(w-z)} \\
V^{(2)++}(z)V^{(2)+-}(w) &\sim -\frac{32(k-1)N(2k+N+2)}{(k+N+2)^2} \frac{1}{(z-w)^4} - \frac{(64(-1+k)N(2+2k+N))}{k(2+k+N)^2} \frac{A^{+3}}{(z-w)^3} \\
&+ \frac{16(-1+k)(1+k)(2+N)(2+2k+N)}{k(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))} \frac{(V^{(1)0}V^{(1)0})}{(z-w)^2} \\
&- \frac{32(-1+k)N(2+2k+N)}{k(2+k+N)^2} \frac{\partial A^{+3}}{(z-w)^2} - \frac{32N(2+2k+N)}{k(2+k+N)^2} \frac{(A^{+3}A^{+3})}{(z-w)^2} \\
&+ \frac{64(-1+k)(1+k)N(2+2k+N)}{(2+k+N)(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))} \frac{C^-}{(z-w)^2} \\
&- \frac{32N(2+2k+N)(-2k^3(2+N)+(2+N)^2+k(4-3N-4N^2)+k^2(-1+N^2))}{k(2+k+N)^2(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))} \frac{C^+}{(z-w)^2} \\
&- \frac{64(-1+k)(1+k)N(2+N)(2+2k+N)}{(2+k+N)(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))} \frac{T-V^{(2)0}}{(z-w)^2} \\
&+ \mathcal{O}((z-w)^{-1}),
\end{aligned}$$

Asymptotic symmetry: linear

- The higher spin theory do **not** capture the free fermions: $A = A_{\mu}^a dx^{\mu} T^a$.
Then what is the relation with coset (I) ?

- 2 steps:

➤ Introduce boundary auxiliary degrees of freedom

$$Q(\gamma) = -\frac{k_{\text{CS}}}{2\pi} \int_{\partial\mathcal{M}} \text{Tr}(\gamma a) \quad \begin{array}{c} \downarrow \\ \longrightarrow \end{array} \quad Q'(\gamma) = -\frac{k_{\text{CS}}}{2\pi} \int \text{Tr}(\gamma a') ,$$

$$a = L_1 + \sum_{s,i} a'^{(s)i} V_{1-s}^{(s)i} , \quad a' = L_1 + \sum_{s,i} a'^{(s)i} V_{1-s}^{(s)i} , \quad a'^{(s)i} = a^{(s)i} + b^{(s)i} ,$$

➤ Determine their interaction with the boundary fields “a”: **Solve[linearity==True]**

➔ asymptotic symmetry algebra contains the linear A_{γ} algebra.

- Matches with coset (I).

Perturbative analysis and ~~HS~~

- BPS state $(f; \bar{f})$, satisfies the BPS bound

$$h \geq \frac{1}{k^+ + k^-} \left[k^+ l^- + k^- l^+ + u^2 + (l^+ - l^-)^2 \right] \quad \text{with} \quad h(f; \bar{f}) = 1/2, \quad l^+ = 1/2 = l^-, \quad u = 0$$

➤ scalar descendent of $(f; \bar{f})$, Φ , is a marginal operator.

- Under perturbation, symmetry generator W is preserved if

$$\mathcal{N}_s \equiv \sum_{l=0}^{s-1} \frac{(-1)^l}{l!} (L_{-1})^l W_{-s+1+l}^s \Phi = 0$$

(Gaberdiel, Jin & Li 2013)

- In our case

$$V_0^{(1)0} \Phi = -\frac{2(N+1)}{(k+N+2)} \Phi \neq 0$$

where $V^{(1)0}$ is the bottom component of the $R^{(1)}$ multiplet.

Truncation of the algebras

- shs₂[μ] can be truncated at special value of the parameter μ :

$$\mu = \gamma = s + 1 \quad \text{shs}_2[s + 1] = D(2, 1 | -\frac{s+1}{s}) \oplus \bigoplus_{i=1}^{s-1} R^{(i)} \oplus \hat{R}_-^{(s)}$$

$$R^{(s)} : \begin{array}{l} s : (1, 1) \\ s + \frac{1}{2} : (2, 2) \\ s + 1 : (3, 1) \oplus (1, 3) \\ s + \frac{3}{2} : (2, 2) \\ s + 2 : (1, 1) . \end{array} \quad \longrightarrow \quad \hat{R}_-^{(s)} : \begin{array}{l} s : (1, 1) \\ s + \frac{1}{2} : (2, 2) \\ s + 1 : (1, 3) \end{array}$$

- Truncation due to null vector $\mathcal{N} = G_{-\frac{1}{2}}^{+-} G_{-\frac{1}{2}}^{++} \Phi_s$ at $\gamma = s+1$, where

$$L_0 \Phi_s = s \Phi_s , \quad L_1 \Phi_s = 0 , \quad G_{\frac{1}{2}}^{\alpha\beta} \Phi_s = 0$$

- This translates to the asymptotic symmetry algebra

$$V^{(2)++}(z)V^{(2)++}(w) \sim \frac{128(\gamma - 2)\gamma^2 a_{(2)++}^2 A^{++}(w) \partial A^{++}(w)}{k_{\text{CS}}(w - z)} - \frac{128(\gamma - 2)\gamma^2 a_{(2)++}^2 A^{++}(w) A^{++}(w)}{k_{\text{CS}}(w - z)^2}$$

Truncation of the algebras

- On the coset side, there exists null vectors

$$\mathcal{N}^+ = \left(G_{-\frac{1}{2}}^{+-} G_{-\frac{1}{2}}^{++} + \frac{4s}{k} A_{-1}^{++} \right) \Phi_s \cong 0 \quad \text{at} \quad sN + (1+s)k + 2s = 0$$

- Chiral algebra truncates, e.g. at $s=1$ (corresponding to $\gamma=1+s=2$)

$$V^{(2)++}(z) V^{(2)++}(w) \sim \frac{\frac{-32N(2k+N+2)}{k(k+N+2)^2} (A^{++} A^{++})(w)}{(z-w)^2} - \frac{\frac{32N(2k+N+2)}{k(k+N+2)^2} (A^{++} \partial A^{++})(w)}{(z-w)}.$$

- Another similar truncation at

➤ HS: $\gamma = -s$, **(3,1)** remains

➤ Coset: $\mathcal{N}^- = \left(G_{-\frac{1}{2}}^{++} G_{-\frac{1}{2}}^{--} - \frac{4s}{N} A_{-1}^{--} \right) \Phi_s \cong 0 \quad \text{at} \quad sk + (1+s)N + 2s = 0 \text{ or } N = 1$

- In the 't Hooft limit, these null vectors reduce to the $\text{shs}_2[\mu]$ null vectors.

Thank you