Symmetries of N=4 higher spin holography

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Why Higher Spin Theory

- Higher Spin theories are interesting:
  - Tensionless/High energy limit of string theory
  - AdS/CFT correspondence

- We focus on the higher spin supergravity theory on AdS_3
  - Simple in many aspects
  - Capture many important features of general higher spin theories

Higher spin/Minimal model Holography

$shs_2[\mu]$ Higher spin theory in AdS_3	$\mathcal{N}=4$ ⟷	2d Wolf space coset models

(Gaberdiel & Gopakumar 2012)

Higher spin/Minimal model Holography

shs$_2[\mu]$ Higher spin theory in AdS$_3$	$\xleftrightarrow{N=4}$	2d Wolf space coset models

(Gaberdiel & Gopakumar 2012)

- Defined by the Chern-Simons action with infinitely dimensional super-higher-spin algebra shs$_2[\mu]$: $I_{\rm HS} = I_{\rm CS}(A, k_{\rm CS}) - I_{\rm CS}(\bar{A}, k_{\rm CS})$

$$I_{\rm CS}(A, k_{\rm cs}) = \frac{k_{\rm cs}}{4\pi}\int_{\mathcal{M}} {\rm Tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right), \qquad k_{CS} = \frac{\ell}{8G_3}\frac{1}{(-{\rm Tr}L_0^2)}, \qquad \Lambda = -\frac{2}{\ell^2}$$

- $A = A_\mu^a dx^\mu T^a$ takes value in shs$_2[\mu]$:

$$sB_2[\mu] = {\rm shs}_2[\mu] \oplus \mathbb{C}, \quad sB_2[\mu] \equiv sB[\mu]\otimes {\rm Mat}_2(\mathbb{C}), \quad sB[\mu] = \frac{U(\mathfrak{osp}(1|2))}{\langle C^{\mathfrak{osp}} - \frac{1}{4}\mu(\mu-1)\mathbb{1}\rangle}$$

- N=4 supersymmetry

$$D(2,1|\alpha), \quad \alpha = \frac{\mu}{1-\mu}$$

$${\rm shs}_2[\mu] = D(2,1|\alpha) \oplus \bigoplus_{s=1}^{\infty} R^{(s)},$$

$$L_0\ ,\ L_{\pm 1}\ , \quad G^a_{\pm\frac{1}{2}}\ , \quad A_0^{+i}, A_0^{-i}$$

$$R^{(s)}: \quad \begin{array}{ll} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{3}) \\ s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\ s+2: & (\mathbf{1},\mathbf{1})\ . \end{array}$$

Higher spin/Minimal model Holography

shs$_2$[μ] Higher spin theory in AdS$_3$ ⟷ (N=4) 2d Wolf space coset models

(Gaberdiel & Gopakumar 2012)

- Defined by the coset (I)

$$\frac{\mathfrak{su}(N+2)^{(1)}_\kappa}{\mathfrak{su}(N)^{(1)}_\kappa} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}} \,, \quad \kappa = k + N + 2$$

(Van Proeyen 1989, Sevrin & Theodoridis 1990)

  - N=4 linear A_γ $(\gamma=\mu)$ algebra generated by $L_m, G_r^{\pm\pm}, A_m^{+,i}, A_m^{-i}, Q_r^{\pm\pm}, u_m$

- An alternative coset (II) $$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \,,$$

  - N=4 nonlinear $\tilde{A}_\gamma$ algebra generated by $L_m, G_r^{\pm\pm}, A_m^{+,i}, A_m^{-i}$
  - The two algebras are related by factoring out the free fermions. (Goddard & Schwimmer 1999)

✓ The spectra: checked

✓ The symmetries: Asymptotic symmetry algebra = Chiral algebra ?

Asymptotic symmetry

- Asymptotic symmetry algebra (ASA)
  - With boundary, $G(\gamma)=\frac{k_{CS}}{4\pi}\int_M Tr(F\gamma)+Q(\gamma),\ \ Q(\gamma)=-\frac{k_{CS}}{2\pi}\int_{\partial M} Tr(\mathrm{a}\,\gamma)$
  - The residual gauge symmetries that preserve some boundary conditions.
  - The asymptotic symmetry of gravity on AdS_3 is 2d virasoro algebra with central charge $c_{BH}=\frac{3l}{2G_3}$. (Brown & Henneaux 1986)
- Boundary conditions
  - Vanishing boundary term in the variation: $A_-|_{\partial\mathcal{M}}=0$
  - Asymptotic AdS: $(A-A_{\mathrm{AdS}})|_{\partial\mathcal{M}}=\mathcal{O}(1)$
- *Directly* imposing the above boundary conditions and gauge fixing to $Q(\gamma)$ leads to the asymptotic symmetry that contains $\tilde{A}_\gamma$.

Asymptotic symmetry: non-linear

- Matches the chiral algebra of coset (II) $\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa}$ in the 't Hooft limit

$$N, k \to \infty \quad \textit{while} \quad \frac{N}{N+k} = \lambda \equiv \gamma \quad \textit{fixed}$$

- E. g. $V^{(2)++}_{\rm CFT} \equiv \frac{i V^{(2)++}_{\rm bulk}}{2a_{(2)++}}$

➢ASA: $$V^{(2)++}_{\rm bulk}(z) V^{(2)++}_{\rm bulk}(w) \sim -\frac{128(\gamma-2)\gamma^2 a^2_{(2)++} A^{++}(w)\, A^{++}(w)}{k_{\rm cs}(w-z)^2} + \frac{128(\gamma-2)\gamma^2 a^2_{(2)++} A^{++}(w)\, \partial A^{++}(w)}{k_{\rm cs}(w-z)}$$

➢Coset: $$V^{(2)++}_{\rm CFT}(z)\, V^{(2)++}_{\rm CFT}(w) \sim \frac{\frac{-32N(2k+N+2)}{k(k+N+2)^2}(A^{++}A^{++})(w)}{(z-w)^2} - \frac{\frac{32N(2k+N+2)}{k(k+N+2)^2}(A^{++}\partial A^{++})(w)}{(z-w)}$$

$$
V^{(2)++}(z)V^{(2)+-}(w) \sim -\frac{128k_{cs}(\gamma-2)a_{(2)++}^2}{(z-w)^4}
$$

$$
+\frac{128(\gamma-2)(\gamma-1)A^{--}(w)\,\partial A^{-+}(w)a_{(2)++}^2}{3k_{cs}(w-z)}+\frac{128(\gamma-2)(\gamma-1)A^{-+}(w)\,\partial A^{--}(w)a_{(2)++}^2}{3k_{cs}(w-z)}
$$

$$
+\frac{256(\gamma-2)(\gamma-1)A^{-3}(w)\,\partial A^{-3}(w)a_{(2)++}^2}{3k_{cs}(w-z)}+\frac{128(\gamma-2)\gamma(2\gamma-1)A^{+-}(w)\,\partial A^{++}(w)a_{(2)++}^2}{3k_{cs}(w-z)}
$$

$$
+\frac{128(\gamma-2)(\gamma-1)\gamma A^{++}(w)\,\partial A^{+-}(w)a_{(2)++}^2}{3k_{cs}(w-z)}+\frac{512(\gamma-2)\gamma A^{+3}(w)\,T(w)a_{(2)++}^2}{3k_{cs}(w-z)}
$$

$$
+\frac{256(\gamma-2)\gamma(3\gamma-1)A^{+3}(w)\,\partial A^{+3}(w)a_{(2)++}^2}{3k_{cs}(w-z)}+\frac{64(\gamma-2)J(w)\,J(w)a_{(2)++}^2}{3k_{cs}(w-z)^2}
$$

$$
+\frac{16(\gamma-2)(5\gamma-7)G^{-+}(w)\,G^{+-}(w)a_{(2)++}^2}{9k_{cs}(w-z)}+\frac{64(\gamma-2)^2G'^{--}(w)\,G'^{++}(w)a_{(2)++}^2}{9k_{cs}(w-z)}
$$

$$
+\frac{512(\gamma-2)(\gamma-1)\gamma A^{--}(w)\,A^{-+}(w)\,A^{+3}(w)a_{(2)++}^2}{3k_{cs}^2(w-z)}+\frac{512(\gamma-2)(\gamma-1)\gamma(A^{-3}(w))^2\,A^{+3}(w)a_{(2)++}^2}{3k_{cs}^2(w-z)}
$$

$$
+\frac{512(\gamma-2)(\gamma-1)\gamma^2 A \cdots}{3 \cdots}
$$

$$
+\frac{256(\gamma-2)\gamma A^{+3}(w)a \cdots}{(w-z)^3}
$$

$$
-\frac{64(\gamma-2)^2G^{--}(w)\,G \cdots}{9k_{cs}(w- \cdots}
$$

$$
-\frac{256(\gamma-2)(\gamma-1)A^{-} \cdots}{3k_{cs}(w \cdots}
$$

$$
-\frac{128(\gamma-2)\gamma(3\gamma-2)A \cdots}{3k_{cs}(\cdots}
$$

$$
-\frac{512(\gamma-2)\gamma A^{+3}(w)\,V \cdots}{3k_{cs}(w-z)}
$$

$$
+\frac{8a_{(2)++}^2}{3(w-z)a_{(2)k}}(16a_{(2)k} \cdots
$$

$$
-32a_{(2)k}\partial^2A^{+3}(w)\gamma+ \cdots
$$

$$
V^{(2)++}(z)\ V^{(2)+-}(w) \sim -\frac{32(k-1)N(2k+N+2)}{(k+N+2)^2}\frac{1}{(z-w)^4}-\frac{(64(-1+k)N(2+2k+N)}{k(2+k+N)^2}\frac{A^{+3}}{(z-w)^3}
$$

$$
+\frac{16(-1+k)(1+k)(2+N)(2+2k+N)}{k(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))}\frac{(V^{(1)0}\ V^{(1)0}\ )}{(z-w)^2}
$$

$$
-\frac{32(-1+k)N(2+2k+N)}{k(2+k+N)^2}\frac{\partial A^{+3}}{(z-w)^2}-\frac{32N(2+2k+N)}{k(2+k+N)^2}\frac{(A^{+3}\ A^{+3})}{(z-w)^2}
$$

$$
+\frac{64(-1+k)(1+k)N(2+2k+N)}{(2+k+N)(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))}\frac{C^-}{(z-w)^2}
$$

$$
-\frac{32N(2+2k+N)(-2k^3(2+N)+(2+N)^2+k(4-3N-4N^2)+k^2(-1+N^2)}{k(2+k+N)^2(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))}\frac{C^+}{(z-w)^2}
$$

$$
-\frac{64(-1+k)(1+k)N(2+N)(2+2k+N)}{(2+k+N)(-(2+N)^2+k^2(-1+4N+3N^2)+k(-4+3N+4N^2))}\frac{T-V^{(2)0}}{(z-w)^2}
$$

$$
+\mathcal{O}\big((z-w)^{-1}\big)\ ,
$$

Asymptotic symmetry: linear

- The higher spin theory do **not** capture the free fermions: $A = A_\mu^a dx^\mu T^a$.
  Then what is the relation with coset (I) ?

- 2 steps:
  - Introduce boundary auxiliary degrees of freedom

$$Q(\gamma) = -\frac{k_{\rm cs}}{2\pi} \int_{\partial\mathcal{M}} {\rm Tr}(\gamma a) \quad \Longrightarrow \quad Q'(\gamma) = -\frac{k_{\rm cs}}{2\pi} \int {\rm Tr}(\gamma\, a') \, ,$$

$$a = L_1 + \sum_{s,i} a'^{(s)\,i} V_{1-s}^{(s)\,i} \, , \qquad a' = L_1 + \sum_{s,i} a'^{(s)\,i} V_{1-s}^{(s)\,i} \, , \quad a'^{(s)\,i} = a^{(s)\,i} + b^{(s)\,i} \, ,$$

  - Determine their interaction with the boundary fields "a": Solve[linearity==True]

$\Longrightarrow$ asymptotic symmetry algebra contains the linear A_γ algebra.

- Matches with coset (I).

Perturbative analysis and ~~HS~~

- BPS state $(\mathrm{f};\bar{\mathrm{f}})$, satisfies the BPS bound

$$h \geq \frac{1}{k^+ + k^-}\left[k^+ l^- + k^- l^+ + u^2 + (l^+ - l^-)^2\right] \quad \text{with} \quad h(f;\bar{f}) = 1/2,\ l^+ = 1/2 = l^-,\ u = 0$$

  - ➢ scalar descendent of $(\mathrm{f};\bar{\mathrm{f}})$, Φ, is a marginal operator.

- Under perturbation, symmetry generator W is preserved if

$$\mathcal{N}_s \equiv \sum_{l=0}^{s-1} \frac{(-1)^l}{l!}(L_{-1})^l\, W^s_{-s+1+l}\,\Phi = 0$$

(Gaberdiel, Jin & Li 2013)

- In our case

$$V_0^{(1)0}\,\Phi = -\frac{2(N+1)}{(k+N+2)}\,\Phi \neq 0$$

where $V^{(1)0}$ is the bottom component of the $R^{(1)}$ multiplet.

Truncation of the algebras

- $\mathrm{shs}_2[\mu]$ can be truncated at special value of the parameter μ:

$$\mu = \gamma = s + 1 \qquad \mathrm{shs}_2[s+1] = D(2,1|-\tfrac{s+1}{s}) \oplus \bigoplus_{i=1}^{s-1} R^{(i)} \oplus \hat{R}^{(s)}_{-}$$

$$R^{(s)} : \quad \begin{array}{ll} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \\ s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\ s+2: & (\mathbf{1},\mathbf{1}) \,. \end{array} \quad \Longrightarrow \quad \hat{R}^{(s)}_{-} : \quad \begin{array}{ll} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{1},\mathbf{3}) \end{array}$$

- Truncation due to null vector $\mathcal{N} = G^{+-}_{-\frac{1}{2}} G^{++}_{-\frac{1}{2}} \Phi_s$ at γ = s+1, where

$$L_0 \Phi_s = s\,\Phi_s \,, \qquad L_1 \Phi_s = 0 \,, \qquad G^{\alpha\beta}_{\frac{1}{2}} \Phi_s = 0$$

- This translates to the asymptotic symmetry algebra

$$V^{(2)++}(z) V^{(2)++}(w) \sim \frac{128(\gamma-2)\gamma^2 a^2_{(2)++} A^{++}(w)\,\partial A^{++}(w)}{k_{\mathrm{cs}}(w-z)} - \frac{128(\gamma-2)\gamma^2 a^2_{(2)++} A^{++}(w)\, A^{++}(w)}{k_{\mathrm{cs}}(w-z)^2}$$

Truncation of the algebras

- On the coset side, there exists null vectors

$$\mathcal{N}^{+} = \left(G^{+-}_{-\frac{1}{2}} G^{++}_{-\frac{1}{2}} + \frac{4s}{k} A^{++}_{-1}\right) \Phi_s \cong 0 \qquad \text{at} \qquad sN + (1+s)k + 2s = 0$$

- Chiral algebra truncates, e.g at s=1 (corresponding to γ=1+s=2)

$$V^{(2)++}(z)\, V^{(2)++}(w) \sim \frac{\frac{-32N(2k+N+2)}{k(k+N+2)^2}(A^{++}A^{++})(w)}{(z-w)^2} - \frac{\frac{32N(2k+N+2)}{k(k+N+2)^2}(A^{++}\partial A^{++})(w)}{(z-w)} \,.$$

- Another similar truncation at
  - HS: γ = - s, **(3,1)** remains
  - Coset: $\mathcal{N}^{-} = \left(G^{++}_{-\frac{1}{2}} G^{-+}_{-\frac{1}{2}} - \frac{4s}{N} A^{-+}_{-1}\right) \Phi_s \cong 0$ at $sk + (1+s)N + 2s = 0$ or $N = 1$

- In the 't Hooft limit, this null vectors reduce to the shs$_2$[μ] null vectors.

Thank you