Deformed supersymmetric gauge theories from String and M-Theory

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based on work with with D. Orlando, S. Hellerman, N. Lambert
arXiv:1106.2097, 1108.0644, 1111.4811, 1204.4192, 1210.7805, 1304.3488, 1309.7350, work in progress

Introduction

In recent years, N=2 supersymmetric gauge theories and their deformations have played an important role in theoretical physics - very active research topic.
Examples:

2d gauge/Bethe correspondence (Nekrasov/Shatashvili): relates 2d gauge theories with twisted masses to integrable spin chains.

4d gauge/Bethe correspondence (Nekrasov/Shatashvili): relates Omega-deformed 4d gauge theories to quantum integrable systems.

AGT correspondence (Alday, Gaiotto, Tachikawa): relates Omega-deformed super-Yang-Mills theory to Liouville theory.

Introduction

All these examples have two things in common:

1. A deformed supersymmetric gauge theory is linked to an integrable system.

    Relation between two very constrained and well-behaved systems that can be studied separately with different methods.

    Transfer insights from one side to the other, cross-fertilization between subjects!

2. The deformed gauge theories in question can be realized in string theory via the fluxtrap background!

    The string theory construction provides a unifying framework and a different point of view on the gauge theory problems.

Introduction

Aim: Realize deformed supersymmetric gauge theories via string theory. Gauge theories encode fluctuations on the world-volume of D-branes. Many parameters can be tuned by varying brane geometry.

Here: Deform the string theory background (“fluxtrap”) into which the branes are placed (Hellerman, Orlando, S.R.)

⇒ different brane set-ups give rise to different gauge theories with seemingly unrelated deformations!

Use the fluxtrap construction to unify and meaningfully relate and reinterpret a large variety of existing results.

Introduction

Our string theoretic approach enables us moreover to generate new deformed gauge theories in a simple and algorithmic way.

Today: short overview over the many applications of the fluxtrap background and some concrete examples.

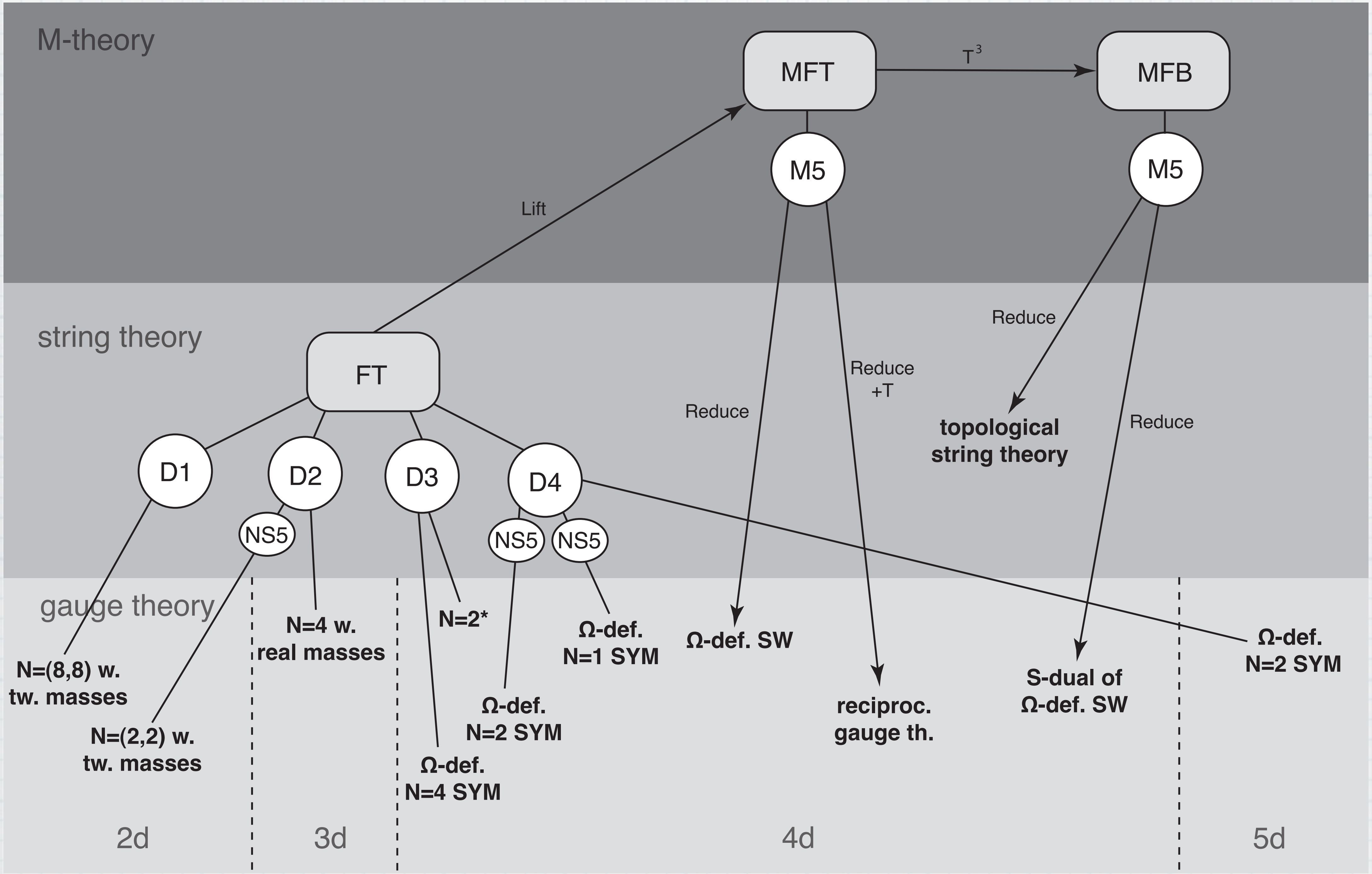
- N=2* theory in 4d

- Omega-deformed SW action from M-theory

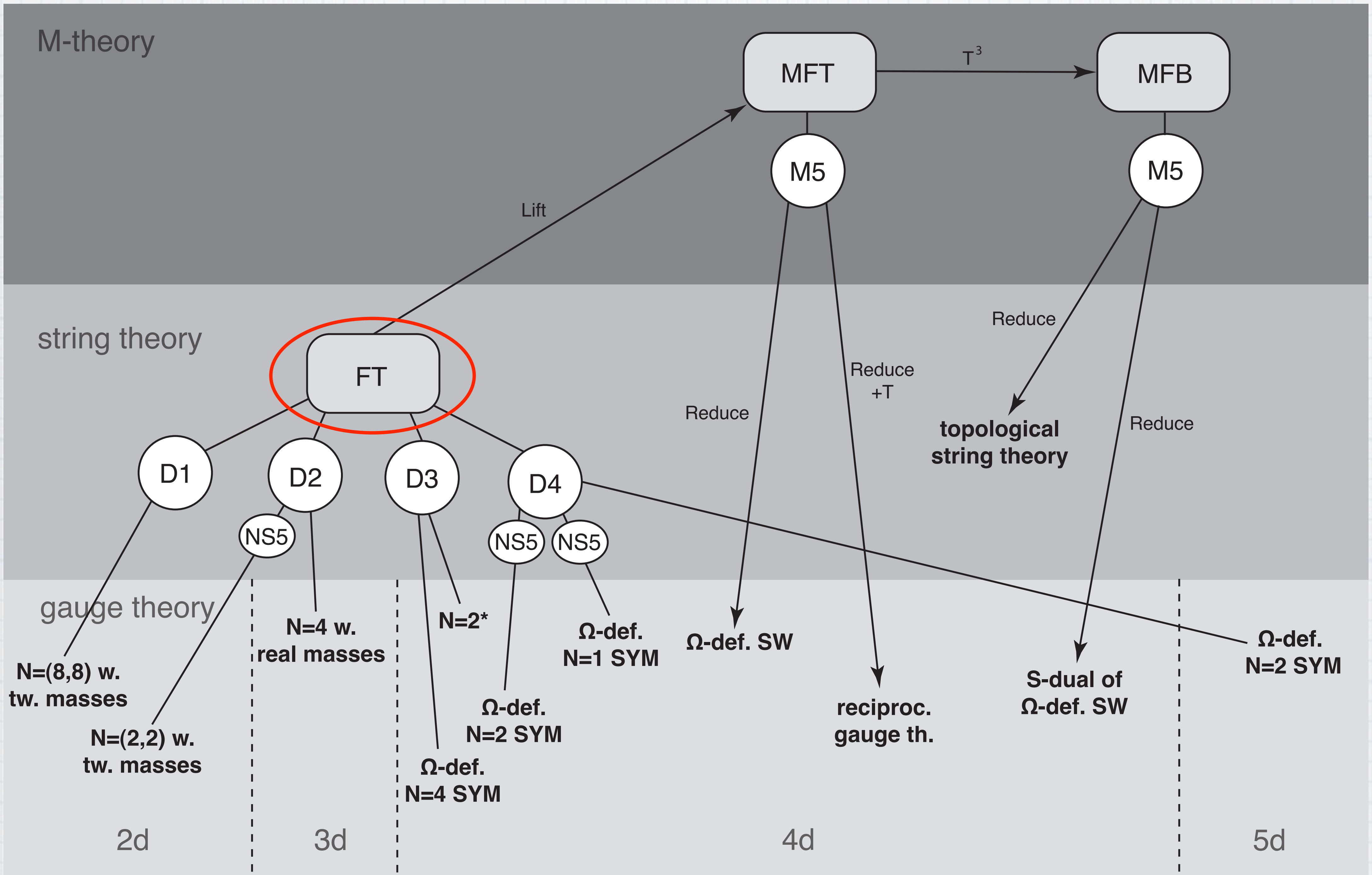
- Alpha-, Omega-deformation and a whole SL(2,Z) worth of deformed theories

Fluxtrap background as toolbox to generate deformed gauge theories and analyze them via string theoretic methods.

Introduction

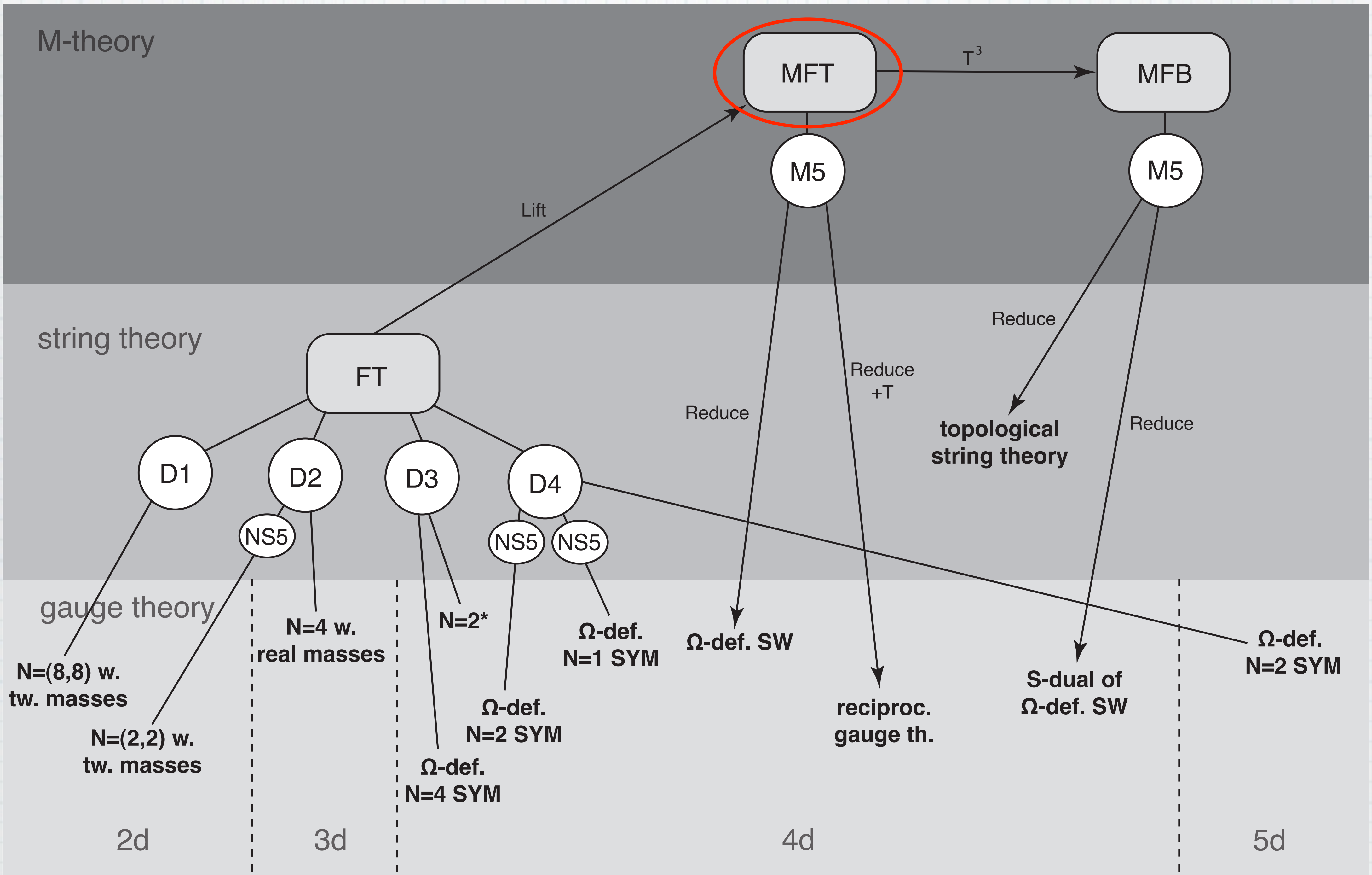


Introduction



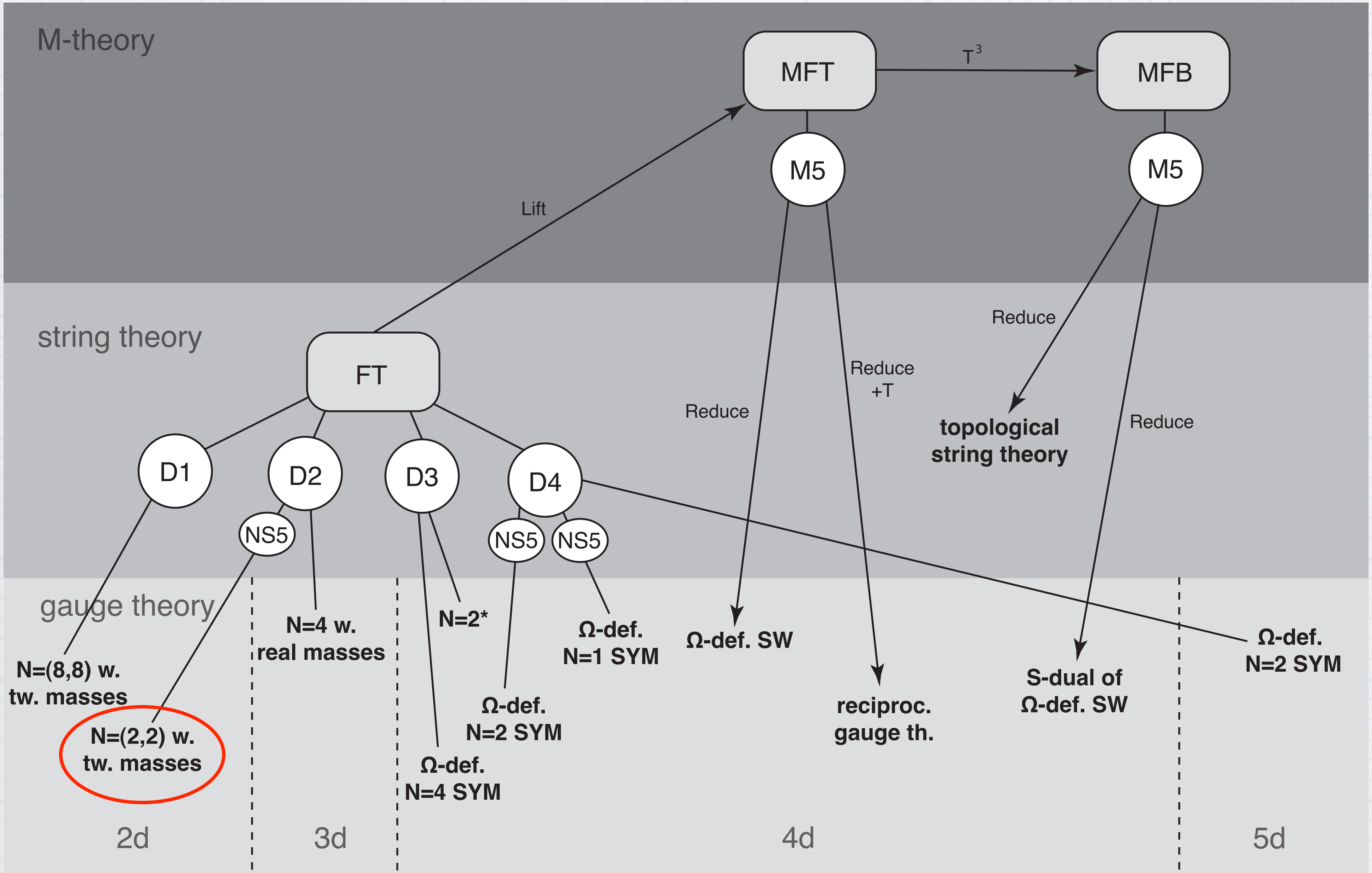
Construct the fluxtrap background in string theory.

Introduction



Can be lifted to M-theory: M-theory Fluxtrap

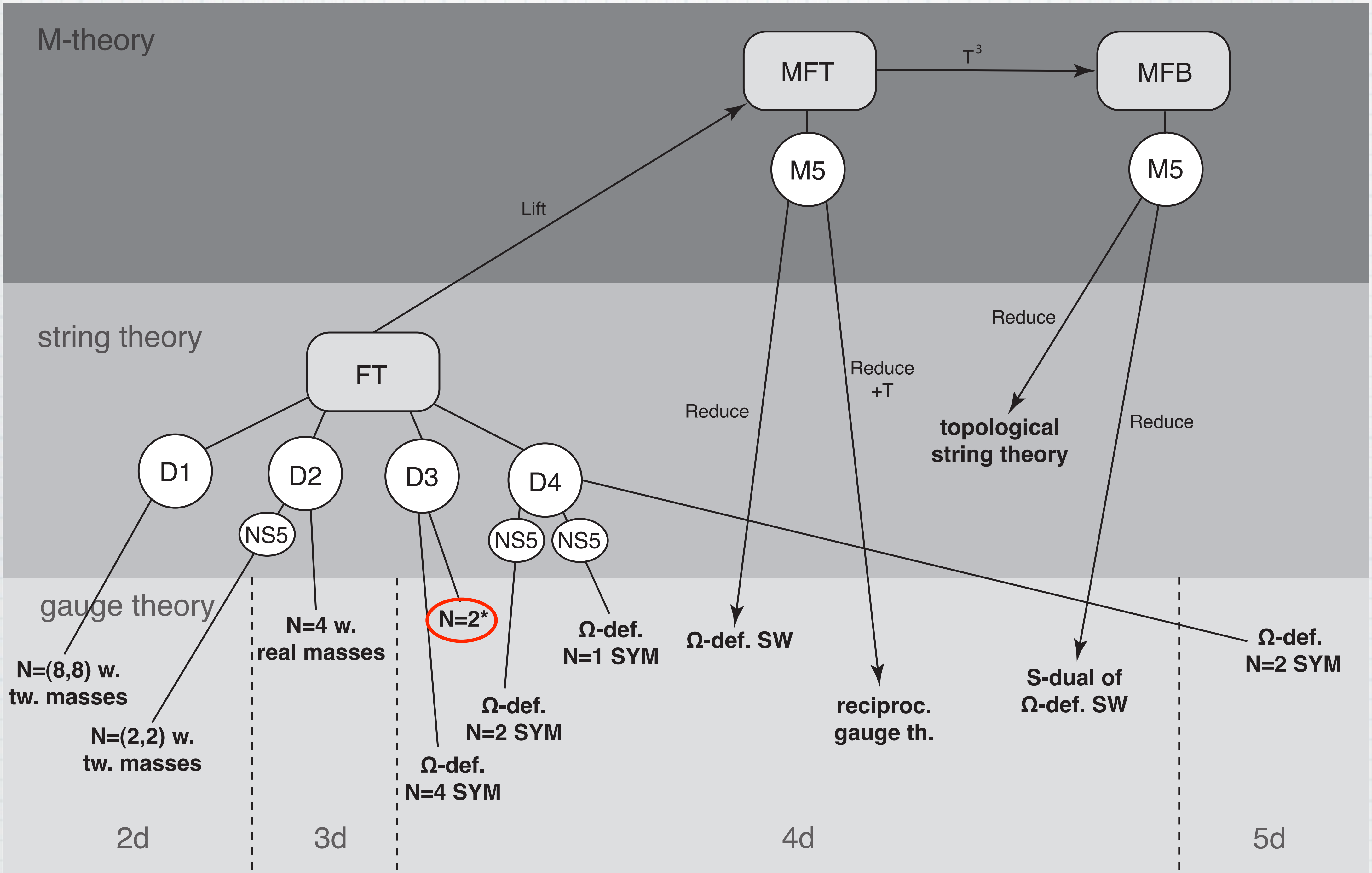
Introduction



It captures the gauge theories with twisted masses of the 2d gauge/Bethe correspondence.

arXiv:1106.0279

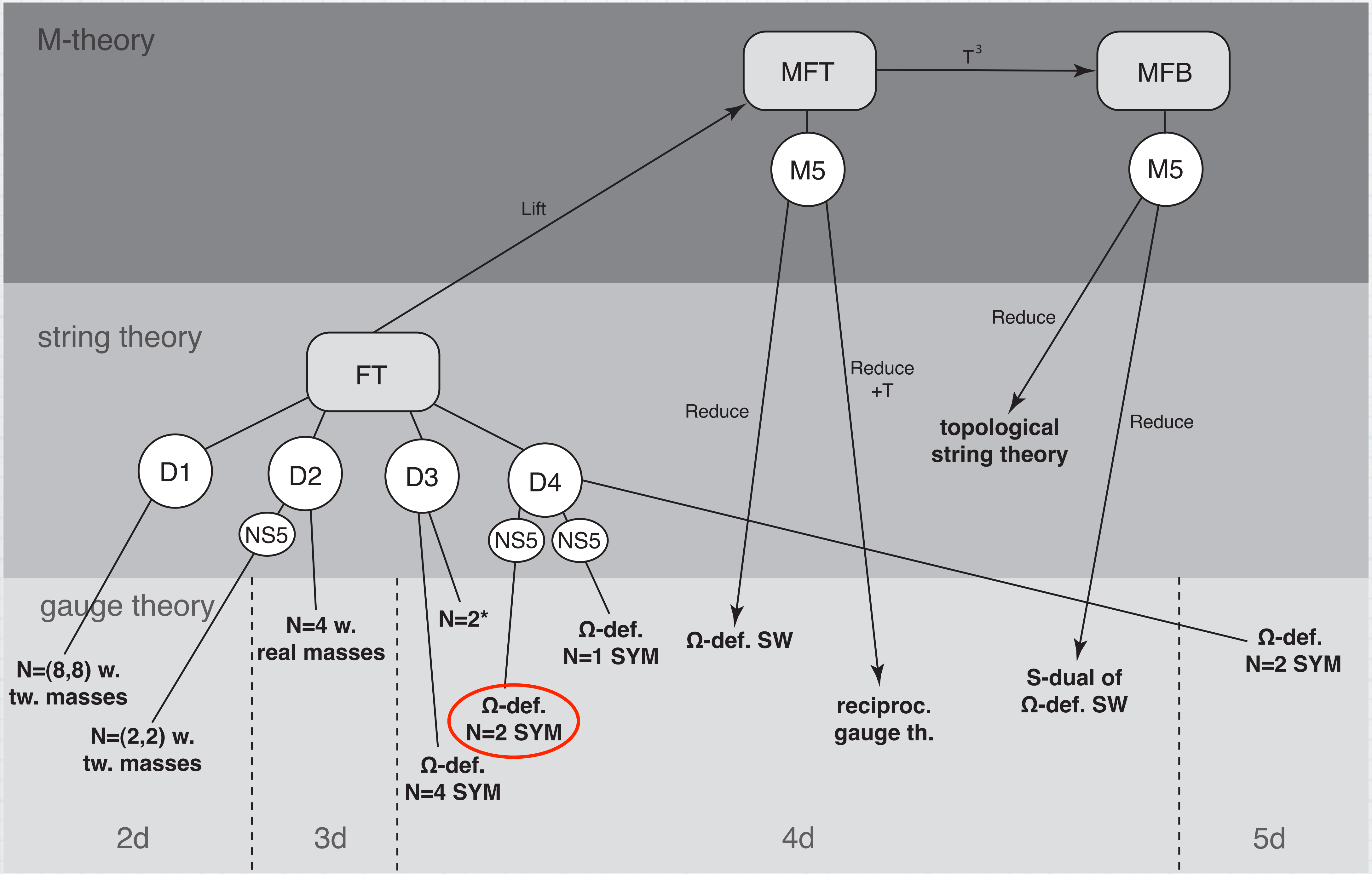
Introduction



We can construct the N=2* theory.

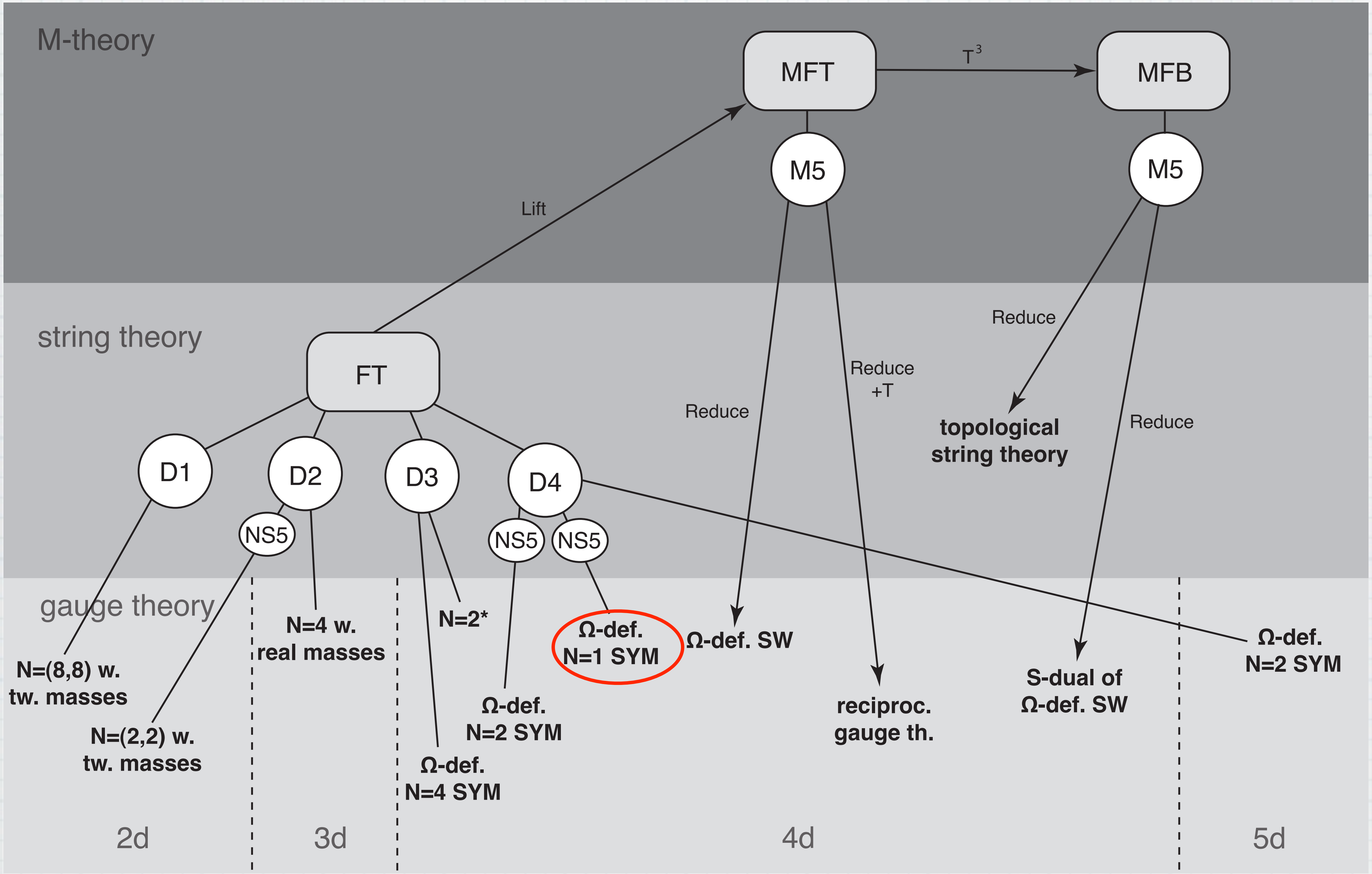
Construct gravity duals of deformed N=4 SYM

Introduction



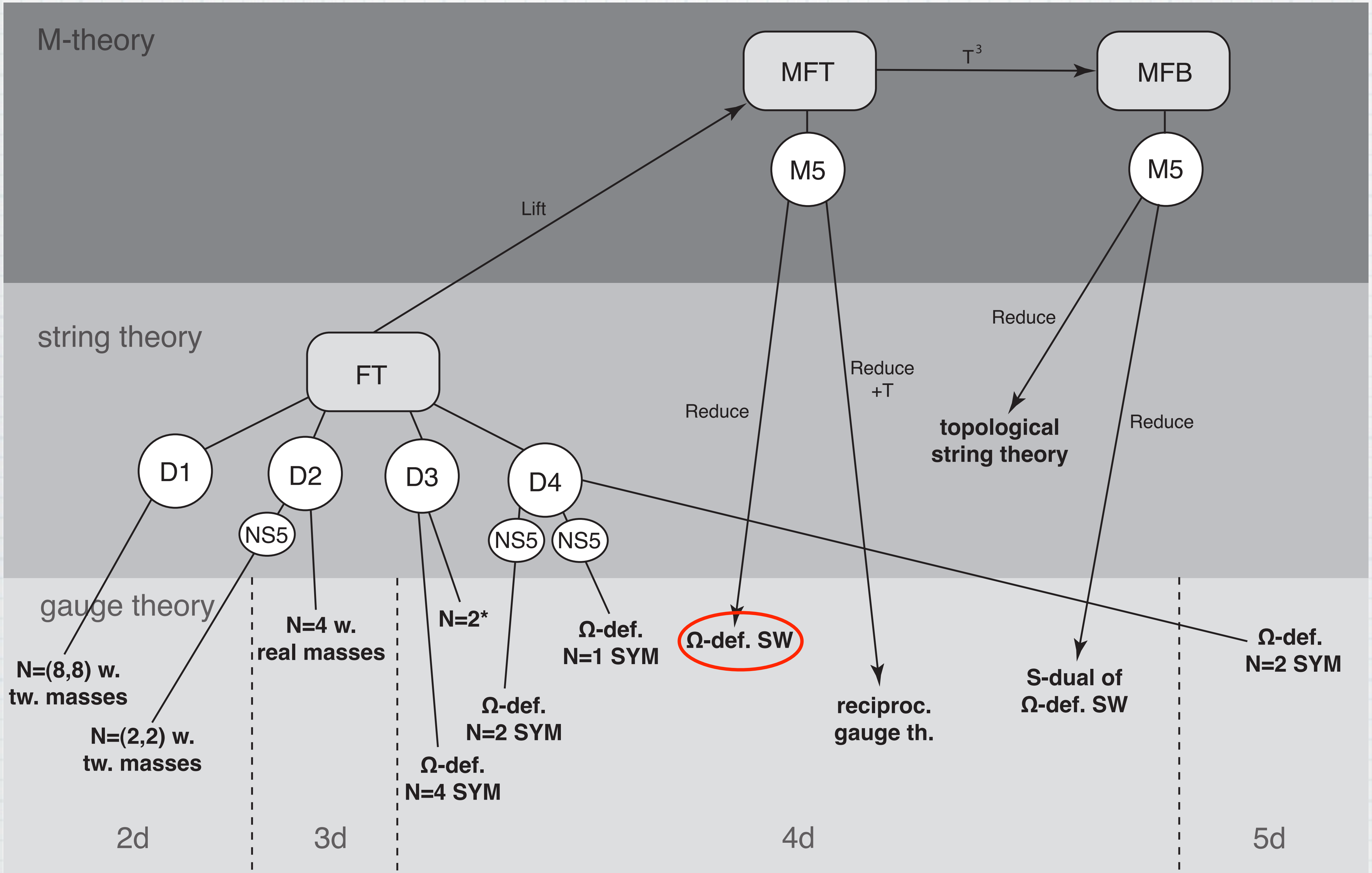
It captures the Omega-deformed gauge theories of the 4d gauge/Bethe correspondence. arXiv:1204.4192

Introduction



Can also construct Omega-deformed N=1 gauge theory.

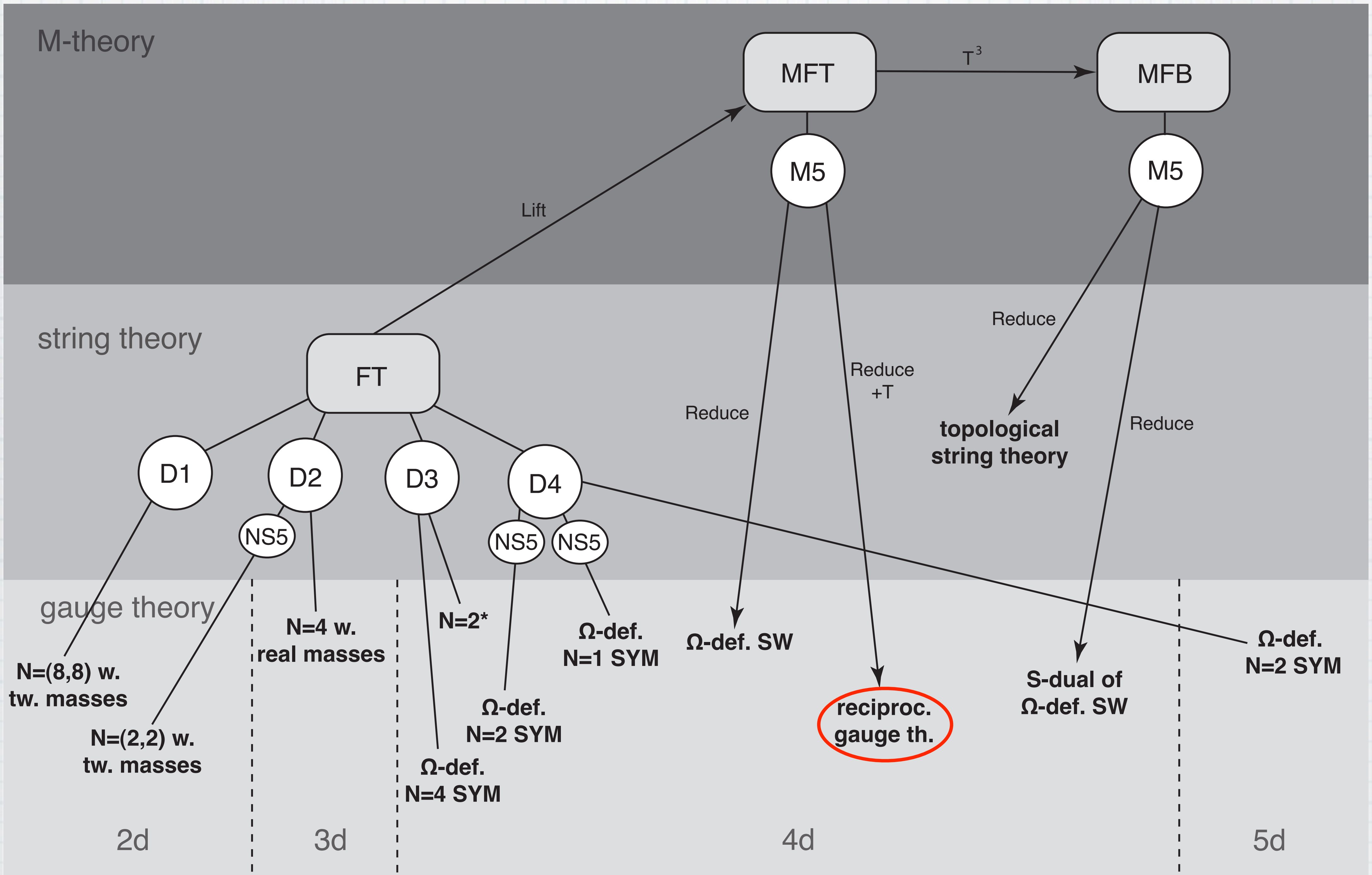
Introduction



Derive Omega-deformed Seiberg-Witten Lagrangian

arXiv:1304.3488

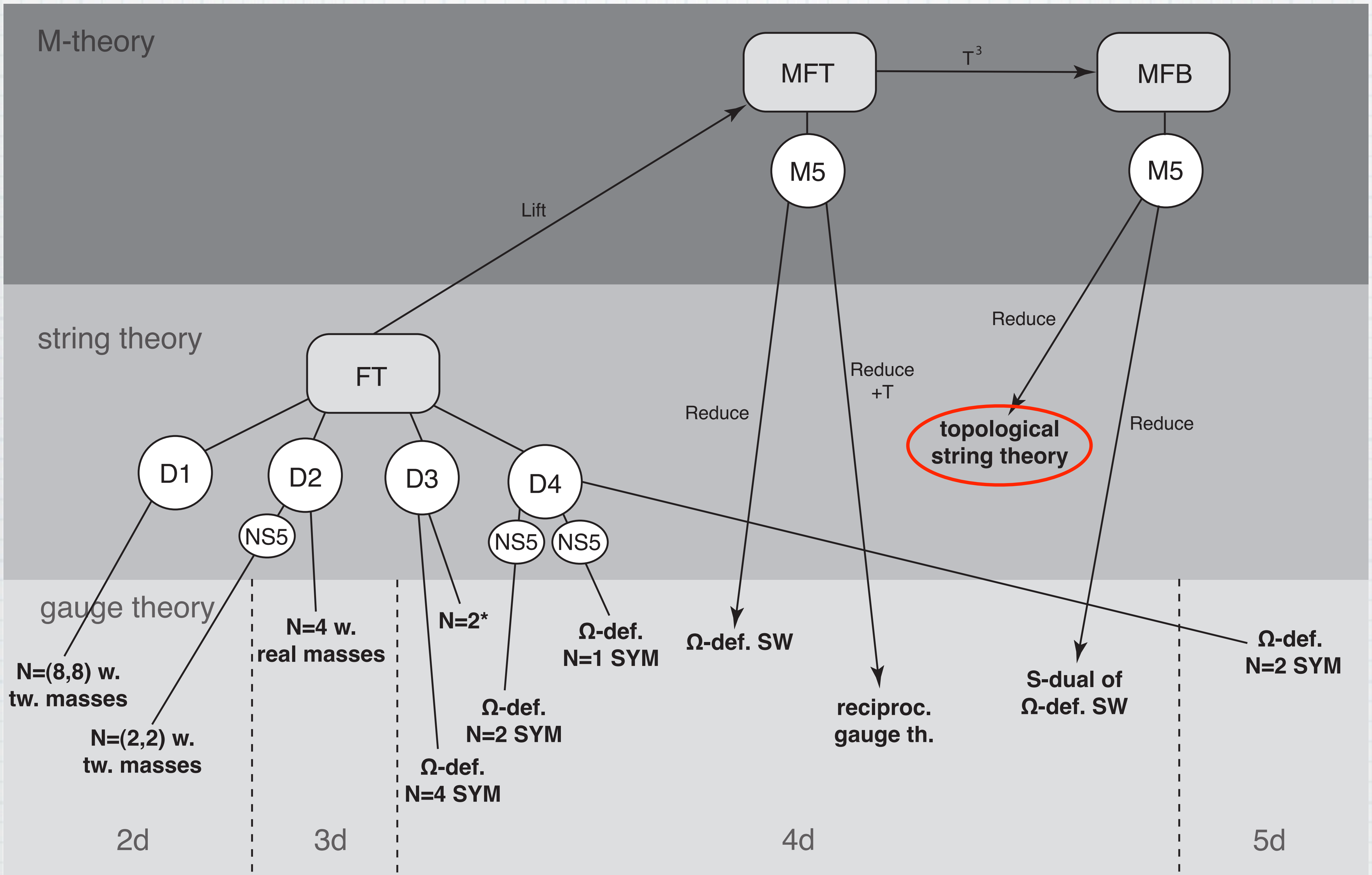
Introduction



Starting point for understanding string theory formulation of AGT correspondence.

arXiv:1210.7805

Introduction



Connection to topological string theory.

The Fluxtrap Background

The Fluxtrap Background

Geometrical realization of Nekrasov's construction of the equivariant gauge theory.

Start with metric with 2 periodic directions and at least a U(1)xU(1) symmetry, no B-field, constant dilaton.

Background with 4 complex independent deformation parameters:

									T^2	
x	0	1	2	3	4	5	6	7	8	9
	(ρ_1, θ_1)		(ρ_2, θ_2)		(ρ_3, θ_3)		(ρ_4, θ_4)		v	
fluxbrane	ϵ_1		ϵ_2		ϵ_3		ϵ_4		$\circ$	$\circ$

$$\widetilde{x}^8 \simeq \widetilde{x}^8 + 2\pi \widetilde{R}_8$$
$$\widetilde{x}^9 \simeq \widetilde{x}^9 + 2\pi \widetilde{R}_9$$

Impose identifications: fluxbrane parameters

$$\begin{cases} \widetilde{x}^8 \simeq \widetilde{x}^8 + 2\pi \widetilde{R}_8 n_8 \\ \theta_k \simeq \theta_k + 2\pi \epsilon_k^R \widetilde{R}_8 n_8 \end{cases} \qquad \begin{cases} \widetilde{x}^9 \simeq \widetilde{x}^9 + 2\pi \widetilde{R}_9 n_9 \\ \theta_k \simeq \theta_k + 2\pi \epsilon_k^I \widetilde{R}_9 n_9 \end{cases}$$

This corresponds to the well-known Melvin or fluxbrane background.

The Fluxtrap Background

Introduce new angular variables with disentangled periodicities: $\phi_k = \theta_k - \epsilon_k^R \widetilde{x}^8 - \epsilon_k^I \widetilde{x}^9 = \theta_k - \mathrm{Re}(\epsilon_k \bar{\widetilde{v}})$

$$\epsilon_k = \epsilon_k^R + \mathrm{i}\,\epsilon_k^I \qquad \widetilde{v} = \widetilde{x}^8 + \mathrm{i}\,\widetilde{x}^9$$

Fluxbrane metric (T^2-fibration over Ω-deformed $\mathbb{R}^8$):

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{d}\vec{x}^2_{0\ldots7} &- \frac{V_i^R V_j^R \,\mathrm{d}x^i\,\mathrm{d}x^j}{1 + V^R\cdot V^R} - \frac{V_i^R V_j^R \,\mathrm{d}x^i\,\mathrm{d}x^j}{1 + V^R\cdot V^R} \\ &+ \left(1 + V^R\cdot V^R\right)\left[\mathrm{d}x^8 - \frac{V_i^R\,\mathrm{d}x^i}{1 + V^R\cdot V^R}\right]^2 \\ &+ \left(1 + V^I\cdot V^I\right)\left[\mathrm{d}x^9 - \frac{V_i^I\,\mathrm{d}x^i}{1 + V^I\cdot V^I}\right]^2 + 2V^R\cdot V^I\,\mathrm{d}x^8\,\mathrm{d}x^9 \end{aligned}$$

Generator of rotations:

$$\begin{aligned} V = V^R + \mathrm{i}\,V^I = {} & \epsilon_1\left(x^1\,\partial_0 - x^0\,\partial_1\right) + \epsilon_2\left(x^3\,\partial_2 - x^2\,\partial_3\right) \\ & + \epsilon_3\left(x^5\,\partial_4 - x^4\,\partial_5\right) + \epsilon_4\left(x^7\,\partial_6 - x^6\,\partial_7\right) \end{aligned}$$

The general case breaks all supersymmetries.

Impose condition $\displaystyle\sum_{k=1}^{N} \pm\epsilon_k = 0$

The Fluxtrap Background

T-dualize along torus directions and take decompactification limit to discard torus momenta:

Fluxtrap background

Before T-duality, locally, the metric was still flat, but some of the rotation symmetries were broken globally.

Bulk fields after T-duality (case $V^R \cdot V^I = 0, \ \epsilon_1 \in \mathbb{R}, \ \epsilon_2 \in \mathrm{i}\,\mathbb{R}, \ \epsilon_3 = \epsilon_4 = 0$):

not anymore flat

$$\mathrm{d}s^2 = \mathrm{d}\rho_1^2 + \frac{\rho_1^2\,\mathrm{d}\phi_1^2 + \mathrm{d}x_8^2}{1+\epsilon_1^2\rho_1^2} + \mathrm{d}\rho_2^2 + \frac{\rho_2^2\,\mathrm{d}\phi_2^2 + \mathrm{d}x_9^2}{1+\epsilon_2^2\rho_2^2} + \sum_{k=4}^{7}(\mathrm{d}x^k)^2,$$

B-field has appeared

$$B = \epsilon_1 \frac{\rho_1^2}{1+\epsilon_1^2\rho_1^2}\,\mathrm{d}\phi_1 \wedge \mathrm{d}x_8 + \epsilon_2 \frac{\rho_2^2}{1+\epsilon_2^2\rho_2^2}\,\mathrm{d}\phi_2 \wedge \mathrm{d}x_9\,,$$

$$\mathrm{e}^{-\Phi} = \frac{\sqrt{\alpha'}\,\mathrm{e}^{-\Phi_0}}{R}\sqrt{(1+\epsilon_1^2\rho_1^2)\,(1+\epsilon_2^2\rho_2^2)}$$

creates a potential that localizes instantons

The Fluxtrap Background

Study resulting geometry.

Space splits into

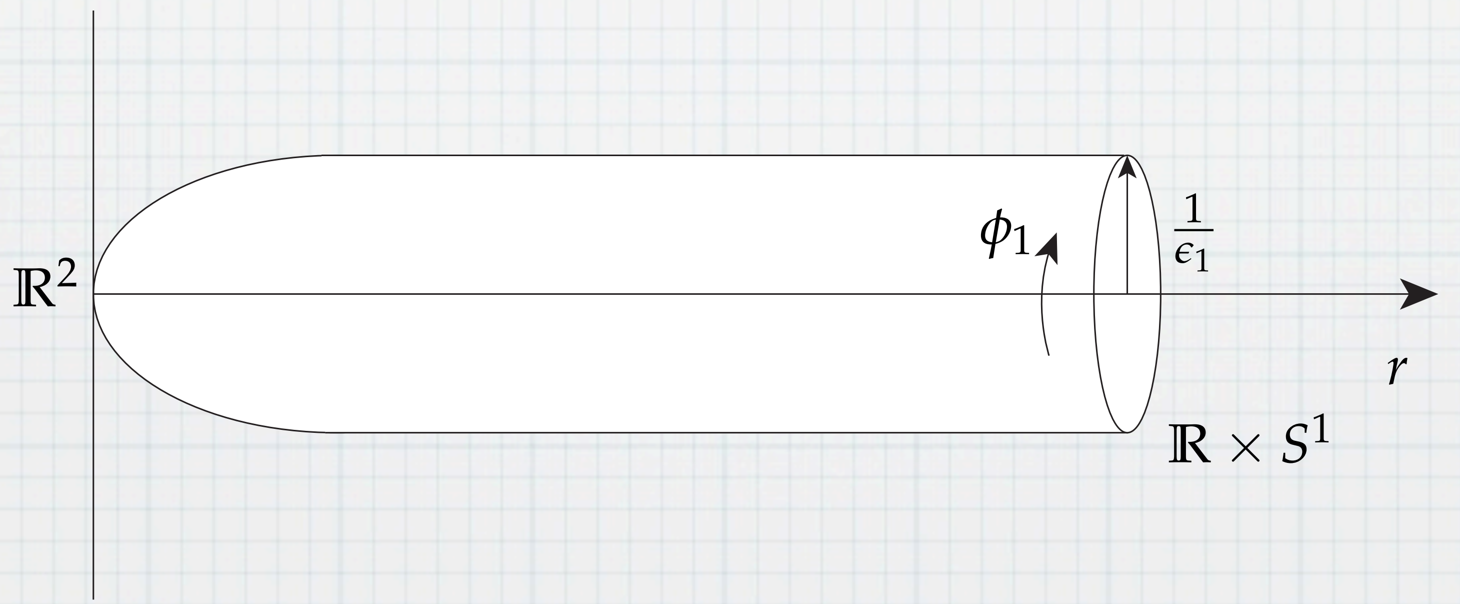
$$M_{10} = M_3(\epsilon_1) \times M_3(\epsilon_2) \times \mathbb{R}^4$$

$$\mathbb{R}\langle x_8 \rangle \longrightarrow M_3(\epsilon_1)$$

$$\downarrow$$

$$\text{cigar } \langle \rho_1, \phi_1 \rangle$$

R-foliation over the cigar



The generator of rotations is bounded (by asymptotic radius).

The Fluxtrap Background

Now we want to lift to M-theory:

$$\mathrm{d}s^2 = (\Delta_1\Delta_2)^{2/3}\left[\mathrm{d}\rho_1^2 + \frac{\epsilon_1^2\rho_1^2}{\Delta_1^2}\,\mathrm{d}\sigma_1^2 + \frac{\mathrm{d}x_8^2}{\Delta_1^2} + \mathrm{d}\rho_2^2 + \frac{\epsilon_2^2\rho_2^2}{\Delta_2^2}\,\mathrm{d}\sigma_2^2 + \frac{\mathrm{d}x_9^2}{\Delta_2^2} + \mathrm{d}x_4^2 + \mathrm{d}x_5^2 + \mathrm{d}x_6^2 + \mathrm{d}x_7^2 + \frac{\mathrm{d}x_{10}^2}{\Delta_1^2\Delta_2^2}\right],$$

$$A_3 = \frac{\epsilon_1^2\rho_1^2}{\Delta_1^2}\,\mathrm{d}\sigma_1\wedge\mathrm{d}x_8\wedge\mathrm{d}x_{10} + \frac{\epsilon_2^2\rho_2^2}{\Delta_2^2}\,\mathrm{d}\sigma_2\wedge\mathrm{d}x_9\wedge\mathrm{d}x_{10},$$

$$\sigma_i = \frac{\phi_i}{\epsilon_i}, \quad \Delta_i^2 = 1 + \epsilon_i^2\rho_i^2, \quad x_{10} = x_{10} + 2\pi R_{10}$$

Consider only linear order in ϵ :

$$g_{MN} = \delta_{MN} + \mathcal{O}(\epsilon^2)\,,$$

$$G_4 = (\mathrm{d}z + \mathrm{d}\bar{z})\wedge(\mathrm{d}s + \mathrm{d}\bar{s})\wedge\omega$$

$$z = x^8 + \mathrm{i}\,x^9 \qquad\qquad s = x^6 + \mathrm{i}\,x^{10}$$

$$\omega = \epsilon_1\,\mathrm{d}x^0\wedge\mathrm{d}x^1 + \epsilon_2\,\mathrm{d}x^2\wedge\mathrm{d}x^3 + \epsilon_3\,\mathrm{d}x^4\wedge\mathrm{d}x^5$$

$$\omega = \mathrm{d}U$$

Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap with respect to the monodromies:

Deformation not on brane world-volume: mass deformation

fluxtrap				ϵ_i	ϵ_j
D–brane	$\times$	$\times$	$\times$	ϕ_i	

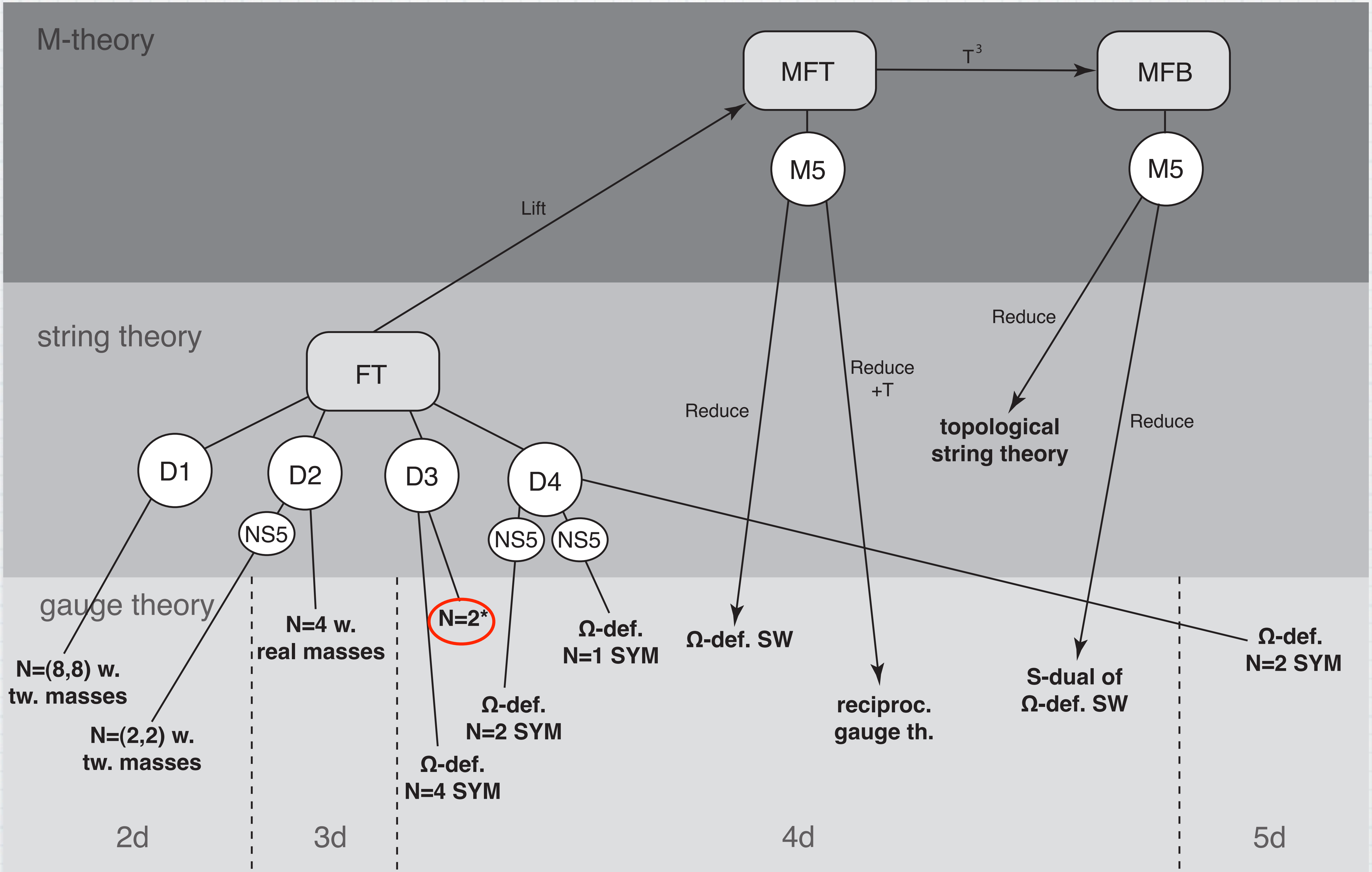
Deformation on brane world-volume: Ω-type deformation, Lorentz invariance broken

fluxtrap		ϵ_i		ϵ_j
D–brane	$\times$	$\times$	$\times$	$\times$

These two cases can be combined.

Examples: N=2* theory

N=2* theory


M-theory
MFT
T³
MFB
M5
M5
Lift
Reduce
string theory
FT
Reduce +T
Reduce
topological string theory
Reduce
D1
D2
D3
D4
NS5
NS5
NS5
gauge theory
N=4 w. real masses
N=2*
Ω-def. N=1 SYM
Ω-def. SW
Ω-def. N=2 SYM
N=(8,8) w. tw. masses
S-dual of Ω-def. SW
Ω-def. N=2 SYM
reciproc. gauge th.
N=(2,2) w. tw. masses
Ω-def. N=4 SYM
2d
3d
4d
5d

N=2* theory

N=2* theory is obtained from N=4 SYM (4d) by giving equal masses to two of the scalar fields.

It is obtained from a D3-brane in the fluxtrap background

x	0	1	2	3	4	5	6	7	8	9
fluxtrap	ϵ_1		ϵ_2		ϵ_3		ϵ_4		$\circ$	$\circ$
D3–brane	$\times$	$\times$	$\times$	$\times$	ϕ_1		ϕ_2		ϕ_3	

Deformation parameters (8 conserved supercharges)

$$\epsilon_1 = \epsilon_2 = 0 \qquad \epsilon_3 = \epsilon_4 = \epsilon$$

Expand DBI action on D3 with up to two derivatives:

$$\mathscr{L}_\Omega = \frac{1}{4g_{\text{YM}}^2}\left[F_{ij}F^{ij} + \frac{1}{2}\sum_{k=1}^{3}\left(\partial^i\phi_k\right)\left(\partial_i\bar{\phi}_k\right) + \frac{1}{2}\left|\epsilon\right|^2\phi_1\bar{\phi}_1 + \frac{1}{2}\left|\epsilon\right|^2\phi_2\bar{\phi}_2\right]$$

Flows to N=2 in the IR (masses become infinite).

Different from Witten's construction (global BC).

AN SL(2,Z) of solutions

Alpha and Omega

Let us revisit the Omega-deformation.

Bulk fields (IIA):

$$ds_{10}^2 = \left[\left(\eta_{\mu\nu} - \frac{U_\mu U_\nu}{\Delta^2}\right) \mathrm{d}x^\mu \, \mathrm{d}x^\nu + (\mathrm{d}x^4)^2 + (\mathrm{d}x^5)^2 + (\mathrm{d}x^6)^2 + (\mathrm{d}x^7)^2 + (\mathrm{d}x^8)^2 + \frac{(\mathrm{d}x^9)^2}{\Delta^2}\right],$$

$$e^\phi = \Delta^{-1},$$

$$B = -\frac{1}{\Delta^2} \mathrm{d}x^9 \wedge U \,,$$

$$\Delta^2 = 1 + U_i U^{=} \epsilon_1^2(x_0^2 + x_1^2) + \epsilon_2^2(x_2^2 + x_3^2) + \epsilon_3^2(x_4^2 + x_5^2)$$

The effective action of a single D4-brane in (0,1,2,3,6) suspended between two parallel NS5s is

$$S_{\mathrm{D}4}^{\Omega} = -\frac{1}{g^2} \int \mathrm{d}^4 x \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu X^8 \partial^\mu X^8 + \frac{1}{2} (\partial_\mu X^9 + F_{\mu\lambda} U^\lambda)(\partial^\mu X^9 + F^{\mu\rho} U_\rho) \right.$$
$$\left. + \frac{1}{2} (U^\lambda \partial_\lambda X^8)^2 \right]$$

Alpha and Omega

We have seen that the Omega-deformation has only the B-field in the bulk.

Perform 9-11 flip: Alpha-deformation has only an RR-background field

$$ds^2_{10} = \Delta\left[\left(\eta_{\mu\nu} - \frac{U_\mu U_\nu}{\Delta^2}\right)\mathrm{d}x^\mu\,\mathrm{d}x^\nu + (\mathrm{d}x^4)^2 + (\mathrm{d}x^5)^2 + (\mathrm{d}x^7)^2 + \frac{(\mathrm{d}x^6)^2 + (\mathrm{d}x^8)^2}{\Delta^2} + (\mathrm{d}x^9)^2\right],$$

$$e^\phi = \Delta^{1/2},$$

$$C^{RR} = \frac{1}{\Delta^2}\,\mathrm{d}x^6 \wedge \mathrm{d}x^8 \wedge U\,.$$

Study D4-brane suspended between two parallel NS5s.

The effective gauge theory action is given by

$$S^A_{\mathrm{D}4} = -\frac{1}{g^2}\int \mathrm{d}^4x\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\Delta^2}\left(\partial_\mu X^8 + iU^\lambda \star F_{\mu\lambda}\right)\left(\partial^\mu X^8 + iU_\rho \star F^{\mu\rho}\right)\right.$$
$$\left. + \frac{1}{2}\partial_\mu X^9 \partial^\mu X^9 + \frac{1}{2\Delta^2}(U^\mu \partial_\mu X^8)^2 + \frac{1}{2}(U^\mu \partial_\mu X^9)^2\right]\,.$$

An SL(2,Z) of solutions

Starting from M-theory lift as before with M5-branes:

$M5:$	0	1	2	3		6				10
$M5:$	0	1	2	3				8	9	

Reduce to 4d: eff. theory on D4 extended between to parallel NS5s.

Reduce instead on new periodic direction y2:

$$\begin{pmatrix} y^1 \\ y^2 \end{pmatrix} = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} x^6/R_6 \\ x^{10}/R_{10} \end{pmatrix} = \Lambda \begin{pmatrix} x^6/R_6 \\ x^{10}/R_{10} \end{pmatrix} , \qquad ad - bc = 1$$

Resulting background contains both B- and C-fields.

$$g = \frac{\sqrt{d^2 R_{10}^2 + c^2 R_6^2 \Delta^2}}{R_2} \left[\left(\delta_{mn} - \frac{U_m U_n}{\Delta^2} \right) \mathrm{d}x^m \, \mathrm{d}x^n + \frac{(\mathrm{d}x^9)^2}{\Delta^2} \right] + \frac{R_{10}^2 R_6^2 (\mathrm{d}y^1)^2}{R_2 \sqrt{d^2 R_{10}^2 + c^2 R_6^2 \Delta^2}},$$

$$B = d \frac{R_{10}}{R_2} \frac{U \wedge \mathrm{d}x^9}{\Delta^2}, \qquad \mathrm{e}^{-\Phi} = \frac{R_2^{3/2} \Delta}{(d^2 R_{10}^2 + c^2 R_6^2 \Delta^2)^{3/4}},$$

$$C_1 = -R_2 \frac{bd\, R_{10}^2 + ac\, R_6^2\, \Delta^2}{d^2 R_{10}^2 + c^2 R_6^2 \Delta^2} \mathrm{d}y^1, \qquad C_3 = -bR_{10} \frac{U \wedge \mathrm{d}x^9 \wedge dy^1}{\Delta^2}$$

An SL(2,Z) of solutions

Expand the DBI action of the D4-brane:

$$S^{\Lambda} = -\frac{1}{g_\Lambda^2}\int \mathrm{d}^4x\Big[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\delta^{\mu\nu}+U^\mu U^\nu\right)\partial_\mu X^8\,\partial_\nu X^8$$
$$+\frac{g_\Lambda^2}{2\Delta g_\Delta^2}\left(\partial_\mu X^9 + d\frac{g_\Omega}{g_\Lambda}F_{\mu\nu}U^\nu - \mathrm{i}\,c\frac{g_A}{g_\Lambda}\star F_{\mu\nu}U^\nu\right)^2 + c^2\frac{g_A^2}{2\Delta g_\Delta^2}\left(U^\mu\,\partial_\mu X^9\right)^2\Big]$$
$$+\frac{\mathrm{i}}{4}\mathrm{Re}[\tau]\int \mathrm{d}^4x\, F^{\mu\nu}\star F_{\mu\nu}$$

$$g_\Omega^2 = \frac{R_{10}}{R_6}\,,\quad g_A^2 = \frac{R_6}{R_{10}} = \frac{1}{g_\Omega^2}\,,\quad g_\Lambda^2 = d^2 g_\Omega^2 + c^2 g_A^2\,,\quad g_\Delta^2 = \frac{d^2 g_\Omega^2}{\Delta} + c^2 g_A^2\Delta$$

$$\tau = \frac{a(\mathrm{i}\,/g_\Omega^2)+b}{c(\mathrm{i}\,/g_\Omega^2)+d}$$

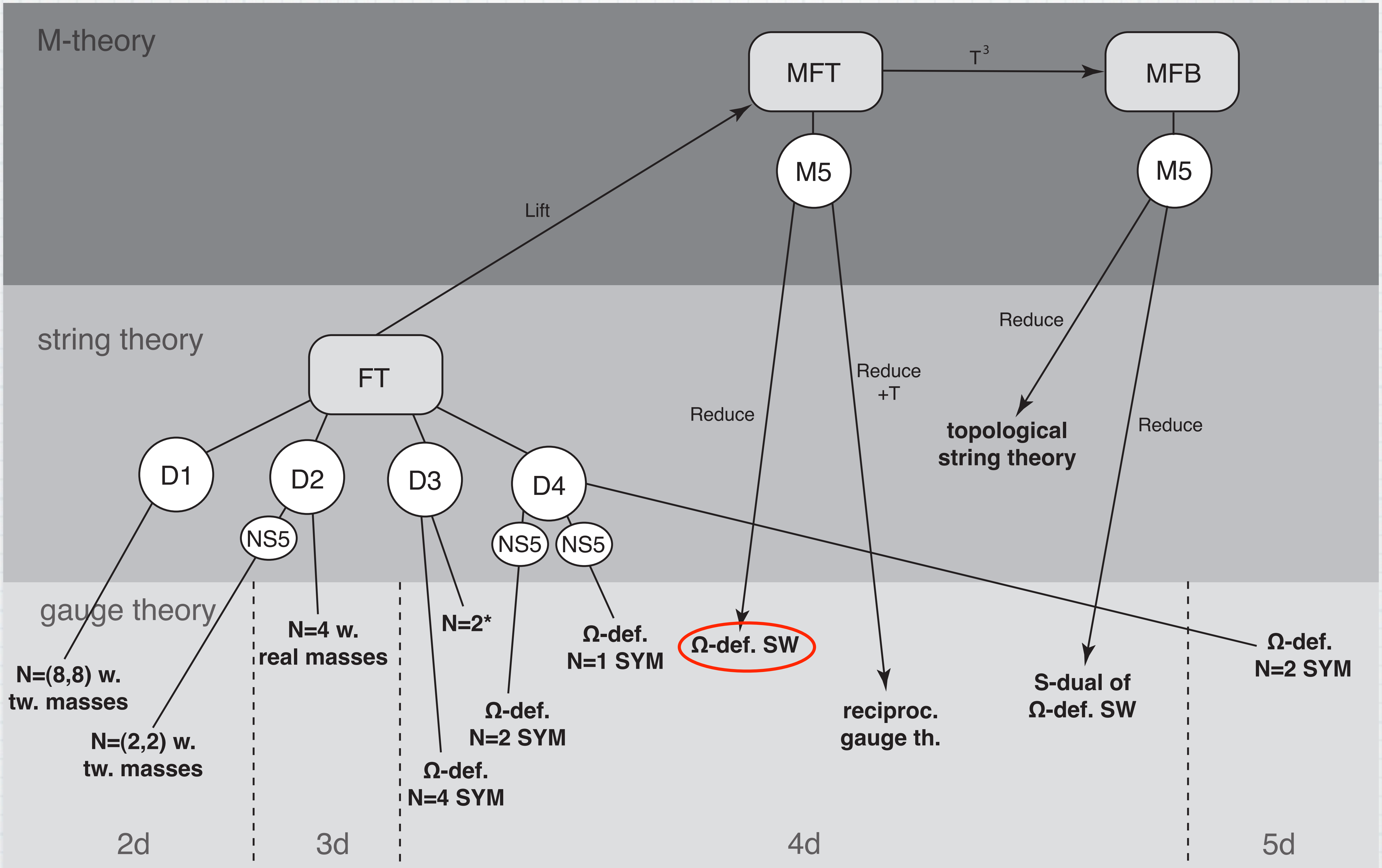
The identity element of SL(2,Z) corresponds to the Omega-deformation: $g_\Lambda^2 = g_\Omega^2,\ g_\Delta^2 = g_\Omega^2/\Delta$

The S-element, $S = \left(\begin{smallmatrix}0 & -1\\ 1 & 0\end{smallmatrix}\right)$, leads to the Alpha-deformation.

$$g_\Lambda^2 = g_A^2 = 1/g_\Omega^2,\ g_\Delta^2 = \Delta g_A^2 = \Delta/g_\Omega^2$$

Examples: Omega-deformed SW action

Omega-deformed SW action



Omega-deformed SW

Use M-theory lift of fluxtrap BG.

Witten: D4 between parallel NS5s lifts to single M5 wrapped on Riemann surface.

Embed M5-brane into fluxtrap BG.

- Self-dual three-form on the brane.
- Still wrapped on a Riemann surface at linear order.

Take vector and scalar equations of motion in 6d (not from an action!).

Integrate equations over Riemann surface.

4d equations of motion are Euler-Lagrange equations of an action.

This action reduces to the Seiberg-Witten action in the undeformed case.

Captures all orders of the 4D gauge theory.

Omega-deformed SW

selfdual

6D e.o.m:

$$(\hat{g}^{mn} - 16 h^{mpq} h^{n}{}_{pq}) \nabla_m \nabla_n X^I = -\frac{2}{3} \hat{G}^I{}_{mnp} h^{mnp} ,$$

$$\mathrm{d}h_3 = -\tfrac{1}{4} \hat{G}_4 ,$$

Howe, Sezgin, West

Integration over the Riemann surface of the e.o.m. results in the 4d e.o.m. for the Omega-deformed SW theory:

Vector equation:

$$(\tau - \bar{\tau}) \left[\partial_\mu F_{\mu\nu} + \tfrac{1}{2} \partial_\mu (a + \bar{a}) \hat{\omega}_{\mu\nu} + \tfrac{1}{2} \partial_\mu (a - \bar{a})^* \hat{\omega}_{\mu\nu} \right] + \partial_\mu (\tau - \bar{\tau}) \left[F_{\mu\nu} + \tfrac{1}{2} (a - \bar{a})\, {}^*\hat{\omega}_{\mu\nu} \right] - \partial_\mu (\tau + \bar{\tau}) \left[{}^*F_{\mu\nu} + \tfrac{1}{2} (a - \bar{a})\, \hat{\omega}_{\mu\nu} \right] = 0$$

Scalar equations:

$$(\tau - \bar{\tau})\, \partial_\mu \partial_\mu a + \partial_\mu a\, \partial_\mu \tau + 2 \frac{\mathrm{d}\bar{\tau}}{\mathrm{d}\bar{a}} \left(F_{\mu\nu} F_{\mu\nu} + F_{\mu\nu} {}^*F_{\mu\nu} \right) + 4 \frac{\mathrm{d}\bar{\tau}}{\mathrm{d}\bar{a}} (a - \bar{a})\, \hat{\omega}^+_{\mu\nu} F_{\mu\nu} - 4 (\tau - \bar{\tau})\, \hat{\omega}^-_{\mu\nu} F_{\mu\nu} = 0 ,$$

$$(\tau - \bar{\tau})\, \partial_\mu \partial_\mu \bar{a} - \partial_\mu \bar{a}\, \partial_\mu \bar{\tau} - 2 \frac{\mathrm{d}\tau}{\mathrm{d}a} \left(F_{\mu\nu} F_{\mu\nu} - F_{\mu\nu} {}^*F_{\mu\nu} \right) + 4 \frac{\mathrm{d}\tau}{\mathrm{d}a} (a - \bar{a})\, \hat{\omega}^-_{\mu\nu} F_{\mu\nu} - 4 (\tau - \bar{\tau})\, \hat{\omega}^+_{\mu\nu} F_{\mu\nu} = 0 .$$

Omega-deformed SW

The vector and scalar e.o.m. are the Euler-Lagrange equations of the following Lagrangian:

generalized covariant derivative for the scalar a, non minimal coupling to the gauge field.

$$\mathrm{i}\,\mathscr{L} = -\left(\tau_{ij} - \bar{\tau}_{ij}\right)\Big[\tfrac{1}{2}\left(\partial_\mu a^i + 2\left(\tfrac{\bar{\tau}}{\tau - \bar{\tau}}\right)_{ik} {}^*F^k_{\mu\nu}\, {}^*\hat{U}_\nu\right)\left(\partial_\mu \bar{a}^j - 2\left(\tfrac{\tau}{\tau - \bar{\tau}}\right)_{jl} {}^*F^l_{\mu\nu}\, {}^*\hat{U}_\nu\right)$$

$$+ \left(F^i_{\mu\nu} + \tfrac{1}{2}\left(a^i - \bar{a}^i\right) {}^*\hat{\omega}_{\mu\nu}\right)\left(F^j_{\mu\nu} + \tfrac{1}{2}\left(a^j - \bar{a}^j\right) {}^*\hat{\omega}_{\mu\nu}\right)\Big]$$

$$+ \left(\tau_{ij} + \bar{\tau}_{ij}\right)\left(F^i_{\mu\nu} + \tfrac{1}{2}\left(a^i - \bar{a}^i\right) {}^*\hat{\omega}_{\mu\nu}\right)\left({}^*F^j_{\mu\nu} + \tfrac{1}{2}\left(a^j - \bar{a}^j\right)\hat{\omega}_{\mu\nu}\right)$$

shift in the gauge field strength

$$\omega = \mathrm{d}U$$

For $\epsilon = 0$, this reproduces the Seiberg-Witten Lagrangian.

Independent of compactification radius to IIA, which is related to gauge coupling in 4d → quantum result (all orders).

Summary

Summary

The fluxtrap construction allows us to study different gauge theories of interest via string theoretic methods.

Omega deformation and (twisted) mass deformations have same origin in string theory.

The construction gives a geometrical interpretation for the Omega BG and its properties, such as localization etc.

The S-dual of the Omega-deformation (Alpha-deformation) has RR-background fields.

The two cases (Alpha, Omega) are two points in a whole SL(2,Z) class of deformed gauge theories.

Open questions:

- string-theoretical realization of the AGT correspondence
- Topological string theory from the fluxtrap BG
- construct gravity duals to deformed gauge theories

Thank you for your attention!