Generalized Global Symmetries

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Ordinary global symmetries

- Generated by operators associated with co-dimension one manifolds M
$$U_g(M)$$
$g \in G$ a group element
- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1 g_2}(M)$
- Local operators $O(p)$ are in representations of G
$$U_g(M)O_i(p) = R_i^j(g)O_j(p)$$
where M surrounds p (Ward identity)
- If the symmetry is continuous,
$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

q-form global symmetries

- Generated by operators associated with co-dimension $q+1$ manifolds M (ordinary global symmetry has $q=0$)
$$U_g(M)$$
$g \in G$ a group element
- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_1 g_2}(M)$. Because of the high co-dimension the order does not matter and G is Abelian.
- The charged operators $\mathrm{V}(L)$ are on dimension q manifolds L. Representations of G – Ward identity
$$U_g(M)V(L) = R(g)V(L)$$
where M surrounds L and $R(g)$ is a phase.

q-form global symmetries

If the symmetry is continuous,

$$U_g(M) = e^{i \int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

Compactifying on a circle, a q-form symmetry leads to a q-form symmetry and a $q-1$-form symmetry in the lower dimensional theory.

- For example, compactifying a one-form symmetry leads to an ordinary symmetry in the lower dimensional theory.

No need for Lagrangian

- Exists abstractly, also in theories without a Lagrangian
- Useful in dualities

q-form global symmetries

- Charged operators are extended (lines, surfaces)
- Charged objects are extended – branes (strings, domain walls)
  - In SUSY BPS bound when the symmetry is continuous
- Selection rules on amplitudes
- Couple to a background classical gauge field (twisted boundary conditions)
- Gauge the symmetry (sum over these background fields)
- The symmetry could be spontaneously broken
- There can be anomalies and anomaly inflow on walls

Example 1: $4d$ $U(1)$ gauge theory

Two global $U(1)$ one-form symmetries:

- Electric symmetry
  - Closed form currents: $\frac{2}{g^2} * F$ (measures the electric flux)
  - Shifts the gauge field A by a flat connection
- Magnetic symmetry
  - Closed form currents: $\frac{1}{2\pi} F$ (measures the magnetic flux)
  - Shifts the magnetic gauge field by a flat connection.

Example 1: $4d$ $U(1)$ gauge theory

The symmetries are generated by surface operators

$$U_{g_E=e^{i\alpha},\, g_M=e^{i\eta}}(M) = e^{\frac{i\,\eta}{2\pi}\int F + \frac{2\,i\,\alpha}{g^2}\int *F}$$

- These are Gukov-Witten surface operators (rescaled α, η).
- They measure the electric and the magnetic flux through the surface M.

The charged objects are dyonic lines

$$W_n(L) H_m(L)$$

($W_n(L)$ are Wilson lines and $H_m(L)$ are 't Hooft lines)

with global symmetry charges n and m under the two global $U(1)$ one-form symmetries.

Example 2: $4d$ $SU(N)$ gauge theory

- Electric $\boldsymbol{Z}_N$ one-form symmetry
  - The Gukov-Witten operator is associated with a conjugacy class in $SU(N)$. When this class is in the center of $SU(N)$ the surface operator is topological.
  - It shifts the gauge field by a flat $\boldsymbol{Z}_N$ connection.
  - It acts on the Wilson lines according to their representation under the $\boldsymbol{Z}_N \in SU(N)$ center.
- No magnetic one-form symmetry.
  - In this theory there are no ‘t Hooft lines – they are not genuine line operators – they need a surface.
  - An open surface operator, whose boundary is an ‘t Hooft line.

Example 3: $4d\ SU(N)$ gauge theory with matter in $\boldsymbol{N}$

The presence of the charged matter explicitly breaks the electric one-form $\boldsymbol{Z}_N$ symmetry.

Hence, there is no global one-form symmetry.

Significance of these symmetries

- Consequence: selection rules, e.g. in compact space the vev of a charged line wrapping a nontrivial cycle vanishes [Witten].
- Dual theories must have the same global symmetries. (They often have different gauge symmetries.)
  - The one-form symmetries are typically electric on one side of the duality and magnetic on the other.
  - $4d\ \ N=1$ SUSY dualities respects the global symmetries.
  - The $SL(2,\boldsymbol{Z})$ orbit of a given $N=4$ theory must have the same global symmetry.

Significance of these symmetries

- Twisted sectors by coupling to background gauge fields
  - An $SU(N)$ gauge theory without matter can have twisted boundary conditions – an $SU(N)/\mathbf{Z}_N$ bundle, which is not an $SU(N)$ bundle – 't Hooft twisted boundary conditions.
- Gauging the symmetry by summing over twisted sectors – like orbifolds.
  - Discrete θ-parameters are analogs of discrete torsion.
- Can characterize phases of gauge theories by whether the global symmetry is broken or not…

Characterizing phases

- In a confining phase the electric one-form symmetry is unbroken.
  - The confining strings are charged and are classified by the unbroken symmetry.
- In a Higgs or Coulomb phase the electric one-form symmetry is broken.
  - Renormalizing the perimeter law to zero, the large size limit of $\langle W \rangle$ is nonzero – vev "breaks the symmetry."
  - It is unbroken in "Coulomb" phase in $3d$ and $2d$.

Example 1: $4d$ $U(1)$ gauge theory

- There are two global $U(1)$ one-form symmetries.
- Both are spontaneously broken:
  - The photon being their Goldstone boson

$$\langle 0|F_{\mu\nu}|\epsilon, p\rangle = (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)e^{i\,px}$$

  - Placing the theory on $\boldsymbol{R}^3 \times \boldsymbol{S}^1$, each one-form global symmetry leads to an ordinary global symmetry and a one-form symmetry.
  - These ordinary symmetries are manifestly spontaneously broken – the moduli space of vacua is $\boldsymbol{T}^2$ parameterized by A_4 and the $3d$ dual photon.

Example 2: $4d$ $SU(N)$ gauge theory

In the standard confining phase the electric $\boldsymbol{Z}_N$ one-form symmetry is unbroken.

- Charged strings
- Area law in Wilson loops
- When compactified on a circle an ordinary ($q = 0$) $\boldsymbol{Z}_N$, which is unbroken [Polyakov]

If no confinement, the global $\boldsymbol{Z}_N$ symmetry is broken

- No charged strings
- Perimeter law in Wilson loops
- When compactified on a circle an ordinary ($q = 0$) $\boldsymbol{Z}_N$, which is broken [Polyakov]

Example 2: $4d$ $SU(N)$ gauge theory

Can also have a phase with confinement index t, where the global one-form symmetry is spontaneously broken $\boldsymbol{Z}_N \to \boldsymbol{Z}_t$.

- W has area law but W^t has a perimeter law [Cachazo, NS, Witten].
- In this case there is a $\boldsymbol{Z}_{N/t}$ gauge theory at low energies – long range topological order.

Example 3: $4d$ $SU(N)$ gauge theory with matter in $\boldsymbol{N}$

No global one-form symmetry.

Hence we cannot distinguish between Higgs and confinement.

This is usually described as screening the loop [Fradkin, Shenker; Banks, Rabinovici].

From our perspective, due to lack of symmetry.

Conclusions

- Higher form global symmetries are ubiquitous.
- They help classify
  - extended objects (strings, domain walls, etc.)
  - extended operators/defects (lines, surfaces, etc.)
- As global symmetries, they must be the same in dual theories.
- They extend Landau characterization of phases based on order parameters that break global symmetries.
  - Rephrase the Wilson/'t Hooft classification in terms of broken or unbroken one-form global symmetries.
- Anomalies
  - 't Hooft matching conditions
  - Anomaly inflow
  - Degrees of freedom on domain walls

Thank you for your attention