Entropy of asymptotically flat black holes in gauged supergravity

Stefan Vandoren, ITF, Utrecht University

with Nava Gaddam, Alessandra Gnecchi (Utrecht),
Oscar Varela (Harvard) - work in progress.

BPS Black Holes

BPS Black holes in flat space (ala extremal Reissner-Nordström (RN)) satisfy a bound,

$$M = |Q|$$

- They preserve $\frac{1}{2}$-susy in e.g. $N = 2, D = 4$ sugra and have zero temperature.
- They have finite entropy which (for large charges) can be computed using the $(0, 4)$ CFT of Maldacena, Strominger and Witten (1997). [Or, in $D = 5$, Strominger-Vafa (1996)]

Near-extremal black holes

Near-extremal RN black holes have also been embedded in string theory [Callan-Maldacena, Horowitz-Strominger, Klebanov-Tseytlin '96,..., Balasubramanian and Larsen '98,...]

- They have finite but small temperature due to Hawking radiation.
- The entropy can be computed by mapping to BTZ and using the Cardy formula [Strominger '97,...].

These is ok for large black holes in sugra and in the absence of light charged matter: ungauged supergravity. What changes in gauged sugra ?

Gauged supergravity

Questions:

- How to construct asymptotically flat black hole solutions in gauged sugra ?
- What is the influence of light charged particles on the black hole ?
- Microstate counting ?

Note:

- Gauged sugra can have AdS_4 vacua. But black holes in AdS_4 is a different story.
- Gauged sugra can have Minkowski vacua, but typically with spontaneously broken supersymmetry. This will be our story.

$\mathbb{R}^{1,3} \times S^1 \times CY_3$ supergravity

Compactification of D=11 sugra on CY_3 yields $N = 2$ $D = 5$ sugra with [Cadavid, Ceresole, D'Auria, Ferrara '95]

- Poincare multiplet
- $h^{1,1} - 1$ vector multiplets
- $h^{2,1} + 1$ hypermultiplets

Further compactification on S^1_R gives at low energies $N = 2$ $D = 4$ sugra with

- Poincare multiplet
- $h^{1,1}$ vector multiplets
- $h^{2,1} + 1$ hypermultiplets (massless and neutral)

Consistency requires $R^6 >> Vol(CY_3)$.

BPS Black Holes and MSW

The M5 brane can be wrapped over a four-cycle to give a BPS string in $D = 5$, with near horizon geometry

$$AdS_3 \times S^2 \times CY_3$$

Upon reducing over S^1 to $D = 4$ it gives a BPS black hole with near horizon geometry

$$AdS_2 \times_f S^1 \times S^2 \times CY_3$$

The entropy of this black hole was computed by Maldacena, Strominger Witten ('97) using AdS_3/CFT_2 and the Cardy formula for the $(0, 4)$ dual CFT.

Non-extremal black holes and BTZ - I

Consider the $STU = T^3$ model ($F = \frac{X_1^3}{X_0}$; $h_{1,1} = 1$). Non-extremal black hole solutions [..., Galli, Ortin, Perz and Shahbazi (2011)]:

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{-2U(r)}f(r)d\Omega^2_{(2)} \, ,$$

with $f(r) = (r - r_+)(r - r_-)$. Warp factor

$$e^{-2U(r)}f(r) = \left[r - r_* + \sqrt{r_0^2 + \frac{2q_0^2}{R_\infty^3}} \right]^{\frac{1}{2}} \left[r - r_* + \sqrt{r_0^2 + 2p_1^2 R_\infty} \right]^{\frac{3}{2}} \, .$$

Inner and outer horizon are $r_\pm = r_* \pm r_0$ with r_0 the nonextremality parameter and $r_*^2 = 2\sqrt{q_0 p_1^3}$ is the horizon radius of the extremal solution. For $R_\infty = \sqrt{\frac{q_0}{p_1}}$ the dilaton is constant and the solution becomes Reissner-Nordström.

Non-extremal black holes and BTZ - II

In fact, in the near-horizon limit of all near-extremal black holes, they all become of the RN type,

$$ds^2_{(4)} = -\left(1 - \frac{2M}{\tilde{r}} + \frac{r_*^2}{\tilde{r}^2}\right) dt^2 + \frac{d\tilde{r}^2}{\left(1 - \frac{2M}{\tilde{r}} + \frac{r_*^2}{\tilde{r}^2}\right)} + \tilde{r}^2 d\Omega^2_{(2)} \ ,$$

with $\tilde{r} = r - r_* + M$ and the mass given by

$$M = \sqrt{r_0^2 + r_*^2} = \sqrt{\frac{1}{2}(r_+^2 + r_-^2)} \ .$$

Near-horizon, near-extremal limit $\epsilon << 1$

$$\tilde{r} = r_* + \epsilon\rho \ , \qquad r_0 = \epsilon\rho_0 \ , \qquad M = r_* + \epsilon^2 \frac{\rho_0^2}{2r_*} \ , \qquad t = \frac{1}{\epsilon} r_*^2 \tau \ ,$$

Non-extremal black holes and BTZ - III

We can uplift to $D = 5$ and take the near-horizon, near-extremal limit [..., Balasubramanian and Larsen '98,...]. Then we get a geometry $(BTZ \times S^2)_\ell$ with $\ell = 2p$ and

$$M^2 = \frac{\ell^2}{2} M_{BTZ} \ , \qquad r_*^2 = \frac{\ell}{4} \left(J_{BTZ} + \ell M_{BTZ} \right) \ ,$$

2D CFT identification, $\ell M_{BTZ} = L_0 + \bar{L}_0$; $J_{BTZ} = L_0 - \bar{L}_0$, so

$$\begin{aligned} n_L &= \frac{1}{2} \left(\ell M_{BTZ} + J_{BTZ} \right) \ , \\ n_R &= \frac{1}{2} \left(\ell M_{BTZ} - J_{BTZ} \right) \ . \end{aligned}$$

BTZ entropy can now be rewritten as Cardy formula [Strominger '97; Brown-Henneaux: $c = \frac{3\ell}{2G_3}$]

$$S = 2\pi \left(\sqrt{\frac{c_L}{6} n_L} + \sqrt{\frac{c_R}{6} n_R} \right) \ .$$

A new twist: Scherk-Schwarz and gauged supergravity

This is all fine and well understood for over 15 years. So let us add an new interesting twist...

- Scherk-Schwarz twist along the S^1 when going from $D = 5$ to $D = 4$.
- It yields gauged $N = 2$ supergravity with $V \geq 0$ (tree level) and a Minkowski vacuum with broken supersymmetry (super-Higgs).
- some of the particles become massive and charged. (At tree level, $m = q$.)
- The susy-breaking scale can be taken very small, so m is small as well.

Scherk-Schwarz Baby version

Consider a massless complex scalar field with $U(1)$ symmetry on $\mathbb{R}^{1,3} \times S^1$ coupled to gravity

$$\begin{aligned} L &= -\partial_{\hat{\mu}}\phi\partial^{\hat{\mu}}\bar{\phi} \\ &= -\left(\partial_\mu\phi\partial^\mu\bar{\phi} + g^{\mu z}\partial_\mu\phi\partial_z\bar{\phi} + g^{z\mu}\partial_z\phi\partial_\mu\bar{\phi} + g^{zz}\partial_z\phi\partial_z\bar{\phi}\right) \end{aligned}$$

Give twisted boundary condition (Scherk-Schwarz)

$$\phi(x, z + 2\pi R) = e^{2\pi i\alpha}\phi(x, z) \Leftarrow \partial_z\phi = i\frac{\alpha}{R}\phi \ .$$

Resulting Lagrangian has KK-charged field and positive definite potential

$$L = -|D_\mu\phi|^2 - V(\phi, \bar{\phi}) \ , \qquad V = \frac{\alpha^2}{R^2}|\phi|^2 \ .$$

Mass and charge

$$m = e = \frac{\alpha}{R} \ .$$

Scherk-Schwarz I

The Scherk-Schwarz twist yields no-scale supergravity in $D = 4$ with $V \geq 0$ [..., Hull, ..., Ferrara et al.; Looyestijn, Plauschinn, SV]. We twist the $U(1) \subset SU(2)_R$ symmetry.
Gravitini and hypers become massive. Vector multiplet scalars and vectors remain massless. We take $h_{1,1} = 1$ and the t^3 model (e.g. quintic CY_3)

$$F(X) = \frac{(X^1)^3}{X^0}$$

The gaugino gets eaten by the gravitino. Freezing the hypers to their VEVs, the resulting bosonic Lagrangian looks like ungauged supergravity for which there are non-extremal black holes known. They are the RN black holes discussed before !!

Scherk-Schwarz potential

The potential is given by a sum of positive terms,

$$\begin{aligned} \frac{V}{\alpha^2} &= \frac{1}{R^3}\mathcal{N}^r\bar{\mathcal{N}}^s\ G_{r\bar{s}} + \frac{1}{4R^3\mathcal{V}^2}\left[\xi^T N^T \xi\right]^2 \\ &\quad \frac{1}{2R^3\mathcal{V}}\xi^T\ N^T\ \mathcal{M}\ (\mathrm{Im}\,\mathcal{M})^{-1}\,\mathcal{M}^T\ N\xi \\ &\quad \frac{1}{4R^6}\ \left(M^A{}_C\phi^C\right)\ \left(M^B{}_D\phi^D\right)\ \mathcal{K}_{\mathcal{AB}}\ . \end{aligned}$$

Finding Minkowski vacua is solving a geometric problem: Finding fixed points of isometries on quaternionic manifolds. Moment map at the fixed point must be non-zero to get non-zero masses. Radius R and Calabi-Yau volume $\mathcal{V}$ are flat directions.

Scherk-Schwarz II

Expanding around the vacuum, we get massive and charged hypermultiplets and gravitini, with $m = q$ and

$$q = n\, q_{1/2}\ , \quad n \in \mathbb{Z}\ , \qquad q_{1/2} = \frac{\alpha}{R}\ , \qquad \alpha << 1\ .$$

Actually, $n \in \{1, 3, 4, 8, 12\}$ for the model at hand with $h_{1,2} = 1$ and hypers span $G_2/SO(4)$.
The set-up we have now is a RN black hole with the same entropy (classically), but with a different quantization condition on the BH charge (due to absorption or emission of light charged particles),

$$q_{BH} = q_0 + n\, q_{1/2} = \frac{m + \alpha\, n}{R}\ ,$$

where q_0 was the Kaluza-Klein charge, $q_0 = \frac{m}{R}$ with $m \in \mathbb{Z}$. The magnetic charge of the BH stays fixed at p_1.

Black holes and Scherk-Schwarz - Micro

- Wrapping the M5 brane over a four-cycle still gives a BPS black string in $D = 5$, where susy is unbroken. Hence there is a still a dual CFT, the (0,4) MSW.
- Wrap the string around the circle with twisted boundary conditions still gives a black hole, but susy is broken by the boundary conditions.
- Compute the thermal partition function of the MSW CFT with twisted boundary conditions. We twist with respect to $U(1) \subset SU(2)_R$ inside right moving $N = 4$ sector. Worldsheet fermions are R-charged.

MSW and Scherk-Schwarz

Microscopically, we must compute the thermal partition function of MSW on the torus with twisted boundary condition on the right moving fermions (R-charged),

$$\psi(\sigma^0, \sigma^1 + 2\pi\tau_1) = e^{+2\pi i\alpha}\psi(\sigma^0, \sigma^1) \quad \text{and}$$
$$\psi(\sigma^0 + 2\pi\tau_2, \sigma^1) = -\psi(\sigma^0, \sigma^1),$$

$\alpha = 1/2$ is antiperiodic, and $\alpha = 0$ is periodic. Shift in mode numbers explains shift in quantization condition.
Sample calculation (free field approximation + preliminary !)

$$Z_R(\tau) = \left(\frac{1}{\sqrt{-i\tau}\eta(\tau)}\right)^{\frac{2c_R}{3}} \left(e^{-2\pi i(\alpha+\frac{1}{2})\alpha} \left(\frac{\vartheta\begin{bmatrix}\alpha+\frac{1}{2}\\ \alpha\end{bmatrix}}{\eta(\tau)}\right)\right)^{\frac{c_R}{3}}.$$

There is only one polar term in the expansion of the partition function, that goes like ($\alpha < \frac{1}{2}$!)

$$q^{\frac{c_R \alpha^2}{6} - \frac{c_R}{24}} , \qquad q \equiv e^{2\pi i \tau}$$

From this one can compute the asymptotic growth of states and hence the entropy:

$$\begin{aligned} S_{CFT} &= S_L + S_R \\ &= 2\pi \left[\sqrt{\frac{c_L n_L}{6}} + \sqrt{\frac{c_R n_R}{6}} \left(1 - 4\alpha^2\right)^{\frac{1}{2}} \right] \end{aligned}$$

For $\alpha = 0$, standard result. Additional correction in $\alpha < \frac{1}{2}$.
Remember: twist parameter is equal to coupling constant in sutra.
Prediction for a macroscopic correction to the entropy ?!

Conclusions

The main question is:

HOW RELIABLE IS ALL THIS ???

- Study stability of the vacuum. Susy breaking parameter is $m_{1/2} = \frac{\alpha}{R}$ and α is continuous and taken very small.
- Instabilities and Schwinger radiation. Threshold: BF-bound in AdS_2:

$$(m^2 - q^2)Q^2 + \frac{1}{4} < 0$$

- Possible emergency plan: Scherk-Schwarz twist that only partially breaks susy. This is possible $N = 4 \rightarrow N = 2$.
- Repeat story for Strominger-Vafa setup (D1-D5).
- Generalizations to AdS vacua/BH with no susy??